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ABSTRACT

Through a research grant funded by the Vocational Division of the Indiana State Department of Public Instruction, a developmental research project was undertaken to develop machine trades-related mathematics materials using the terminology, concepts, and methods of modern mathematics. The two volume set is designed to be utilized by first and second year machine tool technology students. Included in the document are technical information lead-in sheets, machine trades technical information sheets, technical assignment sheets, sample technical operation sheets, and sample technical job sheets. The technical information lead-in sheets present, in simple and direct manner, important terminology, concepts, and methods utilized in modern mathematics. The units may be used for both practice and reference; practice problems with answers are divided with each lead-in sheet. Each of the machine trades technical information sheets presents specific machine tool technology, technical information utilizing the modern mathematics approach, and terminology. As much as possible these units emphasize understanding of the concepts and formulas involved. Technical assignment sheets including assigned problems and answers have been included to provide the student with practice. Appended is a partial listing of books that might be utilized for additional study in the machine trades and in modern mathematics. (Author/BP)

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Modern Mathematics
As Applied To
MACHINE TRADES

U S DEPARTMENT OF HEALTH,
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Volume I

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To the Reader

The materials in this volume were developed under a research grant awarded to Mid-Central Area Vocational School, Elwood, Indiana.

The project was supported by the following personnel in the Indiana State Department of Public Instruction: Mr. Harold Negley, State Superintendent; Mr. Robert E. Howard, Associate Superintendent for Vocational Affairs; Mr. Don Gentry, Executive Officer and State Director, State Board for Vocational-Technical Education; Mr. Monte Janik, Chief Consultant, Industrial Education; and Ms. Carol Hodgson, Coordinator, Research and Exemplary Projects.

PREFACE

Mathematics is one of the areas in education in which a great revolution has taken place. The "new" or "modern" approach to mathematics emphasizes understanding rather than just a series of manipulative techniques with little or no reference to basic properties, laws, and definitions.

A major purpose of the new or modern mathematics is to present mathematics as a consistent, logical, and step by step development. Then, based on this strong foundation the student is helped to develop a firm understanding of mathematics. Analysis of a problem should play a central role. However, it should be emphasized that drill is still an important aspect.

The modern approach to mathematics has developed a new way of teaching mathematics that should be carried on in the vocational related mathematics areas. If vocational machine trades education is to maintain its respectability and to progress in providing real vocational industrial education, it must update its related mathematics. This should involve stimulating the student "to react," "to do," "to discover," and "to explore." The student must have an opportunity to do more than merely follow like a sheep an instructor's command to "listen, watch, and then do as I do."

Through a research grant approved and funded by the Vocational Division of the Indiana State Department of Public Instruction, a developmental research project was undertaken to develop machine trades related mathematics materials using the terminology, concepts, and methods of the modern mathematics.

Volume 1 and Volume 2 of Modern Mathematics as Applied to the Machine Trades were developed through this research project. Included in each of these two volumes are technical information lead in sheets, machine trades technical information sheets, technical assignment sheets, sample technical operation sheets, and sample technical job sheets.

The technical information lead in sheets present in a simple and direct manner important terminology, concepts, and methods utilized in the modern mathematics. So that these units may be used for both practice and reference, practice problems with answers are provided with each technical information lead in sheet.

Each of the machine trades technical information sheets presents specific machine tool technology technical information utilizing the modern mathematics approach and terminology. As much as possible these units emphasize understanding of the concepts and formulas involved. Technical assignment sheets including assigned problems and answers have been included to provide the student with valuable practice.

Volume 1 is designed to be utilized by first year machine tool technology students, and Volume 2 is designed for students in the second year. Each volume was written with the student in mind. That is, ease of reading and understanding was a primary objective.

The two volumes are not designed to be a complete course of study for the machine trades area. However, sample operation sheets and sample job sheets have been included to illustrate the utilization of the technical information sheets in specific operations and jobs.

Since the terminology, concepts, and methods of modern mathematics have been emphasized throughout the two volumes, it is strongly advised that all teachers who plan to utilize the volumes participate either in in-service workshops or take at least one class emphasizing the modern mathematics. As educators we all realize that the success of any course or program depends heavily on the teacher's understanding and enthusiasm.

The writers of Modern Mathematics as Applied to the Machine Trades sincerely believe that these two volumes are a definite advancement and achievement in the area of machine tool related mathematics.

Guy J. Hale
Lester W. Hale
Daniel Rayshich

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MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Algebra of Sets

INTRODUCTION AND/OR OBJECTIVES:

Algebra of sets is considered to be the unifying concept of the modern mathematics. In this section, sets and subsets will be discussed. Also, the concepts of unions and intersections of sets will be explained. At the elementary level, sets are used to help develop an understanding of basic arithmetic operations.

TECHNICAL INFORMATION:

I. SETS

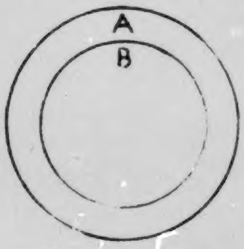
A set is simply a collection of objects. For example, set A may consist of the numbers 1, 2, 3, 4, and 5. Then, $A = \{1, 2, 3, 4, 5\}$. The numbers 1, 2, 3, 4, and 5 are called the elements of set A. The symbol $\in$ denotes "element of." Therefore, we may write $1 \in A$, $2 \in A$, $3 \in A$, $4 \in A$, and $5 \in A$. If $B = \{a, b, c, d\}$, then $a \in B$, $b \in B$, $c \in B$, and $d \in B$.

A set A is equal to a set B if they contain exactly the same elements.

Example 1. If $A = \{1, 2, 3, 4\}$, and $B = \{1, 2, 3, 4\}$, then $A = B$.

II. SUBSETS

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5\}$. Then set B is called a subset of set A. This is denoted as follows: $B \subseteq A$. Notice that every element of set B is an element of set A. B is defined to be a subset of A if every element of B is an element of A. $B \subseteq A$ is read as "B is a subset of A." In Figure 1 B is a subset of A.

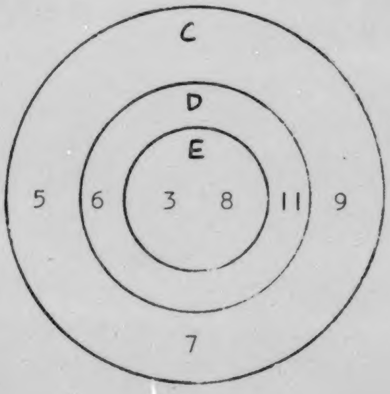


$B \subseteq A$

Figure 1

It is often helpful to use what are called Venn diagrams to provide a visual interpretation of set operations. These were used in Figure 1.

Example 2. If $C = \{3, 5, 6, 7, 8, 9, 11\}$, $D = \{3, 6, 8, 11\}$, and $E = \{3, 8\}$, then $D \subseteq C$, $E \subseteq C$, and $E \subseteq D$. See Figure 2.



$D \subseteq C$

and

$E \subseteq C$

and

$E \subseteq D$

Figure 2

Notice that if $A = \{1, 2, 3, 4\}$, then $A \subseteq A$ by the definition of subset. That is, any set is a subset of itself.

Some textbooks use only the above notation for subsets. Other texts use another notation. As discussed above, $A \subseteq B$ will allow the possibility that $A = B$. The notation, $A \subset B$, (read as "A is a proper subset of B") is used when A is a subset of B, but it is definitely known that A is not equal to B. That is, $A \subset B$ if $A \subseteq B$ and $A \neq B$.

Example 3. $A = \{1, 2, 3, 4\}$, and $B = \{1, 2, 3, 4, 5, 6\}$ will imply that $A \subseteq B$. It is also true that $A \subset B$ (since $A \subseteq B$ and $A \neq B$). Therefore, in this example A is a subset of B, and, also, A is a proper subset of B. See Figure 3.

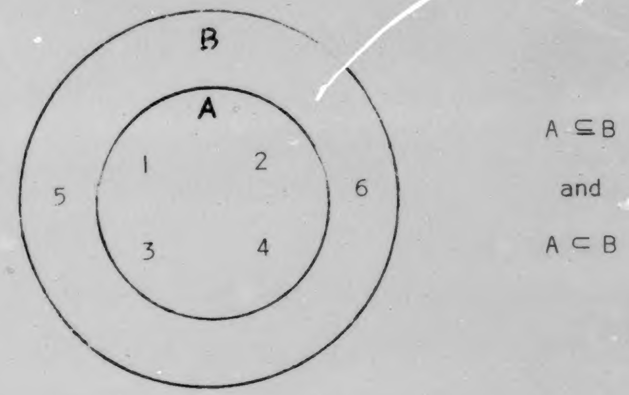


Figure 3

Example 4. If $A = \{-1, 2, 3, 4\}$ and $B = \{-1, 2, 3, 4\}$, then $A \subseteq B$ and $A = B$. Therefore, it is not true that $A \subset B$. See Figure 4.

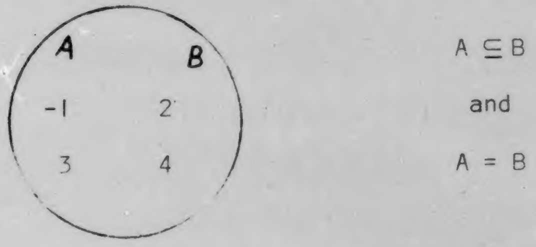


Figure 4

III. UNION OF SETS

If A = {1, 2, 3, 4} and B = {2, 3, 5, 7, 9}, then A U B is defined to be the set {1, 2, 3, 4, 5, 7, 9}. That is, in this example A U B = {1, 2, 3, 4, 5, 7, 9}. A U B is read as "A union B." In the union of two sets, all elements in both sets are listed, but common elements of the two sets are not listed twice.

For any two sets A and B, *A U B is defined to be the set of those elements which are either in A or in B (or in both).*

Example 5. In illustration 1 and in illustration 2 in Figure 5 A U B is represented by the shaded area.

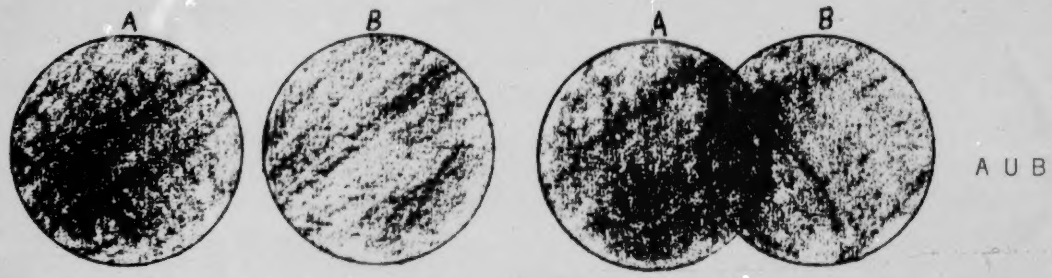


Figure 5

Example 6. If A = {-1, 2, 3, 4} and B = {-3, 0, 2}, then A U B = {-3, -1, 0, 2, 3, 4}. See Figure 6.

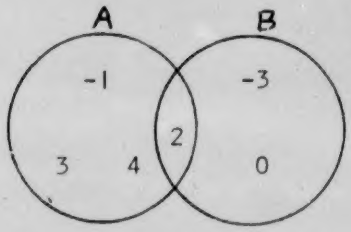
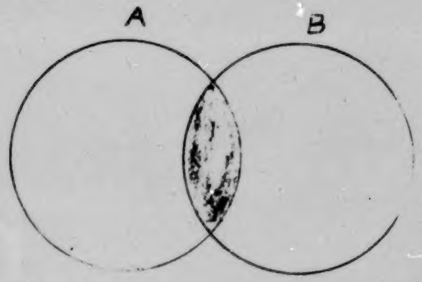


Figure 6

IV. INTERSECTION OF SETS

While the union of two sets lists all of the various elements in either set, the intersection of two sets is the set of elements which are in common to both sets. If $A = \{-1, 0, 1, 2\}$ and $B = \{-1, 1, 2, 4, 5\}$, then $A \cap B = \{-1, 1, 2\}$. $A \cap B$ is read as "A intersection B." The intersection of two sets A and B, written as $A \cap B$, is defined to be the set of elements which are in both sets A and B. If Figure 7, $A \cap B$ is represented by the shaded area.



$A \cap B$

Figure 7

Example 7. If $B = \{-3, -2, 3, 5, 7, 8\}$, and $C = \{-2, 5, 7, 10, 11, 12\}$, then $B \cap C = \{-2, 5, 7\}$. See Figure 8.

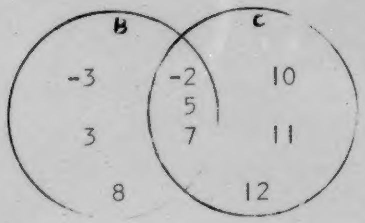


Figure 8

Example 8. If $A = \{-2, -1, 0\}$ and $B = \{2, 3, 4, 5\}$, then $A \cap B$ does not contain any elements. A set containing no elements is called the null set and is denoted by $\emptyset$. Therefore, in this example, $A \cap B = \emptyset$. See Figure 9.

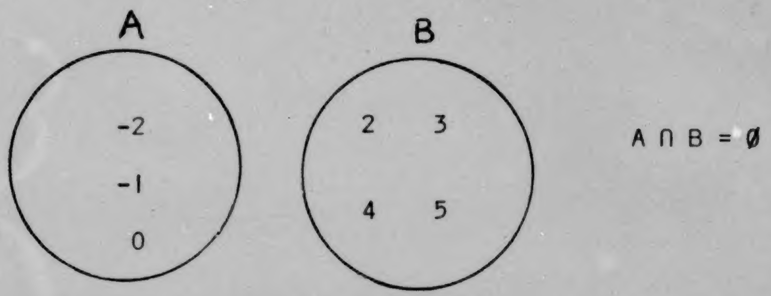


Figure 9

V. FURTHER EXAMPLES

Example 9. If $A = \{3, 4, 7, 8\}$ and $B = \{1, 3, 4, 7, 8, 9\}$, then $A \subseteq B$.

Example 10. If $A = \{-3, 4, 7, 8\}$ and $B = \{2, 3, 4, 7, 8\}$, then $A \cup B = \{-3, 2, 3, 4, 7, 8\}$.

Example 11. If $C = \{4, 5, 9, 10, 12\}$ and $D = \{5, 9, 12\}$, then $D \subseteq C$.

Example 12. If $A = \{-5, 7, 9, 12, 15\}$ and $B = \{-5, 9, 15\}$, then $B \subseteq A$.

Example 13. If $A = \{1, 2, 4, 6, 9\}$ and $B = \{2, 4, 6, 8, 10\}$, then $A \cap B = \{2, 4, 6\}$.

Example 14. If $C = \{-4, 5, 6, 9, 13, 16, 18\}$ and $D = \{6, 9, 16, 18\}$, then $C \cap D = \{6, 9, 16, 18\}$.

Example 15. If $B = \{2, 4, 6, 8\}$ and $C = \{1, 3, 5, 7, 9\}$, then $B \cap C = \emptyset$.

Example 16. If $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

SUMMARY

Symbol	Read As	Definition
$A \subseteq B$	A is a subset of B	If every element of A is an element of B
$A \subset B$	A is a proper subset of B	If $A \subseteq B$ and $A \neq B$
$A \cup B$	A union B	The set of elements either in set A or in set B (or in both)
$A \cap B$	A intersection B	The set of elements common to both sets A and B
$\emptyset$	The null set	The set containing no elements

EXERCISES

1. If $A = \{1, 3, 4, 5\}$ and $B = \{3, 4, 5\}$, is it true that $B \subseteq A$? Is it true that $B \subset A$?

2. If $A = \{-1, 0, 2, 3\}$ and $B = \{-2, -1, 0, 2\}$, is it true that $B \subseteq A$?

3. If $A = \{-1, 2, 3, 4, 5, 7\}$ and $B = \{-1, 0, 2, 4\}$, find $A \cup B$.

4. If $A = \{1, 2, 5, 8, 9\}$ and $B = \{1, 5, 9\}$, is it true that $A \subseteq B$? Find $A \cup B$.

5. If $A = \{-10, -7, -5, 0\}$ and $B = \{-7, -5, 0, 1, 2\}$, find $A \cap B$.

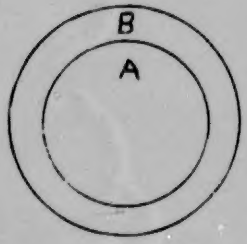
6. If $B = \{3, 5, 7, 9\}$ and $C = \{2, 4, 6, 8\}$, find $A \cup B$ and $A \cap B$.

7. If $A = \{1, 2, 3, 4\}$ and $C = \{1, 2, 3, 4\}$, is it true that $A \subseteq C$? Is it true that $A = C$? Is it true that $A \subset C$?

8. If $A = \{1, 2, 3\}$ and $B = \{2, 4, 8\}$, find $A \cup B$. Find $A \cap B$.

9. If $A = \{-2, 3, 5, 6, 7\}$ and $B = \{-3, -2, 3, 5\}$, find $A \cap B$.

10. If $A \subseteq B$, what is $A \cup B$? What is $A \cap B$?



ANSWERS

1. Yes. Yes.
2. No. (Since not all elements of B are elements of A)
3. $A \cup B = \{-1, 0, 2, 3, 4, 5, 7\}$.
4. No. $A \cup B = \{1, 2, 5, 8, 9\}$.
5. $A \cap B = \{-7, -5, 0\}$.
6. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$. $A \cap B = \emptyset$.
7. Yes. (Since every element of A is an element of C) Yes. No.
8. $A \cup B = \{1, 2, 3, 4, 8\}$. $A \cap B = \{2\}$.
9. $A \cap B = \{-2, 3, 5\}$.
10. $A \cup B = B$. $A \cap B = A$.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead-In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition, Subtraction, Multiplication and Division

INTRODUCTION AND/OR OBJECTIVES:

Of course, the operations of addition, subtraction, multiplication, and division are basic to all mathematical problems. The student should thoroughly understand the relationship between addition and subtraction and the relationship between multiplication and division.

TECHNICAL INFORMATION:

I: ADDITION AND SUBTRACTION

To gain a better understanding of the basic operations of addition and subtraction, we will look at the number line. Suppose we wish to find the value for 3 + 2.

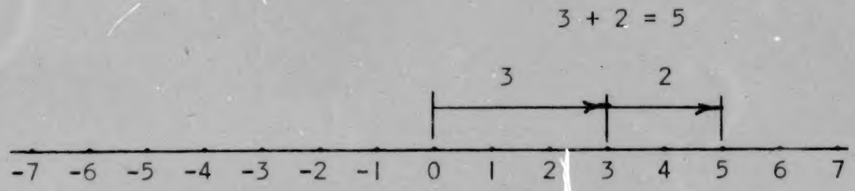


Figure 1

Study Figure 1 and you will see that 3 + 2 = 5. Note that the positive direction is to the right.

Now, suppose we wish to find the value of 5 + (-4). See Figure 2.

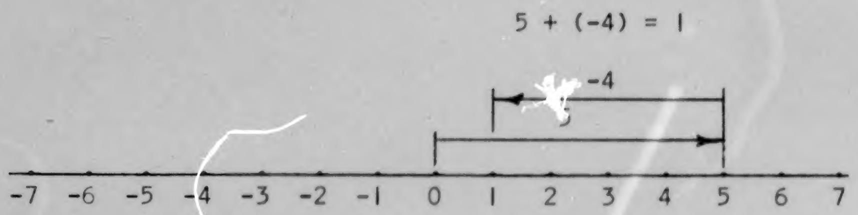


Figure 2

Since the positive direction is to the right and the negative direction is to the left, then 5 + (-4) = 1.

We now make the following definition for subtraction:

a - b = a + (-b)

Example 1. Find the value for 7 - 3.

7 - 3 = 7 + (-3)

= 4

Example 2. Find the value for 2 - 6.

2 - 6 = 2 + (-6)

= -4

Another relationship between addition and subtraction may be noted.

Suppose we wish to find the value of 100 - 98. Many people will solve this problem by noting that we must add 2 to 98 in order to get 100. In other words,

$100 - 98 = \square$ is equivalent to $98 + \square = 100$

or using an x in place of the box,

$100 - 98 = x$ is equivalent to $98 + x = 100$

Of course 2 should be placed in each box and $x = 2$.

We may write the above result in general as follows:

$a - b = \square$ is equivalent to $b + \square = a$

or

$a - b = x$ is equivalent to $b + x = a$

Example 3. Write the equivalent expression for $50 - 3 = \square$, and solve.

$50 - 3 = \square$ is equivalent to $3 + \square = 50$

47 should be placed in each box.

Example 4. Write the equivalent expression for $75 - 71 = x$, and solve.

$75 - 71 = x$ is equivalent to $71 + x = 75$

$x = 4$

II. MULTIPLICATION AND DIVISION

Suppose we wish to find the value of 8/4. The answer is 2 since $4 \cdot 2 = 8$ (4 times 2 equals 8). Here the dot indicates multiplication. In general we have the following definition for division.

$\frac{a}{b} = \square$ is equivalent to $b \cdot \square = a$

or

$\frac{a}{b} = x$ is equivalent to $b \cdot x = a$

Example 5. Write an equivalent expression for $12/3 = \boxed{}$.

$\frac{12}{3} = \boxed{}$ is equivalent to $3 \cdot \boxed{} = 12$

4 should be placed in each box

Example 6. Write an equivalent expression for $20/4 = x$.

$\frac{20}{4} = x$ is equivalent to $4 \cdot x = 20$

$x = 5$

EXERCISES

In each problem write an equivalent expression and solve.

1. $10 - 3$
2. $7 - 9$
3. $12 - 10 = \square$
4. $98 - 95 = \square$
5. $80 - 72 = x$
6. $45 - 39 = x$
7. $\frac{15}{3} = \square$
8. $\frac{12}{6} = \square$
9. $\frac{18}{6} = x$
10. $\frac{24}{3} = x$

ANSWERS

1. $10 + (-3)$
   Answer: 7
2. $7 + (-9)$
   Answer: -2
3. $10 + \square = 12$
   Answer: 2
4. $95 + \square = 98$
   Answer: 3
5. $72 + x = 80$
   Answer: $x = 8$
6. $39 + x = 45$
   Answer $x = 6$
7. $3 \cdot \square = 15$
   Answer: 5
8. $6 \cdot \square = 12$
   Answer: 2
9. $6 \cdot x = 18$
   Answer: 3
10. $3 \cdot x = 24$
    Answer: 8

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Sets of Numbers

INTRODUCTION AND/OR OBJECTIVES:

Exactly what types of numbers are available for working with practical problems? The purpose of this section is to describe the various sets of numbers: the set of counting numbers, the set of whole numbers, the set of integers, the set of rational numbers, and the set of real numbers. It should be noted that as these sets of numbers are developed, each set includes each of the sets of numbers previously discussed. That is, each set of numbers is a subset of each of the sets later discussed. In later technical information sheets, it will be assumed that the set of real numbers is the set of numbers being used.

TECHNICAL INFORMATION:

1. THE SET OF COUNTING NUMBERS

If counting the number of certain objects is all that is desired, the numbers 1, 2, 3, 4, and so on will be sufficient for the purpose. The set of numbers, {1, 2, 3, 4 . . .} , is called the set of counting numbers. This same set is also called the set of natural numbers or the set of positive integers. These numbers continue indefinitely to the right on the number line. See Figure 1.

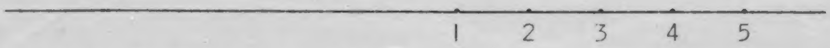


Figure 1

II. THE SET OF WHOLE NUMBERS

If the number 0 is added to the set of counting numbers, then the resulting set, {0, 1, 2, 3, 4 . . .} , is called the set of whole numbers or the set of non-negative integers. See Figure 2.

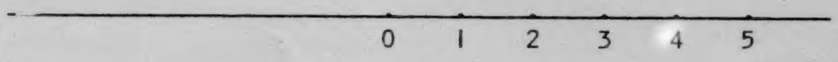


Figure 2

III. THE SET OF INTEGERS

If two whole numbers such as 2 and 3 are added, the result, 5, is again a whole number. However, subtraction will cause problems. The answer to 4 - 7 is not a whole number. Therefore, to perform subtraction, the number -3 as well as the negatives of all counting numbers must be added to the set of whole numbers. The set, {. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . .} , is called the set of integers. The integers continue indefinitely both to the right and to the left on the number line. See Figure 3.

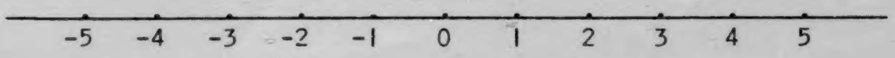


Figure 3

IV. THE SET OF RATIONAL NUMBERS

The set of integers, however, is not sufficient as a set of numbers to use in all applications. For example, in Figure 4, to measure the piece of metal, fractions of an inch are necessary.

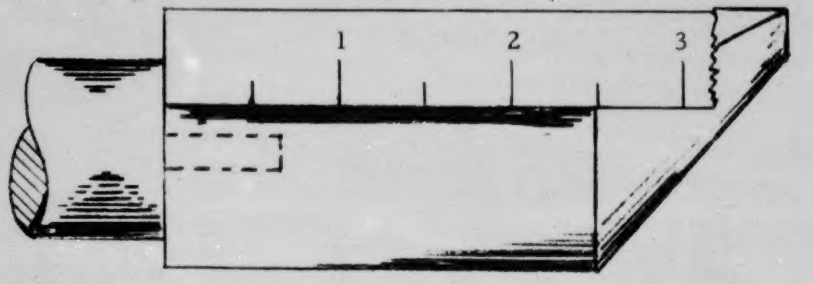


Figure 4

Therefore, to have a more usable set of numbers, positive and negative fractions need to be added to the set of integers. The set of rational numbers is defined to be the set of all numbers which can be expressed in the form $\frac{a}{b}$ where a and b are integers and b is not 0. For example, $\frac{2}{3}$ is a rational number where a = 2 and b = 3. If a = -17 and b = 4, then $\frac{a}{b}$ becomes the rational number $\frac{-17}{4}$.

Is the set of integers a subset of the set of rational numbers? That is, can we tell if the set of integers is included in the set of rational numbers? We are really asking if each integer is a rational number. That is, for example, is the integer 7 a rational number? The answer is "yes" because 7 can be expressed as $\frac{7}{1}$. Therefore, 7 can be expressed in the form $\frac{a}{b}$ where a and b are integers. The set of all rational numbers, then, includes all integers in addition to all fractions such as $\frac{1}{2}$, $\frac{3}{5}$, $\frac{51}{4}$, $\frac{-5}{7}$, $\frac{-35}{8}$, etc. It is impossible to list all of the rational numbers. The list would continue indefinitely. A few of the rational numbers are indicated on the number line in Figure 5.

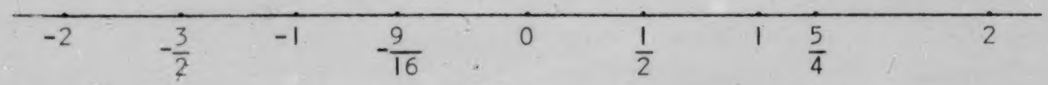


Figure 5

V. THE SET OF REAL NUMBERS

Many applications may be performed without the addition of any numbers other than the rational numbers. However, for such numbers as $\sqrt{2}$, the set of rational numbers is not sufficient. Before considering $\sqrt{2}$, an introductory example may help. In finding $\sqrt{4}$, a positive number x needs to be found such that x times x equals 4. That is, what is the positive number x such that $x \cdot x = 4$? Of course, $x = 2$. Therefore, $\sqrt{4} = 2$. Now, to go back to the problem of $\sqrt{2}$. What is the positive number x such that $x \cdot x = 2$? There is no rational number that will work. The best that can be done is to give an approximate value for $\sqrt{2}$ by using a square root table. A square root table may give the value of $\sqrt{2}$ as 1.414. This is an approximate value correct to three decimal places. Actually, 1.414 times 1.414 equals 1.999 and not 2.

The same is true for many other numbers such as $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$, π, etc. The set of all numbers of this type is called the set of irrational numbers. These comprise all numbers on the number line that are not rational numbers.

The set of real numbers is formed by combining the set of rational numbers with the set of irrational numbers. The set of real numbers comprises all numbers on the number line. It includes all of the types of numbers discussed in the above sections. It includes all integers, all positive and negative fractions, and all irrational numbers such as $\sqrt{2}$ and $\sqrt{11}$. It would certainly again be impossible to list all real numbers. A few of the real numbers are indicated on the number line in Figure 6.

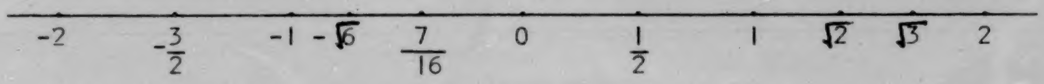


Figure 6

SUMMARY

1. The set, {1, 2, 3, 4 . . .} is called the set of counting numbers, the set of natural numbers, or the set of positive integers.

2. The set, {0, 1, 2, 3, 4 . . .} is called the set of whole numbers or the set of non-negative integers.

3. The set, {. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . .} is called the set of integers.

4. The set of elements that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ is called the set of rational numbers.

5. The set of real numbers is the set consisting of all rational numbers and all irrational numbers. This set contains all numbers on the number line.

EXERCISES

1. Why is 23 a rational number?
2. Is the number 0 a rational number?
3. To what sets does $\frac{-5}{8}$ belong?
4. Is -11 a real number? Is it a rational number? Is it an integer? Is it a counting number?
5. Is the answer to 6 - 11 a whole number? If not, what is it?
6. To what set does any number on the number line belong?

ANSWERS

1. 23 can be expressed in the form $\frac{23}{1}$.
2. Yes. 0 can be written as $\frac{0}{4}$, or $\frac{0}{1}$, etc.
3. Reals and rationals.
4. Yes. Yes. Yes. No.
5. No. An integer, rational number, and real number.
6. The set of real numbers.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead-In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Additive and Multiplicative Properties of Real Numbers

INTRODUCTION AND/OR OBJECTIVES:

This is one of the most important lead-in sections. It is important because it provides much of the basis for the structure involved in modern mathematics. This section should be well mastered. Later work with equations and formulas in the technical information sheets will depend heavily on a thorough understanding of this material.

TECHNICAL INFORMATION:

I. COMMUTATIVE AND ASSOCIATIVE PROPERTIES OF ADDITION

In Figure I, the dimension D may be found by evaluating 2 + 1 or 1 + 2. The answer in either case is certainly 3. This expresses the property that $a + b = b + a$.

The property that for any two real numbers a and b,

$$a + b = b + a$$

is called the commutative property of addition.

To find the value of dimension E in Figure I, 2 + 1 may first be determined, and then the result added to 3, or 2 could be added to the

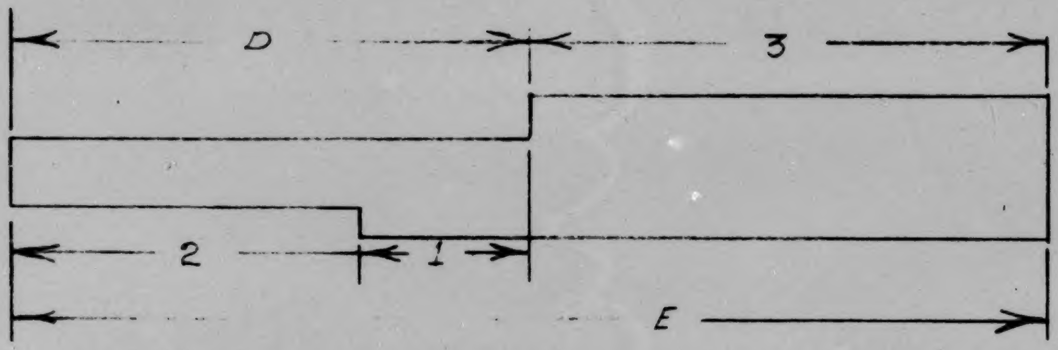


Figure 1

result of 1 + 3. In the first method, (2 + 1) + 3 = 3 + 3 = 6. Note that in evaluation, the sum 2 + 1 on the inside of the parentheses is first found to be 3. Then, this 3 is added to the second 3 to get the final answer of 6. In the second method of finding E, 2 + (1 + 3) = 2 + 4 = 6. Note that again the quantity 1 + 3 on the inside of the parentheses is first found to be 4. Then 2 is added to 4 to find the final answer of 6. This example demonstrates the property that (a + b) + c = a + (b + c).

$$E = (2 + 1) + 3 \quad \text{or} \quad E = 2 + (1 + 3)$$
$$= 3 + 3 \qquad\qquad = 2 + 4$$
$$= 6 \qquad\qquad = 6$$

Therefore, (2 + 1) + 3 = 2 + (1 + 3)

The property that for any three real numbers a, b, and c, then

$$(a + b) + c = a + (b + c)$$

is called the associative property of addition.

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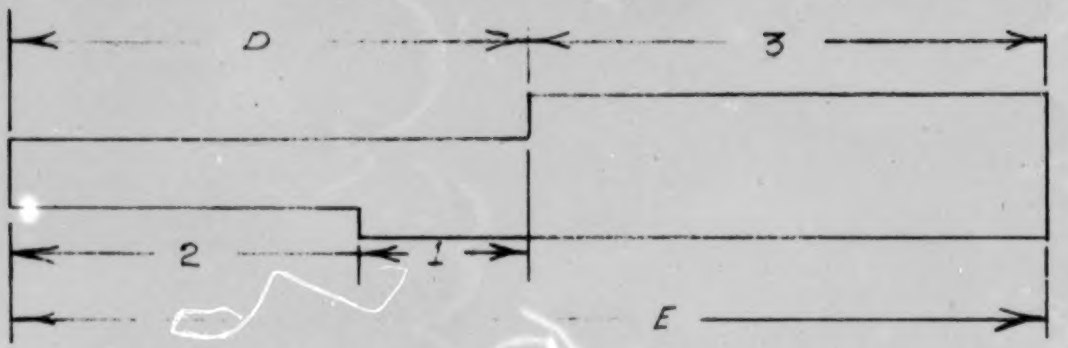

Figure 1

result of $1 + 3$. In the first method, $(2 + 1) + 3 = 3 + 3 = 6$. Note that in evaluation, the sum $2 + 1$ on the inside of the parentheses is first found to be 3. Then, this 3 is added to the second 3 to get the final answer of 6. In the second method of finding E, $2 + (1 + 3) = 2 + 4 = 6$. Note that again the quantity $1 + 3$ on the inside of the parentheses is first found to be 4. Then 2 is added to 4 to find the final answer of 6. This example demonstrates the property that $(a + b) + c = a + (b + c)$.

$$\begin{array}{lcl} E = (2 + 1) + 3 & \text{or} & E = 2 + (1 + 3) \\ \quad = \quad 3 \quad + 3 & & \quad = 2 + \quad 4 \\ \quad = 6 & & \quad = 6 \end{array}$$

Therefore, $(2 + 1) + 3 = 2 + (1 + 3)$

The property that for any three real numbers a, b, and c, then

$$(a + b) + c = a + (b + c)$$

is called the associative property of addition.

II. COMMUTATIVE AND ASSOCIATIVE PROPERTIES OF MULTIPLICATION

In the multiplication of two numbers such as 2 and 3, it is true that $2 \cdot 3 = 3 \cdot 2$ since both sides of the equation equal 6. This illustration demonstrates the property that $ab = ba$.

$2 \cdot 3 = 6$ and $3 \cdot 2 = 6$

Therefore, $2 \cdot 3 = 3 \cdot 2$.

The property that for any two real numbers a and b, then

$$ab = ba$$

is called the commutative property of multiplication.

Note the similarity between the commutative property of addition and the commutative property of multiplication. The properties demonstrate that regardless of the order, the result is the same.

Similar to the associative property of addition, there is a comparable property for multiplication. In the evaluation of $(2 \cdot 3)4$ it is first found that $2 \cdot 3 = 6$. Then the product of $6 \cdot 4$ is found to be 24. Therefore, $(2 \cdot 3)4 = 24$. If the placement of the parentheses is changed, the problem becomes $2(3 \cdot 4)$. The product of $3 \cdot 4$ is first found to be 12. Then 2 is multiplied times 12 to get the final answer of 24. Note the procedure below:

$$(2 \cdot 3)4 = 6 \cdot 4 = 24 \quad \text{and} \quad 2(3 \cdot 4) = 2 \cdot 12 = 24$$

This example demonstrates the property that $(ab)c = a(bc)$.

The property that for any three real numbers a, b, and c, then

$$(ab)c = a(bc)$$

is called the associative property of multiplication.

III. DISTRIBUTIVE PROPERTIES

In the evaluation of $2(3 + 5)$, first of all, the sum of $3 + 5$ is determined to be 8. Then 2 is multiplied times 8 to get the final answer of 16. This result is the same as finding $2 \cdot 3 + 2 \cdot 5$. In this expression, $2 \cdot 3 = 6$ and $2 \cdot 5 = 10$. Then $2 \cdot 3 + 2 \cdot 5 = 6 + 10 = 16$.

$$2(3 + 5) = 2 \cdot 8 = 16 \quad \text{and} \quad 2 \cdot 3 + 2 \cdot 5 = 6 + 10 = 16$$

This illustrates the property that $a(b + c) = ab + ac$.

Note that similarly $(3 + 5)2 = 3 \cdot 2 + 5 \cdot 2$.

$$(3 + 5)2 = 8 \cdot 2 = 16 \quad \text{and} \quad 3 \cdot 2 + 5 \cdot 2 = 6 + 10 = 16$$

This illustrates the property that $(b + c)a = ba + ca$.

The properties that for any three real numbers a, b, and c, then

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca$$

are called the distributive properties of multiplication over addition.

IV. ADDITIVE IDENTITY AND ADDITIVE INVERSES

It is certainly true that $5 + 0 = 5$ and also that $0 + 5 = 5$. This demonstrates the property that for 0 and any real number a, then a + 0 = a and 0 + a = a.

5 + 0 = 5 and 0 + 5 = 5

The property that for any real number a, it is true that

a + 0 = a and 0 + a = a

is called the additive property of zero. 0 is called the additive identity.

It is true that for any given number, for example 5, there exists exactly one number such that 5 plus that number is equal to 0. The number in this example is -5 since 5 + (-5) = 0. Also, (-5) + 5 = 0. Given the number, -6, then there exists the number 6 such that (-6) + 6 = 0 and 6 + (-6) = 0. In other words, for any given real number a, there exists a number -a such that a + (-a) = 0 and (-a) + a = 0.

The property that for any given real number a, there exists exactly one real number -a such that

a + (-a) = 0 and (-a) + a = 0

is called the property of additive inverses. The number -a is called the additive inverse of a, and a is the additive inverse of -a.

Example 1. Find the additive inverse of 2.

Solution: The additive inverse of 2 is -2 since,

2 + (-2) = 0 and (-2) + 2 = 0

Example 2. Find the additive inverse of $\frac{2}{3}$.

Solution: The additive inverse of $\frac{2}{3}$ is $-\frac{2}{3}$ since,

$$\frac{2}{3} + (-\frac{2}{3}) = 0 \text{ and } (-\frac{2}{3}) + \frac{2}{3} = 0$$

Example 3. Find the additive inverse of -4.

Solution: The additive inverse of -4 is 4 since,

$$(-4) + 4 = 0 \text{ and } 4 + (-4) = 0$$

V. MULTIPLICATIVE IDENTITY AND MULTIPLICATIVE INVERSES

In multiplication, it is true that $5 \cdot 1 = 5$ and $1 \cdot 5 = 5$. This demonstrates the property that for the number 1 and any real number a, then $a \cdot 1 = 1 \cdot a = a$.

$$5 \cdot 1 = 5 \qquad \text{and} \qquad 1 \cdot 5 = 5$$

The property that for any real number a it is true that

$$a \cdot 1 = 1 \cdot a = a$$

is called the multiplicative property of 1. 1 is called the multiplicative identity.

It is true that for a given number, for example 2, there exists a number such that 2 times that number is equal to 1. The number in this example is $\frac{1}{2}$ since $2 \cdot \frac{1}{2} = 1$. Also, note that $\frac{1}{2} \cdot 2 = 1$. For the number 4 there exists what number such that 4 times that number is 1? The answer is $\frac{1}{4}$ since $4 \cdot \frac{1}{4} = 1$. In other words for any given number a except 0, there exists a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.

The property that for any real number a except 0 there exists a real number 1/a such that

$$a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$$

is called the property of multiplicative inverses. 1/a is called the multiplicative inverse of a, and a is called the multiplicative inverse of 1/a.

Example 4. Find the multiplicative inverse of 3.

Solution: The multiplicative inverse of 3 is $\frac{1}{3}$.

$$3 \cdot \frac{1}{3} = 1 \text{ and } \frac{1}{3} \cdot 3 = 1$$

Example 5. Find the multiplicative inverse of $\frac{2}{3}$.

Solution: The multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \text{ and } \frac{3}{2} \cdot \frac{2}{3} = 1$$

VI. A NOTE ON NOTATION

Referring back to the associative property of addition, since $(a + b) + c = a + (b + c)$, there is no confusion in writing $a + b + c$. Similarly, since from the associative property of multiplication, $a(bc) = (ab)c$, we may write abc.

SUMMARY

For any real numbers a, b, and c

$a + b = b + a$	the commutative property of addition
$ab = ba$	the commutative property of multiplication
$(a + b) + c = a + (b + c)$	the associative property of addition
$(ab)c = a(bc)$	the associative property of multiplication
$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	the distributive properties of multiplication over addition
$a + 0 = 0 + a = a$	the additive property of zero (0 is called the additive identity)
$a \cdot 1 = 1 \cdot a = a$	the multiplicative property of 1 (1 is called the multiplicative identity)
For any real number a, there exists a real number -a such that $a + (-a) = (-a) + a = 0$	the property of additive inverses (a and -a are additive inverses)
For any real number a except 0, there exists a real number 1/a such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$	the property of multiplicative inverses (a and 1/a are multiplicative inverses)

EXERCISES

For problems 1 through 7 state the property being used. Check the property by evaluating each side of the equation. (commutative and associative properties)

PROPERTY

1. 3 + 10 = 10 + 3 ____________________
2. 4 · 10 = 10 · 4 ____________________
3. 3 + (4 + 5) = (3 + 4) + 5 ____________________
4. 4(3 · 4) = (4 · 3)4 ____________________
5. 2 · 8 = 8 · 2 ____________________
6. 2 + (4 + 1) = (2 + 4) + 1 ____________________
7. 5(3 · 4) = (5 · 3)4 ____________________

For problems 8 to 14 complete using the property indicated. Again, find the value for each side of the equation.

8. 3 · 5 = ____________________ (commutative property of mult.)
9. 3 + (5 + 7) = ____________________ (associative property of add.)
10. 4 + 5 = ____________________ (commutative property of add.)
11. 2(6 · 3) = ____________________ (associative property of mult.)
12. 2.4 + (2.2 + 3.1) = ____________________ (associative property of add.)
13. 7.1 + 2.3 = ____________________ (commutative property of add.)
14. 2(5 · 3) = ____________________ (associative property of mult.)

For problems 15 to 18, complete using the property indicated. Evaluate each side of the equation. (distributive properties)

15. 3(5 + 6) = ____________________ (distributive property)
16. (2 + 1)3 = ____________________ (distributive property)

17. 4(3.1 + 3.5) = ____________________ (distributive property)

18. (3 + 5)x = ____________________ (distributive property)

Complete (in problems 19 through 27)
(identities and inverses)

19. The additive identity is ____________________.

20. The multiplicative identity is ____________________.

21. The additive inverse of 3 is ____________________.

22. The additive inverse of 100 is ____________________.

23. The multiplicative inverse of 7 is ____________________.

24. The multiplicative inverse of 14 is ____________________.

25. The additive inverse of -6 is ____________________.

26. The multiplicative inverse of -4 is ____________________.

27. The multiplicative inverse of 3/4 is ____________________.

ANSWERS

For problems 1 through 7

	Property	Value of each side
1.	Commutative property of addition	13
2.	Commutative property of multiplication	40
3.	Associative property of addition	12
4.	Associative property of multiplication	48
5.	Commutative property of multiplication	16
6.	Associative property of addition	7
7.	Associative property of multiplication	60

For problems 8 through 14

	Completion using property	Value of each side
8.	$5 \cdot 3$	15
9.	$(3 + 5) + 7$	15
10.	$5 + 4$	9
11.	$(2 \cdot 6)3$	36
12.	$(2.4 + 2.2) + 3.1$	7.7
13.	$2.3 + 7.1$	9.4
14.	$(2 \cdot 5)3$	30

For problems 15 through 18

	Completion using property	Value of each side
15.	$3 \cdot 5 + 3 \cdot 6$	33
16.	$2 \cdot 3 + 1 \cdot 3$	9
17.	$4(3.1) + 4(3.5)$	26.4

18. 3x + 5x

For problems 19 through 27

19. 0

20. 1

21. -3

22. -100

23. 1/7

24. 1/14

25. 6

26. -1/4

27. 4/3

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Equivalence of Fractions

INTRODUCTION AND/OR OBJECTIVES:

Since all measurements cannot be limited to working with whole numbers, fractions become very important. A certain measurement may be noted as 12/16, 24/32, or 3/4. These are equivalent fractions. How do we know if one fraction is equivalent to another? This question will be answered in this section. Also, in dealing with equivalent fractions, which is the simplest fraction to use? For example, it would certainly be burdensome to work with the fraction 228/304. It would in most cases be much nicer to work with the fraction 3/4, which is equivalent to 228/304. This section deals with finding for any given fraction the simplest fraction name or what may be called "reducing the fraction to lowest terms."

TECHNICAL INFORMATION:

Suppose that we need to measure the width of a piece of metal. In Figure 1, we use a rule in which each inch is divided into four equal parts. The width is measured as 3 of 4 equal parts. Therefore, the width is 3/4 inch.

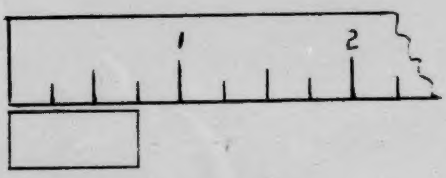


Figure 1

In Figure 2 the same piece of metal is being measured by a rule on which each inch is divided into 8 equal parts. In this case the width is measured as 6 of the 8 equal parts. Therefore, the width is 6/8 of an inch.

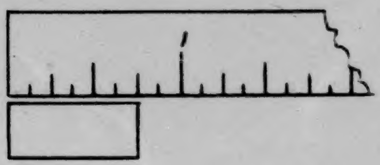


Figure 2

In Figure 3 the same piece of metal is being measured by a rule in which each inch is divided into 16 equal parts. In this case the width is measured as 12 of the 16 equal parts. Therefore, the width is 12/16 of an inch.

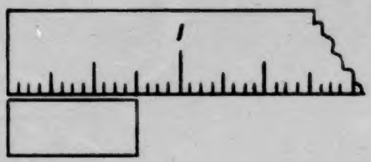


Figure 3

Similarly, if a rule divided into 32 equal parts per inch were used, the width would be measured as 24/32 of an inch.

All of these measurements were for the same piece of metal. Therefore, all four measurements should be the same. This means that: $\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{24}{32}$. These fractions are called "equivalent fractions".

There are two interesting relationships that may be noted between these equivalent fractions. First of all, note that for the equivalent fractions 3/4 and 6/8 it is true that $3 \times 8 = 4 \times 6$. Similarly, 6/8 and 24/32 are equivalent fractions, and it is true that $6 \times 32 = 8 \times 24$. This suggests the following:

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if $a \times d = b \times c$.

Example 1. Are $\frac{2}{3}$ and $\frac{8}{12}$ equivalent fractions?

That is, can we check to see if it is true that:

$2 \times 12 \overset{?}{=} 3 \times 8$

$24 = 24$

Therefore, since $2 \times 12 = 3 \times 8$, then $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent fractions.

Example 2. Are $\frac{5}{8}$ and $\frac{8}{32}$ equivalent fractions?

$5 \times 32 \overset{?}{=} 8 \times 8$

$160 \neq 64$

Therefore, since $5 \times 32 \neq 8 \times 8$, then $\frac{5}{8}$ and $\frac{8}{32}$ are not equivalent.

Therefore: $\frac{5}{8} \neq \frac{8}{32}$

Example 3. Are $\frac{5}{8}$ and $\frac{20}{32}$ equivalent fractions?

$5 \times 32 \overset{?}{=} 8 \times 20$

$160 = 160$

Therefore, $\frac{5}{8}$ and $\frac{20}{32}$ are equivalent fractions.

Then: $\frac{5}{8} = \frac{20}{32}$

Now, remember that previously we noted that $\frac{3}{4} = \frac{6}{8}$.

$\frac{3}{4} = \frac{6}{8}$

And $\frac{6}{8} = \frac{3 \times 2}{4 \times 2}$

Then $\frac{3}{4} = \frac{3 \times 2}{4 \times 2}$

Similarly, we previously recognized that $\frac{3}{4} = \frac{12}{16}$.

$\frac{3}{4} = \frac{12}{16}$

And $\frac{12}{16} = \frac{3 \times 4}{4 \times 4}$

Then $\frac{3}{4} = \frac{3 \times 4}{4 \times 4}$

These examples suggest the property that:

$$\frac{a \times c}{b \times c} = \frac{a}{b} \qquad (\text{if } c \neq 0)$$

That is, we may obtain an equivalent fraction from a/b by multiplying both the numerator a and the denominator b by a common factor c. This is the second relationship that we may notice about equivalent fractions.

For example,

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Similarly, we may use the idea in order to simplify a fraction by finding common factors in the numerator and the denominator.

For example,

$$\frac{14}{18} = \frac{7 \times 2}{9 \times 2} = \frac{7}{9}$$

Example 4. Find the simplest fractional name for $\frac{15}{20}$.

What do we mean to find the simplest fractional name? A fraction is said to have the simplest fractional name or be reduced to lowest terms if the numerator and the denominator have no common factors except for 1 (or -1). (Note that every number has a factor of 1 or -1 since, for example, $4 = 4 \times 1$ and $4 = -4 \times -1$.) For example 4/6 is not in lowest terms since $\frac{4}{6} = \frac{2 \times 2}{3 \times 2}$. The numerator 4 and the denominator have a common factor which is 2.

$$\frac{4}{6} = \frac{2 \times 2}{2 \times 3} = \frac{2}{3}$$

Now, 2/3 is in lowest terms.

In Example 4,

$$\frac{15}{20} = \frac{3 \times 5}{5 \times 5} = \frac{3}{5}$$ (3/5 is the simplest fractional name)

Example 5. Find the simplest fractional name for $\frac{12}{18}$.

$$\frac{12}{18} = \frac{3 \times 2 \times 2}{3 \times 3 \times 2} = \frac{2}{3}$$ (Note that the common factors are 2 and 3.)

Example 6. Find the simplest fractional name for $\frac{7}{21}$.

$$\frac{7}{21} = \frac{1 \times 7}{3 \times 7} = \frac{1}{3}$$

Example 7. Reduce $-\frac{10}{15}$ to lowest terms.

$$-\frac{10}{15} = -\frac{2 \times 5}{3 \times 5} = -\frac{2}{3}$$

Example 8. Reduce $\frac{18}{30}$ to lowest terms.

$$\frac{18}{30} = \frac{3 \times 3 \times 2}{5 \times 3 \times 2} = \frac{3}{5}$$

Or, if we can see that 6 is a common factor we could solve as follows:

$$\frac{18}{30} = \frac{3 \times 6}{5 \times 6} = \frac{3}{5}$$

EXERCISES

1. Are $\frac{2}{3}$ and $\frac{4}{6}$ equivalent fractions?

2. Is $\frac{6}{9} = \frac{8}{12}$?

3. Is $-\frac{3}{2} = -\frac{15}{10}$?

4. Is $\frac{3}{4} = \frac{8}{12}$?

In problems 5 through 12 find the simplest fractional name (or in other words, reduce to lowest terms).

5. $\frac{15}{25}$

6. $\frac{6}{10}$

7. $\frac{18}{27}$

8. $\frac{16}{24}$

9. $\frac{125}{150}$

10. $-\frac{10}{14}$

11. $\frac{33}{55}$

12. $\frac{32}{50}$

ANSWERS

1. Yes.
2. Yes.
3. Yes
4. No.
5. $\frac{3}{5}$
6. $\frac{3}{5}$
7. $\frac{2}{3}$
8. $\frac{2}{3}$
9. $\frac{5}{6}$
10. $-\frac{5}{7}$
11. $\frac{3}{5}$
12. $\frac{16}{25}$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition and Subtraction of Rational Numbers

INTRODUCTION AND/OR OBJECTIVES:

In determining certain measurements it is frequently necessary to add or subtract numbers involving fractions. This section will deal with the addition and subtraction of rational numbers. The number line is used to provide a better understanding of the procedures.

TECHNICAL INFORMATION:

I. ADDITION OF RATIONAL NUMBERS

In Figure I each unit is divided into 8 equal parts just as each inch on a rule could be divided into 8 equal parts. If we wish, for example, to add 3/8 to 4/8, we can first move to the right from the origin 3 of the 8 equal parts. Then, we move to the right 4 more of the equal parts. Thus, we have totally moved 7 of the 8 equal parts. Therefore, 3/8 + 4/8 = 7/8.

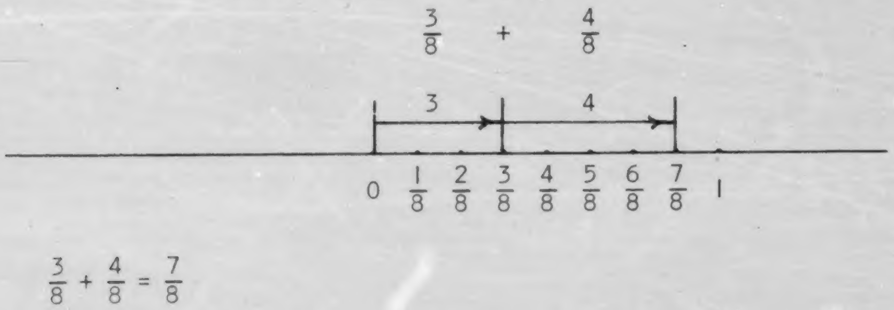
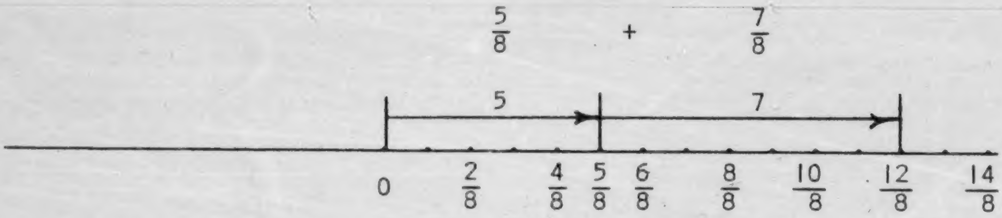


Figure I

In Figure 2 we have added 5/8 to 7/8 to get the total of 12/8.



$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8}$$

Figure 2

These illustrations suggest a method for addition of rational numbers.

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

Example 1. Find: $\frac{5}{7} + \frac{3}{7}$

$$\frac{5}{7} + \frac{3}{7} = \frac{5 + 3}{7} = \frac{8}{7}$$

Example 2. Find: $\frac{10}{21} + \frac{7}{21}$

$$\frac{10}{21} + \frac{7}{21} = \frac{10 + 7}{21} = \frac{17}{21}$$

Example 3. Find: $\frac{3}{4} + \frac{11}{8}$

$\frac{3}{4} + \frac{11}{8} = \frac{6}{8} + \frac{11}{8} = \frac{6 + 11}{8} = \frac{17}{8}$ (Note that here we have changed $\frac{3}{4}$ to the equivalent form $\frac{6}{8}$)

This example involved the addition of fractions with different denominators. In order to add (or subtract) fractions with different denominators, we must find what is called a common denominator. It is usually best to find what is called the least common denominator. Then we find an equivalent fraction utilizing the least common denominator for each of the fractions in the original problem.

For example, suppose that we wish to find: $\frac{3}{8} + \frac{5}{18}$

To find the least common denominator (LCD) we first factor each of the denominators, 8 and 18, into prime factors. (A number is called a prime factor if it cannot be factored further except using 1 (or -1).) The numbers 2 3 5, and 7, for example, are prime.

$$8 = 2 \times 2 \times 2$$

$$18 = 2 \times 3 \times 3$$

Now, we use the product of all the different factors involved in the 2 denominators, 8 and 18. We repeat a factor if it is repeated in either the 8 or the 18. Each factor is entered the largest number of times that it appears in either 8 or 18.

Therefore:

LCD (for 8 and 18) = $2 \times 2 \times 2 \times 3 \times 3 = 72$ (Note that we enter 3 twice since it appears in 18 twice. We enter 2 three times since it appears in 8 three times.)

Now, we find equivalent fractions so that the denominator of each is 72.

$$\frac{3}{8} = \frac{3 \times ?}{72} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$$

$$\frac{5}{18} = \frac{5 \times ?}{72} = \frac{5 \times 4}{18 \times 4} = \frac{20}{72}$$

Therefore:

$$\frac{3}{8} + \frac{5}{18} = \frac{27}{72} + \frac{20}{72} = \frac{27 + 20}{72} = \frac{47}{72}$$

The student should note that the least common denominator is a multiple of each denominator. In this previous illustration: 8 x 9 = 72 and 18 x 4 = 72. In some problems it is easy to identify the least common denominator without going through the procedure of factoring each denominator. For example to find the value of $\frac{2}{3} + \frac{1}{2}$, we would use 6 as the least common denominator. 6 is the smallest number that is a multiple of 3 and also a multiple of 2.

$$\frac{2}{3} = \frac{2 \times ?}{6} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{1 \times ?}{6} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Therefore:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{4 + 3}{6} = \frac{7}{6}$$

Example 4. Find: $\frac{3}{14} + \frac{1}{6}$

$$14 = 2 \times 7$$

$$6 = 2 \times 3$$

$$\text{LCD} = 2 \times 3 \times 7 = 42$$

$$\frac{3}{14} = \frac{3 \times ?}{42} = \frac{3 \times 3}{14 \times 3} = \frac{9}{42}$$

$$\frac{1}{6} = \frac{1 \times ?}{42} = \frac{1 \times 7}{6 \times 7} = \frac{7}{42}$$

Therefore:

$$\frac{3}{14} + \frac{1}{6} = \frac{9}{42} + \frac{7}{42} = \frac{9 + 7}{42} = \frac{16}{42}$$

Example 5. Find: $\frac{1}{4} + \frac{3}{16}$

Here, it should be seen that we can use 16 as the LCD.

$$\frac{1}{4} = \frac{1 \times ?}{16} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$$

Therefore:

$$\frac{1}{4} + \frac{3}{16} = \frac{4}{16} + \frac{3}{16} = \frac{4 + 3}{16} = \frac{7}{16}$$

Example 6. Find: $\frac{1}{4} + \frac{1}{10} + \frac{3}{8}$

$4 = 2 \times 2$

$10 = 2 \times 5$

$8 = 2 \times 2 \times 2$

LCD = $2 \times 2 \times 2 \times 5 = 40$

$$\frac{1}{4} = \frac{1 \times ?}{40} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}$$

$$\frac{1}{10} = \frac{1 \times ?}{40} = \frac{1 \times 4}{10 \times 4} = \frac{4}{40}$$

$$\frac{3}{8} = \frac{3 \times ?}{40} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

Therefore:

$$\frac{1}{4} + \frac{1}{10} + \frac{3}{8} = \frac{10}{40} + \frac{4}{40} + \frac{15}{40} = \frac{10 + 4 + 15}{40} = \frac{29}{40}$$

Example 7. $\frac{3}{8} + \frac{3}{5} + \frac{1}{6}$

$8 = 2 \times 2 \times 2$

$5 = 5$

$6 = 2 \times 3$

LCD = $2 \times 2 \times 2 \times 3 \times 5 = 120$

$$\frac{3}{8} = \frac{3 \times ?}{120} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$$

$$\frac{3}{5} \quad \frac{3 \times ?}{120} \quad \frac{3 \times 24}{5 \times 24} \quad \frac{72}{120}$$

$$\frac{1}{6} = \frac{1 \times ?}{120} = \frac{1 \times 20}{6 \times 20} = \frac{20}{120}$$

$$\frac{3}{8} + \frac{3}{5} + \frac{1}{6} = \frac{45}{120} + \frac{72}{120} + \frac{20}{120} = \frac{45 + 72 + 20}{120} = \frac{137}{120}$$

We could perform these additions in column form. For example, we could have performed Example 7 as follows:

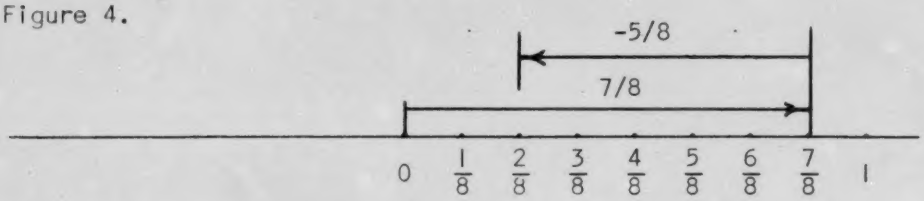
$$\begin{array}{ccc} \frac{3}{8} & \frac{3 \times 15}{8 \times 15} & \frac{45}{120} \\[2ex] \frac{3}{5} & \frac{3 \times 24}{5 \times 24} & \frac{72}{120} \\[2ex] \frac{1}{6} & \frac{1 \times 20}{6 \times 20} & \frac{20}{120} \\ \hline & & \frac{137}{120} \end{array}$$

(Note that we multiply the numerators and denominators in column 2 by numbers so that the denominators will all become 120, the LCD)

II. SUBTRACTION OF RATIONAL NUMBERS

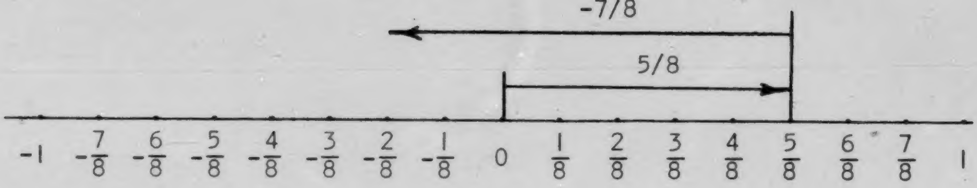
The main difference between addition and subtraction is that of direction on the number line. To find 7/8 + (-5/8) we first move to the right 7 of 8 equal parts of the unit. See Figure 3. Then, we move back to the left 5 of 8 equal parts. The result is 2/8. Therefore, 7/8 + (-5/8) = 7/8 - 5/8 = 2/8.

Similarly, to find 5/8 - 7/8, first move 5 to the right and then 7 to the left. The result is -2/8. Therefore, 5/8 + (-7/8) = 5/8 - 7/8 = -2/8. See Figure 4.



$$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$$

Figure 3



$\frac{5}{8} - \frac{7}{8} = -\frac{2}{8}$

Figure 4

The above examples suggest the following results:

$$\frac{a}{b} - \frac{c}{b} = \frac{a + (-c)}{b} \qquad \text{or} \qquad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

Example 8. Find: $\frac{2}{3} - \frac{4}{3}$

$$\frac{2}{3} - \frac{4}{3} = \frac{2 + (-4)}{3} = \frac{-2}{3} = -\frac{2}{3}$$

Example 9. Find: $\frac{15}{8} - \frac{5}{8}$

$$\frac{15}{8} - \frac{5}{8} = \frac{15 - 5}{8} = \frac{10}{8} = \frac{5 \times 2}{4 \times 2} = \frac{5}{4}$$

Example 10. Find: $\frac{5}{8} - \frac{1}{4}$

$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{5 - 2}{8} = \frac{3}{8}$ (Note that here the LCD = 8, and $\frac{1}{4} = \frac{2}{8}$)

Example 11. Find: $\frac{9}{10} - \frac{3}{4}$

$10 = 2 \times 5$

$4 = 2 \times 2$

$\text{LCD} = 2 \times 2 \times 5$

$$\frac{9}{10} = \frac{9 \times ?}{20} = \frac{9 \times 2}{10 \times 2} = \frac{18}{20}$$

$$\frac{3}{4} = \frac{3 \times ?}{20} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Therefore:

$$\frac{9}{10} - \frac{3}{4} = \frac{18}{20} - \frac{15}{20} = \frac{18 - 15}{20} = \frac{3}{20}$$

Or, in column form:

$$\begin{array}{ccc} \frac{9}{10} & \frac{9 \times 2}{10 \times 2} & \frac{18}{20} \\ - \frac{3}{4} & - \frac{3 \times 5}{4 \times 5} & - \frac{15}{20} \\ \hline & & \frac{3}{20} \end{array}$$

III. MIXED NUMERALS, PROPER FRACTIONS, AND IMPROPER FRACTIONS

A number which consists of a whole number and a fraction is called a *mixed numeral*. For example, $1\frac{1}{2}$, $3\frac{2}{3}$, and $5\frac{7}{8}$ are mixed numerals.

A positive fraction in which the numerator is less than the denominator is called a *proper fraction*. For example, $\frac{2}{3}$ is a proper fraction since the numerator, 2, is less than the denominator, 3.

An *improper fraction* is a fraction in which the numerator is larger than or equal to the denominator. For example $\frac{7}{4}$ is an improper fraction since the numerator, 7, is larger than the denominator, 4. Likewise, $\frac{6}{6}$ is an improper fraction since the numerator, 6, is equal to the denominator, 6

We can replace an improper fraction by an equivalent mixed numeral as illustrated below.

Example 12. Change $\frac{11}{8}$ to a mixed numeral.

$$\frac{11}{8} = \frac{8+3}{8} = \frac{8}{8} + \frac{3}{8} = 1 + \frac{3}{8} = 1\frac{3}{8}$$

Note that $\frac{8}{8} = 1$, or in general:

$$\frac{a}{a} = 1 \qquad \text{if } a \neq 0$$

Example 13. Change $\frac{9}{4}$ to a mixed numeral.

$$\frac{9}{4} = \frac{4+4+1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 1 + 1 + \frac{1}{4} = 2\frac{1}{4}$$

Or $$\frac{9}{4} = \frac{8+1}{4} = \frac{8}{4} + \frac{1}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$$

Example 14. Convert $1\frac{7}{8}$ to an improper fraction.

$$1\frac{7}{8} = 1 + \frac{7}{8} = \frac{8}{8} + \frac{7}{8} = \frac{15}{8}$$

Example 15. Convert $5\frac{3}{4}$ to an improper fraction.

$$5\frac{3}{4} = 5 + \frac{3}{4} = \frac{5}{1} + \frac{3}{4} = \frac{5 \times 4}{1 \times 4} + \frac{3}{4} = \frac{20}{4} + \frac{3}{4} = \frac{23}{4}$$

EXERCISES

1. $\frac{3}{7} + \frac{2}{7} =$
2. $\frac{7}{16} + \frac{13}{16} =$
3. $\frac{3}{8} + \frac{5}{16} =$
4. $\frac{3}{4} + \frac{3}{16} =$
5. $\frac{1}{2} + \frac{7}{16} =$
6. $\frac{7}{8} - \frac{3}{8} =$
7. $\frac{5}{16} - \frac{10}{16} =$
8. $\frac{11}{16} - \frac{3}{8} =$
9. $\frac{7}{10} - \frac{2}{5} =$
10. $\frac{5}{6} + \frac{3}{8} =$
11. $\frac{3}{8} + \frac{1}{16} + \frac{1}{4} =$
12. $\frac{1}{10} + \frac{1}{5} + \frac{1}{4} =$
13. $\frac{1}{3} + \frac{2}{5} + \frac{3}{10} =$
14. Convert $\frac{13}{9}$ to a mixed numeral.
15. Convert $\frac{17}{7}$ to a mixed numeral.
16. Convert $1\frac{3}{16}$ to an improper fraction.
17. Convert $2\frac{5}{8}$ to an improper fraction.
18. Convert $4\frac{1}{4}$ to an improper fraction.

ANSWERS

1. $\frac{5}{7}$
2. $\frac{20}{16}$ or $\frac{5}{4}$ or $1\frac{1}{4}$
3. $\frac{11}{16}$
4. $\frac{15}{16}$
5. $\frac{15}{16}$
6. $\frac{4}{8}$ or $\frac{1}{2}$
7. $-\frac{5}{16}$
8. $\frac{5}{16}$
9. $\frac{3}{10}$
10. $\frac{29}{24}$
11. $\frac{11}{16}$
12. $\frac{11}{20}$
13. $\frac{31}{30}$
14. $1\frac{4}{9}$
15. $2\frac{3}{7}$
16. $\frac{19}{16}$
17. $\frac{21}{16}$
18. $\frac{17}{4}$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Multiplication of Rational Numbers

INTRODUCTION AND/OR OBJECTIVES:

Just as the addition of rational numbers is important for application problems; so also is the multiplication of rational numbers and the multiplication of whole numbers and rational numbers.

TECHNICAL INFORMATION:

To find the product of two rational numbers the following property is used:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \qquad \text{or} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Note that the · indicates multiplication just as the x sign.

Example 1. Find: $\frac{3}{4} \times \frac{5}{7}$

$$\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

Example 2. Find: $\frac{7}{5} \times \frac{5}{6}$

$$\frac{7}{5} \times \frac{5}{6} = \frac{7 \times 5}{5 \times 6} = \frac{7}{6}$$ (Note that there is a common factor of 5 in the numerator and the denominator)

Example 3. Find: $-\frac{6}{5} \times \frac{4}{5}$

$$-\frac{6}{5} \times \frac{4}{5} = -\frac{6 \times 4}{5 \times 5} = -\frac{24}{25}$$

Example 4. Find: $\frac{4}{5} \times 7$

$$\frac{4}{5} \times 7 = \frac{4}{5} \times \frac{7}{1} = \frac{4 \times 7}{5 \times 1} = \frac{28}{5}$$

Notice that in Example 4:

$$\frac{4}{5} \times 7 = \frac{4 \times 7}{5}$$

This suggests the following property:

$$\frac{a}{b} \times c = \frac{a \times c}{b} \qquad \text{or} \qquad \frac{a}{b} \cdot c = \frac{a \cdot c}{b}$$

Also, very similarly:

$$c \times \frac{a}{b} = \frac{c \times a}{b} \qquad \text{or} \qquad c \cdot \frac{a}{b} = \frac{c \cdot a}{b}$$

Example 5. Find: $\frac{2}{3} \times 8$

$$\frac{2}{3} \times 8 = \frac{2 \times 8}{3} = \frac{16}{3} = \frac{15 + 1}{3} = \frac{15}{3} + \frac{1}{3} = 5 + \frac{1}{3} = 5\frac{1}{3}$$

Example 6. Find: $5 \times \frac{3}{8}$

$$5 \times \frac{3}{8} = \frac{5 \times 3}{8} = \frac{15}{8} = \frac{8 + 7}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8} = 1\frac{7}{8}$$

EXERCISES

1. $\frac{2}{3} \times \frac{4}{7} =$
2. $\frac{1}{3} \times \frac{5}{8} =$
3. $\frac{3}{7} \times \frac{5}{6} =$
4. $\frac{11}{3} \times \frac{2}{7} =$
5. $3 \times \frac{2}{5} =$
6. $\frac{5}{6} \times 4 =$
7. $5 \times \frac{1}{8} =$
8. $\frac{3}{32} \times 8 =$

ANSWERS

1. $\frac{8}{21}$
2. $\frac{5}{24}$
3. $\frac{5}{14}$
4. $\frac{22}{21}$ or $1\frac{1}{21}$
5. $\frac{6}{5}$ or $1\frac{1}{5}$
6. $\frac{10}{3}$ or $3\frac{1}{3}$
7. $\frac{5}{8}$
8. $\frac{3}{4}$

MODERN MATHEMATICS
As Related To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Solution of Equations

INTRODUCTION AND/OR OBJECTIVES:

One of the basic operations in algebra is that of solving for an unknown quantity x (or whatever it may be) in an equation. This is an extremely important and necessary section as a lead up to the solution of unknowns in application formulas.

TECHNICAL INFORMATION:

I. ADDITIVE PROPERTY OF EQUALITY

We can easily see that since $8 = 4 \times 2$, then:

$$8 + 3 = 4 \times 2 + 3$$

$$11 = 11$$

Also, since $16 = 8 \times 2$, then:

$$16 + (-4) = 8 \times 2 + (-4)$$

$$16 - 4 = 16 - 4$$

$$12 = 12$$

In other words we may add (or subtract) the same quantity to both sides of an equation.

We will refer to the property that we may add the same quantity (either a positive or negative number) to both sides of an equation as the Additive Property of Equality (abbreviated as a. p. e.).

II. MULTIPLICATIVE PROPERTY OF EQUALITY

Since $8 = 5 + 3$, then:

$$6 \cdot 8 = 6(5 + 3)$$

$$48 = 6 \cdot 8$$

$$48 = 48$$

Also, since $8 = 5 + 3$, then:

$$\frac{1}{2} \cdot 8 = \frac{1}{2} \cdot (5 + 3)$$

$$\frac{8}{2} = \frac{5 + 3}{2}$$

$$4 = 4$$

In other words we may multiply both sides of an equation by the same number.

> We will refer to the property that we may multiply both sides of an equation by the same quantity as the Multiplicative Property of Equality (abbreviated as m. p. e.).

Notice that in the second illustration above, multiplying both sides of an equation by 1/2 is the same as dividing both sides of the equation by 2. Therefore, we can divide both sides of an equation by the same nonzero number.

III. SOLUTION OF EQUATIONS

Example 1. Find x in the following equation: $x + 3 = 8$

$$x + 3 = 8$$

$$x + 3 - 3 = 8 - 3 \qquad \text{(Subtract 3 from both sides of the equation)}$$

$$x + 0 = 5 \qquad \text{(-3 is the additive inverse of 3)}$$

$$x = 5 \qquad \text{(0 is the additive identity)}$$

Example 2. Find x in the following equation: $x - 5 = 7$

$x - 5 = 7$

$x - 5 + 5 = 7 + 5$ (Add 5 to both sides of the equation)

$x = 12$

Example 3. Find x in the following equation: $2x = 16$

$2x = 16$

$\frac{1}{2}\cdot 2x = \frac{1}{2}\cdot 16$ (Multiply both sides by 1/2)

$1\cdot x = \frac{16}{2}$ (1/2 is the multiplicative inverse of 2, $\frac{a}{b}\cdot c = \frac{a\cdot c}{b}$)

$x = 8$ (1 is the multiplicative identity)

Example 4. Find x in the following equation: $\frac{x + 3}{4} = 2$

$\frac{x + 3}{4} = 2$

$\frac{x + 3}{4}\cdot 4 = 2\cdot 4$ (Multiply both sides by 4)

$x + 3 = 8$ (Multiplicative inverses (1/4 and 4))

$x + 3 - 3 = 8 - 3$ (Subtract 3 from both sides)

$x = 5$ (-3 is the additive inverse of 3)

Example 5. Find x in the following equation: $\frac{x - 2}{8} = \frac{3}{4}$

$\frac{x - 2}{8} = \frac{3}{4}$

$\frac{x - 2}{8}\cdot 8 = \frac{3}{4}\cdot 8$ (Multiply both sides by 8)

$x - 2 = \frac{3\cdot 8}{4}$ (Multiplicative inverse, $\frac{a}{b}\cdot c = \frac{a\cdot c}{b}$)

$x - 2 = \frac{3\cdot 2\cdot 4}{4}$

$x - 2 = 3\cdot 2$

$x - 2 = 6$

$x - 2 + 2 = 6 + 2$ (Add 2 to both sides)

$x = 8$ (2 is the additive inverse of -2)

Example 6. Find x in the following equation: $2x - 4 = x + 8$

$2x - 4 = x + 8$

$2x - 4 + 4 = x + 8 + 4$ (Add 4 to both sides)

$2x = x + 12$ (4 is the additive inverse of -4)

$2x - x = x + 12 - x$ (Subtract x from both sides)

$x = 12$ ($-x$ is the additive inverse of x)

EXERCISES

1. Solve for x: $x + 2 = 6$
2. Solve for x: $x - 5 = 7$
3. Solve for x: $x - 2 = 10$
4. Solve for x: $2x + 3 = 7$
5. Solve for x: $3x - 2 = 10$
6. Solve for x: $5x \div 7 = 32$
7. Solve for x: $\frac{x + 1}{3} = 4$
8. Solve for x: $2x - 5 = x + 3$
9. Solve for x: $\frac{x - 2}{6} = \frac{2}{3}$
10. Solve for x: $\frac{2x - 1}{4} = \frac{1}{3}$
11. Solve for x: $\frac{x + 1}{3} = \frac{2}{5}$

ANSWERS

1. $x = 4$
2. $x = 12$
3. $x = 12$
4. $x = 2$
5. $x = 4$
6. $x = 5$
7. $x = 11$
8. $x = 8$
9. $x = 6$
10. $x = \frac{7}{6}$
11. $x = \frac{1}{5}$

MODERN MATHEMATICS
As Related To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Division of Rational Numbers

INTRODUCTION AND/OR OBJECTIVES:

Just as well as it is frequently necessary to add, subtract, or multiply rational numbers in solving for dimensions or solving application problem formulas, it is frequently necessary to divide rational numbers. As will be seen, division of rational numbers can be accomplished in terms of multiplication of rational numbers.

TECHNICAL INFORMATION:

Remember that with whole numbers, $\frac{16}{8} = 2$ because $8 \times 2 = 16$.

In the division of rational numbers, $\dfrac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2}$ because $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$.

Therefore:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{x}{y} \quad \text{means that} \quad \frac{c}{d} \times \frac{x}{y} = \frac{a}{b}$$

Or

$$\frac{a}{b} \div \frac{c}{d} = \frac{x}{y} \quad \text{means that} \quad \frac{c}{d} \times \frac{x}{y} = \frac{a}{b}$$

Example 1. Find $\frac{2}{3} \div \frac{4}{5}$.

$$\frac{2}{3} \div \frac{4}{5} = \frac{x}{y}$$

$$\frac{4}{5} \times \frac{x}{y} = \frac{2}{3}$$

$$\frac{5}{4} \times \frac{4}{5} \times \frac{x}{y} = \frac{5}{4} \times \frac{2}{3} \qquad \text{(Multiply both sides by } \tfrac{5}{4}\text{)}$$

$$1 \times \frac{x}{y} = \frac{5}{4} \times \frac{2}{3} \qquad (\tfrac{5}{4} \text{ is the multiplicative inverse of } \tfrac{4}{5})$$

$$\frac{x}{y} = \frac{2}{3} \times \frac{5}{4} \qquad \text{(1 is the multiplicative identity)}$$

Before finishing, note that the problem was to find: $\frac{x}{y} = \frac{2}{3} \div \frac{4}{5}$. In the last step above we found that: $\frac{x}{y} = \frac{2}{3} \times \frac{5}{4}$. This example suggests the fact that dividing by a rational number other than zero may be accomplished by multiplying by the multiplicative inverse of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \quad (\text{if } c \neq 0)$$

Therefore, in the above example:

$$\begin{aligned} \frac{2}{3} \div \frac{4}{5} &= \frac{2}{3} \times \frac{5}{4} \\ &= \frac{2 \times 5}{3 \times 4} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$

Example 2. Find $\frac{7}{8} \div \frac{5}{16}$.

80

$$\frac{7}{8} \div \frac{5}{16} = \frac{7}{8} \times \frac{16}{5}$$

$$= \frac{7 \times 16}{8 \times 5}$$

$$= \frac{7 \times 8 \times 2}{8 \times 5}$$

$$= \frac{7 \times 2}{5}$$

$$= \frac{14}{5}$$

Example 3. Find $5 \div \frac{3}{8}$.

$$5 \div \frac{3}{8} = 5 \times \frac{8}{3}$$

$$= \frac{5}{1} \times \frac{8}{3}$$

$$= \frac{5 \times 8}{1 \times 3}$$

$$= \frac{40}{3}$$

Example 4. Find $\frac{3}{4} \div 8$.

$$\frac{3}{4} \div 8 = \frac{3}{4} \div \frac{8}{1}$$

$$= \frac{3}{4} \times \frac{1}{8}$$

$$= \frac{3 \times 1}{4 \times 8}$$

$$= \frac{3}{32}$$

EXERCISES

1. Find $\frac{4}{5} \div \frac{2}{3}$
2. Find $\frac{6}{7} \div \frac{3}{4}$
3. Find $\frac{3}{4} \div \frac{5}{8}$
4. Find $\frac{5}{2} \div \frac{3}{4}$
5. Find $\frac{2}{3} \div 4$
6. Find $\frac{3}{16} \div 6$
7. Find $4 \div \frac{3}{8}$
8. Find $8 \div \frac{1}{2}$

ANSWERS

1. $\frac{6}{5}$ or $1\frac{1}{5}$

2. $\frac{8}{7}$ or $1\frac{1}{7}$

3. $\frac{6}{5}$ or $1\frac{1}{5}$

4. $\frac{10}{3}$ or $3\frac{1}{3}$

5. $\frac{1}{6}$

6. $\frac{1}{32}$

7. $\frac{32}{3}$ or $10\frac{2}{3}$

8. 16

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition and Subtraction of Decimal Numbers

INTRODUCTION AND/OR OBJECTIVES:

In working with measurements it is mandatory that the student be able to work with decimal numbers. The student should understand the meaning of the placement of numbers in a decimal. In determining a various dimension it is frequently necessary to add or subtract various decimal numbers.

TECHNICAL INFORMATION:

In a decimal number, the first digit to the right of the decimal point indicates tenths, the next digit indicated hundredths, the next indicates thousandths, the next indicates ten thousandths, the next indicates hundred thousandths, and so forth. See Figure 1 below.

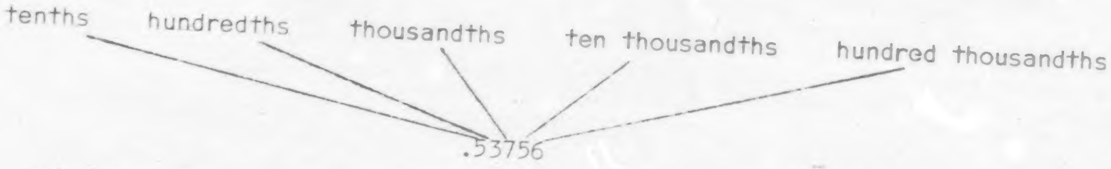

Figure 1

Therefore:

$$.53756 = \frac{5}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{5}{10{,}000} + \frac{6}{100{,}000}$$

Consider the number .37

$$.37 = \frac{3}{10} + \frac{7}{100}$$

Now:

$$\frac{3}{10} = \frac{3}{10} \times \frac{10}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$$

Therefore:

$$\begin{aligned} .37 &= \frac{30}{100} + \frac{7}{100} \\ &= \frac{30 + 7}{100} \\ &= \frac{37}{100} \end{aligned}$$

Thus, we read .37 as "37 hundredths" (thirty seven hundredths).

Similarly:

$$.432 = \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}$$

Now, we need to use the common denominator 1000.

$$\frac{4}{10} = \frac{4 \times 100}{10 \times 100} = \frac{400}{1000}$$

$$\frac{3}{100} = \frac{3 \times 10}{100 \times 10} = \frac{30}{1000}$$

Then:

$$\begin{aligned} .432 &= \frac{400}{1000} + \frac{30}{1000} + \frac{2}{1000} \\ &= \frac{400 + 30 + 2}{1000} \\ &= \frac{432}{1000} \end{aligned}$$

Therefore, .432 is read as "432 thousandths" (four hundred thirty two thousandths).

Example 1. Read .275

.275 is read as "275 thousandths"

Example 2. Read .63

.63 is read as "63 hundredths"

Example 3. Read .0625

.0625 is read as "625 ten thousandths"

Example 4. Find the value of .325 + .432

$$\begin{array}{r} .325 \\ +\ \underline{.432} \\ .757 \end{array}$$

Step 1. Add the 5 thousandths to the 2 thousandths to get 7 thousandths.

Step 2. Add the 2 hundredths to the 3 hundredths to get 5 hundredths.

Step 3. Add the 3 tenths to the 4 tenths to get 7 tenths.

The answer is .757 (read as "757 thousandths")

Example 5. Find the value of .47 + .36

$$\begin{array}{r} .4\ 7 \\ \underline{.3\ 6} \end{array}$$

Note that when we add the numbers in the hundredths column we get 13.

$$\frac{13}{100} = \frac{10 + 3}{100}$$

$$= \frac{10}{100} + \frac{3}{100}$$

$$= \frac{1 \times 10}{10 \times 10} + \frac{3}{100}$$

$$= \frac{1}{10} + \frac{3}{100}$$

Therefore, we have a result of 3 hundredths and 1 tenth. We, then, enter 3 in the hundredths column in the answer and add 1 to the tenths column.

$$\begin{array}{r} / \\ .4\ 7 \\ \underline{.3\ 6} \\ .8\ 3 \end{array}$$

Now, add the numbers in the tenths column to get a total of 8 tenths.

Example 6. Find the value of .234 + .341 + .256

$$\begin{array}{r} /\ / \\ .2\ 3\ 4 \\ .3\ 4\ 1 \\ \underline{.2\ 5\ 6} \\ .8\ 3\ 1 \end{array}$$

Step 1. Add the numbers in the thousandths column. The total is 11. Enter 1 in the thousandths column of the answer and add 1 to the hundredths column.

Step 2. Add the numbers in the hundredths column. The total is 13. Enter 3 in the hundredths column of the answer and add 1 to the tenths column.

Step 3. Add the numbers in the tenths column. Enter the total 8 in the tenths column of the answer.

Example 7. Find the value of .124 + .311 + .245 + .422

$$\begin{array}{r} /\ / \\ .1\ 2\ 4 \\ .3\ 1\ 1 \\ .2\ 4\ 5 \\ \underline{.4\ 2\ 2} \\ 1.1\ 0\ 2 \end{array}$$

Example 8. Find the value of .457 - .232

$$\begin{array}{r} .4\ 5\ 7 \\ -\ \underline{.2\ 3\ 2} \\ .2\ 2\ 5 \end{array}$$

Step 1. Subtract in the thousandths column.
Step 2. Subtract in the hundredths column.
Step 3. Subtract in the tenths column.

Example 9. Find the value of .65 - .17

$$\begin{array}{r} .6\ 5 \\ -\ \underline{.1\ 7} \end{array}$$

Step 1. We cannot subtract 7 hundredths from 5 hundredths. We, therefore, change .65 to .5 + .15 . We have, thus, taken 1 from the tenths column (from 6). This 1 tenth is equal to 10 hundredths. The 10 hundredths plus the 5 hundredths is 15 hundredths. We now subtract the 7 hundredths from the 15 hundredths. We enter the result 8 in the hundredths column of the answer.

$$\begin{array}{r} .5\ 15 \\ -\ \underline{.1\ 7} \\ .4\ 8 \end{array}$$

Step 2. Subtract 1 from 5 in the tenths column. Enter the result 4 in the tenths column of the answer.

Example 10. Find the value of .432 - .216

$$\begin{array}{r} .4\ 3\ 2 \\ -\ \underline{.2\ 1\ 6} \end{array} \qquad \begin{array}{r} .4\ 2\ 12 \\ -\ \underline{.2\ 1\ 6} \\ .2\ 1\ 6 \end{array}$$

Example 11. Find the value of .522 - .237

$$\begin{array}{r} .5\ 2\ 2 \\ -\ \underline{.2\ 3\ 7} \end{array} \qquad \begin{array}{r} .5\ 1\ 12 \\ -\ \underline{.2\ 3\ 7} \\ 5 \end{array} \qquad \begin{array}{r} .4\ 11\ 2 \\ -\ \underline{.2\ 3\ 7} \\ .2\ 8\ 5 \end{array}$$

Example 12. Find the total length L in Figure 2.

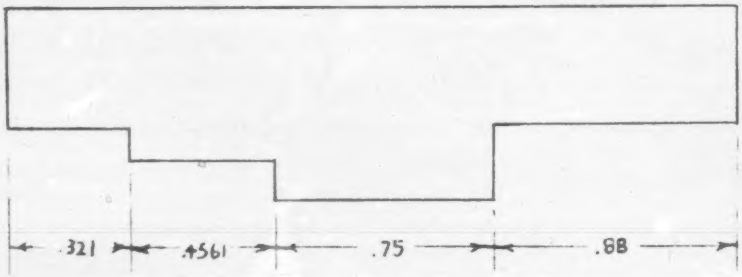

Figure 2

L = .321 + .4561 + .75 + .88

$$\begin{array}{r@{}l} & \overset{2}{.3}\,2\,1 \\ & .4\,5\,6\,1 \\ & .7\,5 \\ & .8\,8 \\ \hline 2&.4\,0\,7\,1 \end{array}$$

L = 2.4071

or 2.41 (correct to two decimal places)

EXERCISES

In problems 1 to 4 read the decimal value.

1. .23
2. .152
3. .275
4. .3752

In problems 5 to 16 find the value.

5. .23 + .35
6. .34 + .47
7. .234 + .312
8. .337 + .435
9. .312 + .214 + .456
10. .124 + .326 + .125 + .427
11. .32 - .11
12. .72 - .37
13. .342 - .125
14. .455 - .278
15. .567 - .289
16. .682 - .391

ANSWERS

1. 23 hundredths (twenty three hundredths)
2. 152 thousandths (one hundred fifty two thousandths)
3. 275 thousandths (two hundred seventy five thousandths)
4. 3,752 ten thousandths (three thousand seven hundred fifty two ten thousandths)
5. .58
6. .81
7. .546
8. .772
9. .982
10. 1.002
11. .21
12. .35
13. .217
14. .177
15. .278
16. .291

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Multiplication and Division of Decimal Numbers

INTRODUCTION AND/OR OBJECTIVES:

Just as it is important to be able to add and subtract decimal numbers, it is necessary to be able to multiply and divide decimal numbers. In dealing with approximate numbers obtained from measurements, we should consider the number of significant digits.

TECHNICAL INFORMATION:

Suppose we wish to find .3 × .7 .

$$.3 = \frac{3}{10}$$

$$.7 = \frac{7}{10}$$

Therefore:

$$\begin{aligned} .3 \times .7 &= \frac{3}{10} \times \frac{7}{10} \\ &= \frac{3 \times 7}{100} \\ &= \frac{21}{100} \\ &= .21 \end{aligned}$$

Note that there is one decimal place in .3, one decimal place in .7, and the answer .21 has two decimal places.

Now, let us find the value of .4 × .43 .

$$.4 = \frac{4}{10}$$

$$.43 = \frac{43}{100}$$

Then:

$$\begin{aligned} .4 \times .43 &= \frac{4}{10} \times \frac{43}{100} \\ &= \frac{4 \times 43}{10 \times 100} \\ &= \frac{172}{1000} \\ &= .172 \end{aligned}$$

Note that there is one decimal place in .4, two decimal places in .43, and three decimal places in the answer .172 .

These two examples suggest that if two numbers are multiplied, the number of decimal places in the answer is the sum of the numbers of decimal places in the two numbers multiplied.

Example 1. Find the value of .35 × .42

$$\begin{array}{r} .35 \\ \underline{.42} \\ 70 \\ \underline{140} \\ .1470 \end{array}$$

Since .35 has 2 decimal places and .42 has 2 decimal places, then the answer should have 2 + 2 or 4 decimal places.

Example 2. Find the value of .241 × .37

$$\begin{array}{r} .241 \\ \underline{.37} \\ 1687 \\ \underline{723} \\ .08917 \end{array}$$

Since .241 has 3 decimal places and .37 has 2 decimal places, then the answer should have 3 + 2 or 5 decimal places.

Example 3. Find the value of .372 x 24

$$\begin{array}{r} .372 \\ \underline{24} \\ 1488 \\ \underline{744} \\ 8.928 \end{array}$$

Since .372 has 3 decimal places and 24 has none, then the answer should have 3 + 0 or 3 decimal places.

Example 4. Find the value of .24 x 3.7

$$\begin{array}{r} .24 \\ \underline{3.7} \\ 168 \\ \underline{72} \\ .888 \end{array}$$

Since .24 has 2 decimal places and 3.7 has one decimal place, then the answer should have 2 + 1 or 3 decimal places.

Now, let us turn to the division of decimal numbers. Let us find the value of $\frac{.543}{.3}$. In order to determine the position of the decimal point in the answer, we shall begin by multiplying the numerator and denominator by 10 in order to change the denominator to a whole number. This should help us to determine the correct position for the decimal point.

$$\frac{.543}{.3} = \frac{.543 \times 10}{.3 \times 10}$$

$$= \frac{5.43}{3}$$

$$\begin{array}{r} 181 \\ 3\,\overline{)543} \\ \underline{3} \\ 24 \\ \underline{24} \\ 03 \\ \underline{3} \end{array}$$

We have 181 as the digits in the answer, but where does the decimal point go? Since we are dividing a number over 5 (5.43) by the number 3, the answer should be between 1 and 2. Therefore, in 181, the decimal point must go after the 1. Therefore:

$$\frac{.543}{.3} = 1.81$$

Note that in this example we multiplied by 10 so that the denominator .3 will become the whole number 3. However, we also had to multiply .543 by 10. Now, how does this affect the decimal point placement if we use long division?

```
.3  √.543
```

When we multiplied by 10 we changed .3 to 3 and .543 to 5.43 . In the long division form we could accomplish this by moving the decimal point one place for both numbers .3 and .543 . This will then determine the correct position for the decimal point in the answer.

```
         1.81
.3.  √.5.43
        3
        2 4
        2 4
          03
           3
```

This illustration suggests that in the long division of decimals, we think of moving the decimal point in the divisor (.3 in our example) so that the divisor is a whole number. We then move the decimal point in the dividend (.543 in the example) the same number of places. This will position the decimal point correctly for the quotient (1.81 in the example).

Example 5. Find the value of $\frac{.9375}{2.5}$

```
             .375
2.5.  √.9.375
         7 5
         1 87
         1 75
           125
           125
```

Example 6. Find the value of $\frac{.8625}{.345}$

```
           2.5
.345   √.862 5
         690
         172 5
         172 5
```

Example 7. Find the value of $\frac{20}{.34}$.

```
         58.823529  etc.
.34   √20.00 0000000
       17 0
        3 00
        2 72
          28 0
          27 2
             80
             68
             120
             102
              180
              170
               100
                68
                320
                306
                 140
```

Therefore:

$\frac{20}{.34}$ = 58.8235 (correct to four decimal places)

EXERCISES

Find the value in each of the following problems.

1. $.4 \times .8 =$

2. $.23 \times .41 =$

3. $17 \times 3.4 =$

4. $321 \times .61 =$

5. $4.2 \times 7.3 =$

6. $\frac{.2}{.1} =$

7. $\frac{.84}{2.1} =$

8. $\frac{4.62}{2.3} =$

9. $\frac{87.3}{45.2} =$

10. $\frac{3.214}{22} =$

ANSWERS

1. .32
2. .0943
3. 57.8
4. 195.81
5. 30.66
6. 2
7. .4
8. 2.0087 (to the nearest ten thousandth)
9. 1.9314 (to the nearest ten thousandth)
10. .1461 (to the nearest ten thousandth)

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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Equivalent Fractional and Decimal Names

INTRODUCTION AND/OR OBJECTIVES:

In most situations a chart is available for converting a decimal to a fraction or a fraction to a decimal. However, the student should be capable of changing a number in fractional form to its equivalent decimal form or from a decimal form to a fractional form in case a conversion chart is not available.

TECHNICAL INFORMATION:

Example 1. Convert $\frac{3}{8}$ to its decimal equivalent.

To convert from the factional form to the equivalent decimal form, we use long division.

$$\begin{array}{r} .375 \\ 8\,\overline{)3.0} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Therefore:

$\frac{3}{8} = .375$

Example 2. Convert $\frac{5}{16}$ to its decimal equivalent.

$$\begin{array}{r} .3125 \\ 16\,\overline{)5.0} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \end{array}$$

Therefore:

$$\frac{5}{16} = .3125$$

Example 3. Convert $\frac{1}{3}$ to its decimal equivalent.

$$\begin{array}{r} .3333 \\ 3\,\overline{)1.0} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \end{array}$$

Note that we continue to get 3's. Therefore, we present our answer correct to the number of decimal places desired.

Therefore:

$\frac{1}{3} = .333$ (correct to the nearest thousandth)

or $\frac{1}{3} = .3333$ (correct to the nearest ten thousandth)

Example 4. Convert .625 to its fractional equivalent.

$$.625 = \frac{625}{1000}$$

$$= \frac{25 \times 25}{25 \times 40}$$

$$= \frac{25}{40}$$

$$= \frac{5 \times 5}{8 \times 5}$$

$$= \frac{5}{8}$$

Example 5: Convert .125 to its fractional equivalent.

$$.125 = \frac{125}{1000}$$

$$= \frac{25 \times 5}{25 \times 40}$$

$$= \frac{5}{40}$$

$$= \frac{1 \times 5}{8 \times 5}$$

$$= \frac{1}{8}$$

Example 6: Convert .25 to its fractional equivalent.

$$.25 = \frac{25}{100}$$

$$= \frac{1 \times 25}{4 \times 25}$$

$$= \frac{1}{4}$$

EXERCISES

In problems 1 to 5 convert to the decimal equivalent.

1. $\frac{3}{5}$

2. $\frac{7}{16}$

3. $\frac{5}{8}$

4. $\frac{2}{3}$

5. $\frac{3}{32}$

In problems 6 to 10 convert to the fractional equivalent.

6. .500

7. .875

8. .125

9. .750

10. .0625

ANSWERS

1. .600

2. .4375

3. .625

4. .6667 (correct to the nearest ten thousandth)
   or .667 (correct to the nearest thousandth)

5. .09375
   or .094 (correct to the nearest thousandth)

6. $\frac{1}{2}$

7. $\frac{7}{8}$

8. $\frac{1}{8}$

9. $\frac{3}{4}$

10. $\frac{1}{16}$

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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Calculations Involving Approximate Numbers

INTRODUCTION:

If there are four people in a particular room, then the number of people is exactly four. However, when we measure something with a rule, micrometer, or a similar instrument, the value we get for the measurement is an approximation of the exact value. For example, we may measure something with an ordinary pair of micrometers and find the measurement is .376. We have indicated the measurement to the nearest thousandth. However, the actual exact measurement might be .37582, or it might be .37612. The measurement of .376 is an approximation which is accurate to the nearest thousandth. Note that there are three different digits in .376. We denote this by saying .376 has three significant digits.

A constant problem is the number of decimal places and the number of digits to include in answers which are the result of addition, subtraction, multiplication, and division involving approximate numbers.

OBJECTIVES:

1. To provide the student with an understanding of how to determine the number of significant digits in an approximate number.
2. To provide the student with an understanding of how to determine the number of decimal places and the number of significant digits which should be involved in the answer to a problem which includes addition, subtraction, multiplication, and division with approximate numbers.

TECHNICAL INFORMATION:

I. SIGNIFICANT DIGITS

First of all, we will call a digit a significant digit if it is known to be correct within the limits of the type of measurement used. For example, if we measure a piece of round stock and find that the diameter is .446 to the nearest thousandth of an inch, then there are three significant digits (4, 4, and 6) in the number .446.

Example 1. How many significant digits are there in each of the following numbers? 3.54, 4.3251, 2.001, 3.21, 3.210, .523, .21, and 321.251

	Given Approximate Number	Answer: Number of Significant Digits
a.	3.54	3
b.	4.3251	5
c.	2.001	4
d.	3.21	3
e.	3.210	4 (since the 0 placed on the end tells us that the measurement was to the nearest thousandth)
f.	.523	3
g.	.21	2
h.	321.251	6

Now, let us consider the approximate number .002 with regard to the number of significant digits. A temptation is to say that there are three significant digits. However, there is only 1 significant digit, namely 2. Notice that the number .002 is read as 2 thousandths. The two zeros in front of the 2 are merely decimal place holders and are not considered as significant digits.

Example 2. How many significant digits are in the following numbers? .032, .005, .132, 4.032, 52.001, 654.32, 654.320, .0123

	Given Approximate Number	Answer: Number of Significant Digits
a.	.032	2
b.	.005	1
c.	.132	3
d.	4.032	4 (since the 0 is preceded and followed by nonzero digits, the 0 is more than a place holder)

e.	52.001	5
f.	654.32	5
g.	654.320	6
h.	.0123	3

If the number 230 is an approximate number correct to the nearest 10, then there are only two significant digits in 230, namely 2 and 3. If 230 is measured to the nearest 1, then there are three significant digits, namely 2, 3, and 0. Therefore, we must know the accuracy of the measurement before we can exactly determine the number of significant digits in such numbers as 230, 3500, 234,00, etc. Normally, unless we know the exact method of measurement, we will indicate that 234,000, if it is an approximate number, has three significant digits, 372,500 has 4 significant digits, and 5,200,000 has 2 significant digits.

II. ADDITION AND SUBTRACTION OF APPROXIMATE NUMBERS

Suppose we wish to add 3.234 and 2.44 and we know that both of the numbers are approximate.

$$\begin{array}{r} 3.234 \\ +\ 2.44 \\ \hline 5.674 \end{array}$$

When we add, we get 4 digits in the answer. However, the number 2.44 is accurate only to the nearest hundredth. Therefore, we cannot expect the answer to be correct to the nearest thousandth. Thus, we should round the answer to the nearest hundredth, so that the answer will be 5.67 instead of 5.674. There are various ways of rounding numbers. Probably the most frequently used is to round up if the following digit is 5 to 9 and round down if the digit following is 0 to 4. Since the digit following 7 in this problem is 4 we round down to 7.

Suppose we add 5.21 and 3.747.

$$\begin{array}{r} 3.747 \\ +\ 5.21 \\ \hline 8.957 \end{array}$$

The answer should be rounded to the nearest hundredth since 5.21 is correct to only the nearest hundredth. How do we round 8.957 so that it is expressed as a number correct to the nearest hundredth (two decimal places)? That is, do we write the number as 8.95 or as 8.96? In this case we round up to 8.96 since the 7 in 8.957 is between 5 and 9. Therefore, we round up from 5 to 6 in the hundredths position in the answer.

A helpful concept to remember is that the number of decimal places in the answer of an addition (or subtraction) problem involving approximate numbers should be the same as the number of decimal places in the number in the original problem with the fewest number of decimal places.

Example 1. Add the approximate numbers 4.238,and 5.21.

$$\begin{array}{r} 4.238 \\ +\ 5.21 \\ \hline 9.448 \end{array}$$

We will round up to 9.45. (Note that the answer is now correct to the nearest hundredth as was the least accurate number 5.21 in the original problem.)

Answer: 9.45

Example 2. Add the approximate numbers 32.21 and 4.4.

$$\begin{array}{r} 32.21 \\ +\ 4.4 \\ \hline 36.61 \end{array}$$

We round down to 36.6.

Answer: 36.6

Subtraction will follow the same process as for addition.

Example 3. If 3.732 and 2.41 are approximate, find the value of 3.732 - 2.41.

$$\begin{array}{r} 3.732 \\ -\ 2.41 \\ \hline 1.322 \end{array}$$

We will round to 1.32.

Answer: 1.32

Example 4. If 5.7477 and 2.352 are approximate, find the value of 5.7477 - 2.352.

$$\begin{array}{r} 5.7477 \\ -\ \underline{2.352} \\ 3.3957 \end{array}$$

We will round to 3.396.

Answer: 3.396

III. MULTIPLICATION AND DIVISION OF APPROXIMATE NUMBERS

In multiplication and division we can expect the answer to have no larger number of significant digits than the number in the original problem with the least number of significant digits. Therefore, we examine the original problem to determine the least number of significant digits in any number in the original problem. Our answer will then be rounded to that number of significant digits.

Example 1. If 32.1 and 2.4 are approximate numbers, find the value of 32.1 x 2.4.

$$\begin{array}{r} 32.1 \\ \times\ \underline{2.4} \\ 1284 \\ \underline{642} \\ 77.04 \end{array}$$

Since 32.1 has 3 significant digits and 2.4 has 2 significant digits, our answer should have the smaller number, 2, of significant digits. Therefore, the answer should be rounded to 77.

Answer: 77

Example 2. If 3.45 and 4.321 are both approximate numbers, find the value of 3.45 x 4.321.

$$\begin{array}{r} 3.45 \\ \times\ \underline{4.321} \\ 345 \\ 690 \\ 1035 \\ \underline{1380} \\ 14.90745 \end{array}$$

Since 3.45 has only 3 significant digits, the answer should have only 3 significant digits. Therefore, the answer should be rounded to 14.9.

Answer: 14.9

Example 3. If 3.34 and 2.2 are approximate numbers, find the value of $\frac{3.34}{2.2}$.

```
         1.51 etc.
  2.2 √3.34
       2 2
       1 14
       1 10
          40
          22
         180
```

Since 2.2 has only 2 significant digits, the answer should have only 2 significant digits. Therefore, we round 1.51 to 1.5.

Answer: 1.5

Example 4. If 4.321 and 3.45 are approximate numbers, find the value of $\frac{4.321}{3.45}$.

```
          1.252 etc.
  3.45 √4.321
        3 45
          871
          690
         1810
         1725
           850
           690
```

Since 3.45 has only three significant digits, the answer should have only 3 significant digits. Therefore, the answer should be rounded to 1.25.

Answer: 1.25

IV. CALCULATIONS WITH BOTH EXACT AND APPROXIMATE NUMBERS

If one or more numbers are exact and one or more are approximate, then in determining the number of decimal places and the number of significant digits in the answer, we need only to consider the decimal places and significant digits in the approximate numbers. This is true since the exact number is not restricted as to decimal place or significant digit accuracy.

Example 1. Suppose 7 is exact and 2.32 is approximate, find:

a. $7 + 2.32$

b. $7 - 2.32$

c. 7×2.32

d. $\frac{7}{2.32}$

Since 7 is exact we may add zeros if it is helpful in the problem.

For part a:

$$\begin{array}{r} 7.00 \\ +\ 2.32 \\ \hline 9.32 \end{array}$$

Since 2.32 is accurate to the nearest hundredth (two decimal places), then the answer should be correct to the nearest hundredth.

Answer: 9.32

For part b:

$$\begin{array}{r} 7.00 \\ -\ 2.32 \\ \hline 4.68 \end{array}$$

Answer: 4.68

For part c:

$$\begin{array}{r} 2.32 \\ \times\quad 7 \\ \hline 16.24 \end{array}$$

Since 2.32 has 3 significant digits, the answer should have three significant digits. Therefore, the answer should be rounded to 16.2.

Answer: 16.2

For part d:

$$\begin{array}{r} 3.017\ \text{etc.} \\ 2.32\overline{)7.0000} \\ 6\ 96 \\ \hline 400 \\ 232 \\ \hline 1680 \\ 1624 \\ \hline \end{array}$$

Since 2.32 has 3 significant digits, the answer should have three significant digits. Therefore, the answer should be rounded to 3.02.

Answer: 3.02

Since the methods of handling the number of decimal places and significant digits depends on a knowledge of how the numbers were obtained (with regard to the accuracy), the constant checking of significant digits will not be stressed in this book. However, in dealing with measurements performed by the students, the knowledge of the accuracy of numbers will allow utilization of the methods in this section. Also, the teacher might wish to indicate the accuracy of particular numbers in various assignments presented in this book.

PROBLEMS

1. Indicate the number of significant digits in each of the following numbers:

   a. 35.1
   b. .021
   c. .201
   d. 36.223
   e. 2.2
   f. 2.20
   g. 3.001

2. Carry out the indicated operations involving the given approximate numbers and round your answers so that the correct number of digits appears in the answer.

   a. 35.221 + 7.25
   b. 32.238 + 5.23
   c. 14.221 + 2.32 + 3.21
   d. 31.271 - 3.14
   e. 2.778 - 1.314
   f. 9.3248 - 7.217
   g. 3.421 x 2.3
   h. 4.32 x 7.41
   i. 1.221 x 2.25
   j. 5.22/3.1
   k. 7.223/3.21
   l. 5.22/.331

3. If we know that 5 is an exact number and 2.31 is an approximate number, find each of the following:

   a. 5 + 2.31
   b. 5 - 2.31
   c. 5 x 2.31
   d. 5/2.31

ANSWERS

1.

a. 3
b. 2
c. 3
d. 5
e. 2
f. 3
g. 4

2.

a. 42.47
b. 37.47
c. 19.75
d. 28.13
e. 1.464
f. 2.108
g. 7.9
h. 32.0
i. 2.75
j. 1.7
k. 2.25
l. 15.8

3.

a. 7.31
b. 2.69
c. 11.6
d. 2.16

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL INFORMATION TITLE: Reading and Calculations--Rule

INTRODUCTION:

The student needs to be able to read various rules and interpret the values. The rule is the most common measuring tool, and skill depends on the ability to read it as well as to solve problems involving fractions.

OBJECTIVES:

1. To learn the values of the increments of a rule.
2. To learn how to add, subtract, and divide using fractions on a rule.
3. To learn various applications of a rule.

TECHNICAL INFORMATION:

In Figure I note that each unit of the rule is divided into 32 equal divisions. Therefore, each space or subdivision represents 1/32 of an inch. Thus, the reading on the rule in Figure I is $3 + \frac{3}{32} = 3\frac{3}{32}$ inches.

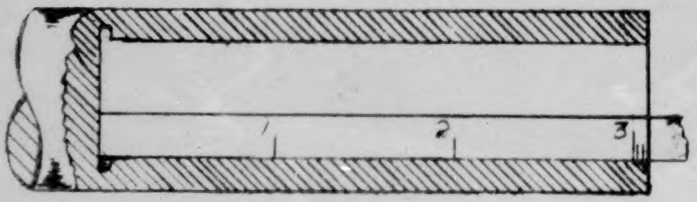


Figure I

APPLICATION OF THE RULE:

Example 1. Solve for dimension B. (See Figure 2)

From the figure:

$$B + \frac{13}{32} = 1\frac{7}{16}$$

Therefore:

$$B = 1\frac{7}{16} - \frac{13}{32}$$

Vertically, the solution can be found as follows:

$$\begin{array}{rcr} 1\frac{7}{16} & 1 + \frac{7}{16} & 1 + \frac{14}{32} \\ - \quad \frac{13}{32} & - \quad \frac{13}{32} & - \quad \frac{13}{32} \\ \hline & & 1 + \frac{1}{32} = 1\frac{1}{32} \end{array}$$

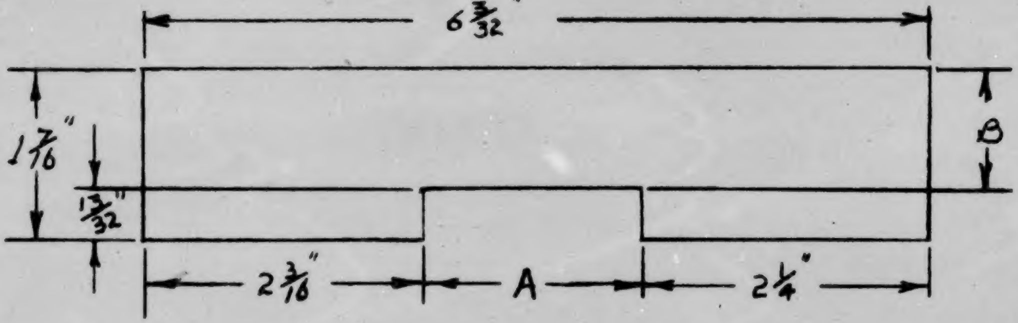


Figure 2

Example 2. Solve for dimension A. (See Figure 2)

From the figure:

$$2\frac{3}{16} + 2\frac{1}{4} + A = 6\frac{3}{32}$$

Therefore:

$$A = 6\frac{3}{32} - (2\frac{5}{16} + 2\frac{1}{4})$$

Find:

$$2\frac{5}{16} + 2\frac{1}{4}$$

$$\begin{array}{r} 2\frac{5}{16} \\ +\ 2\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} 2 + \frac{5}{16} \\ +\ 2 + \frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{l} \ \ 2 + \frac{5}{16} \\ +\ 2 + \frac{4}{16} \\ \hline \ \ 4 + \frac{9}{16} = 4\frac{9}{16} \end{array}$$

Then:

$$A = 6\frac{3}{32} - 4\frac{9}{16}$$

$$\begin{array}{r} 6\frac{3}{32} \\ -\ 4\frac{9}{16} \\ \hline \end{array} \qquad \begin{array}{r} 6 + \frac{3}{32} \\ -4 - \frac{9}{16} \\ \hline \end{array} \qquad \begin{array}{r} 6 + \frac{3}{32} \\ -4 - \frac{18}{32} \\ \hline \end{array} \qquad \begin{array}{r} 5 + 1 + \frac{3}{32} \\ -4 \qquad - \frac{18}{32} \\ \hline \end{array} \qquad \begin{array}{r} 5 + \frac{32}{32} + \frac{3}{32} \\ -4 \qquad\quad - \frac{18}{32} \\ \hline \end{array}$$

$$\begin{array}{l} \ \ 5 + \frac{35}{32} \\ -4 - \frac{18}{32} \\ \hline \ \ 1 + \frac{17}{32} = 1\frac{17}{32} \end{array}$$

Example 3. In estimating the stock to make a high speed plain mill cutter, a machinist must allow 9/16 inch for each cutter. How many cutters can be made from a piece of stock 9 1/8 inches long? See Figure 3.

This becomes a problem of dividing $9\frac{1}{8}$ by $\frac{9}{16}$.

$$9\frac{1}{8} = 9 + \frac{1}{8} = \frac{72}{8} + \frac{1}{8} = \frac{73}{8}$$

Therefore:

$$\frac{9\frac{1}{8}}{\frac{9}{16}} = \frac{\frac{73}{8}}{\frac{9}{16}}$$

$$= \frac{73}{8} \cdot \frac{16}{9}$$

$$= \frac{73 \cdot 16}{8 \cdot 9}$$

$$= \frac{73 \cdot 8 \cdot 2}{8 \cdot 9}$$

$$= \frac{73 \cdot 2}{9}$$

$$= \frac{146}{9}$$

$$= 16\frac{2}{9}$$

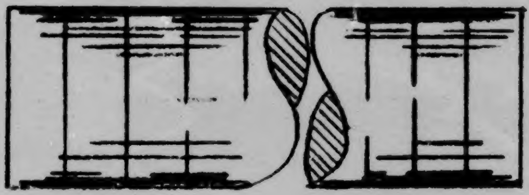


Figure 3

Example 4. If each arbor spacer in Figure 4 measures 2 1/64 inches, what is the total length if 6 spacers are used on an arbor?

This problem becomes that of multiplying $2\frac{1}{64}$ by 6.

$$6(2\frac{1}{64}) = 6(2 + \frac{1}{64}) = 6 \cdot 2 + 6 \cdot \frac{1}{64} = 12 + \frac{6}{64} = 12 + \frac{3}{32} = 12\frac{3}{32}$$

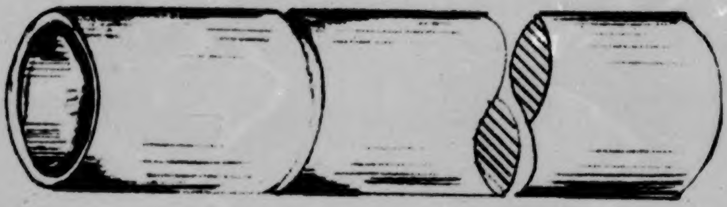


Figure 4

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Calculations Involving Rational Numbers (Fractions)

INTRODUCTION:

Calculations involving addition, subtraction, multiplication, and division of proper fractions, improper fractions, and mixed numerals is an everyday occurance in the machine trades. Much practice is necessary so that proficiency in these operations is attained.

OBJECTIVE:

To provide the student practice in calculations involving rational numbers (fractions).

ASSIGNMENT:

Addition

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} =$
2. $\frac{3}{32} + \frac{5}{16} + \frac{3}{4} =$
3. $\frac{1}{32} + \frac{3}{64} =$
4. $\frac{5}{24} + \frac{5}{12} + \frac{9}{16} =$
5. $\frac{7}{40} + \frac{1}{2} + \frac{31}{32} =$
6. $\frac{15}{16} + \frac{1}{12} + \frac{5}{8} =$
7. Find the length of the bolt in Figure I.

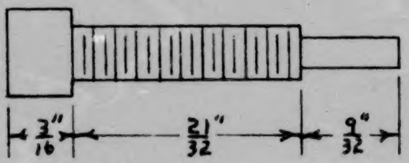

3/16"
21/32"
9/32"

Figure 1

8. $5\frac{1}{4} + 7\frac{3}{4} + 2\frac{7}{8} =$

9. $1\frac{11}{16} + 2\frac{5}{8} + 3\frac{1}{2} =$

10. $1\frac{7}{8} + 3\frac{11}{32} + 2\frac{3}{8} =$

11. Find the value for dimension A in Figure 2.

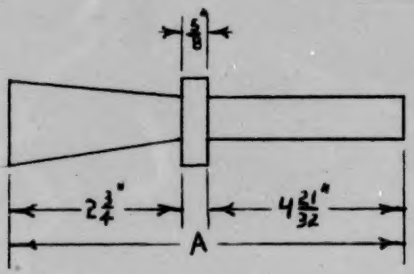

5/8"
2 3/4"
4 21/32"
A

Figure 2

Subtraction

12. $\frac{39}{64} - \frac{3}{8} =$

13. $\frac{27}{40} - \frac{3}{8} =$

14. $\frac{5}{12} - \frac{9}{32} =$

15. Find the value of dimension A in Figure 3.

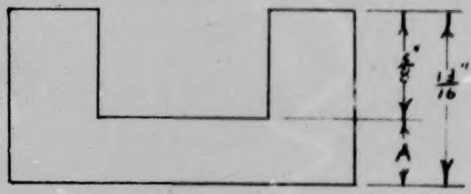


Figure 3

16. $7\frac{5}{8} - 2\frac{7}{16} =$

17. $9\frac{15}{32} - 4\frac{7}{8} =$

18. Find the value of dimension C in Figure 4.

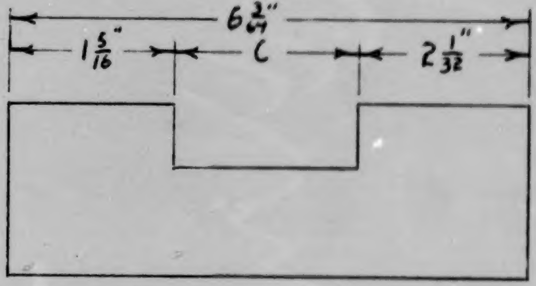


Figure 4

19. $\frac{7}{8} \times \frac{2}{3} =$

20. $6\frac{1}{2} \times 5\frac{7}{8} =$

21. $7\frac{1}{12} \times 3\frac{3}{8} \times 8\frac{3}{16} =$

Division

22. $\frac{7}{8} \div \frac{3}{4} =$

23. $\dfrac{\frac{3}{16}}{\frac{7}{12}} =$

24. $\dfrac{15}{\frac{1}{2}} =$

25. Find the value of D, the distance between the centers of the two holes in Figure 5.

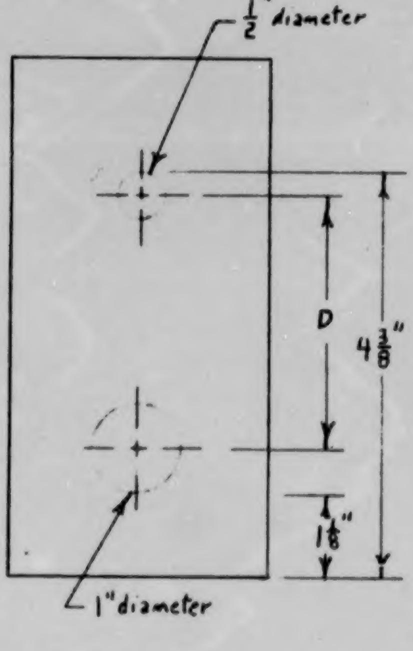

½" diameter
D
4⅜"
1⅛"
1" diameter

Figure 5

ANSWERS

1. $\frac{77}{60}$ or $1\frac{17}{60}$
2. $\frac{37}{32}$ or $1\frac{5}{32}$
3. $\frac{5}{64}$
4. $\frac{57}{48} = \frac{19}{16}$ or $1\frac{3}{16}$
5. $\frac{263}{160}$ or $1\frac{103}{160}$
6. $\frac{79}{48}$ or $1\frac{31}{48}$
7. $\frac{36}{32} = \frac{9}{8}$ or $1\frac{1}{8}$
8. $15\frac{7}{8}$
9. $7\frac{13}{16}$
10. $7\frac{19}{32}$
11. $8\frac{1}{32}$
12. $\frac{15}{64}$
13. $\frac{12}{40} = \frac{3}{10}$
14. $\frac{13}{96}$
15. $\frac{3}{16}$
16. $5\frac{3}{16}$
17. $4\frac{19}{32}$
18. $2\frac{45}{64}$
19. $\frac{7}{12}$
20. $38\frac{3}{16}$
21. $195\frac{375}{512}$
22. $\frac{7}{6}$ or $1\frac{1}{6}$
23. $\frac{9}{28}$
24. 30
25. $2\frac{1}{2}$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL INFORMATION TITLE: Reading and Calculations--Micrometer

INTRODUCTION:

The use of micrometers is an essential skill for toolmakers, machinists, machine operators, and inspectors. Micrometers are made for measuring inside and outside dimensions as well as depth. They range in various sizes and have many applications. The basic components of a micrometer consists of the frame, anvil, spindle, sleeve, thimble, lock nut, and ratchet. See Figure I below.

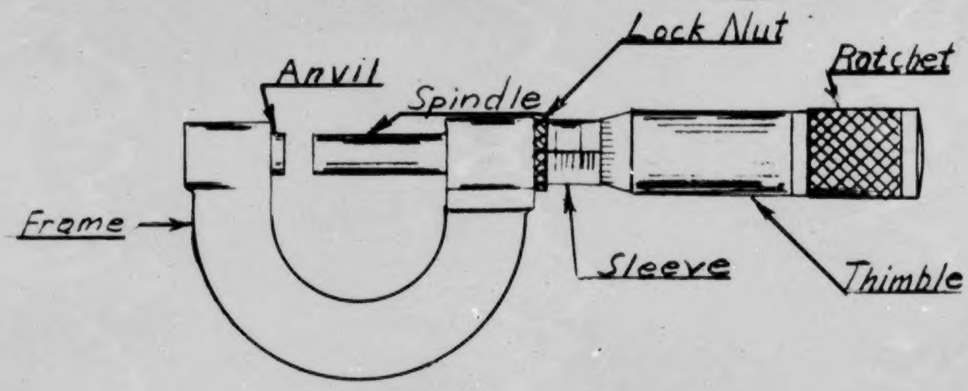


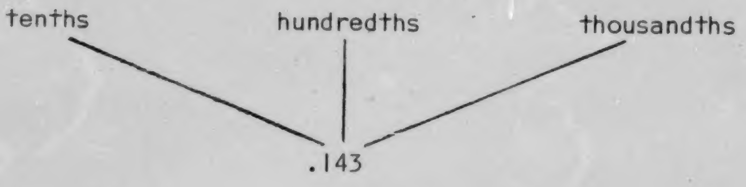
Figure I

OBJECTIVES:

1. To provide the student an opportunity to learn the technical terms and nomenclature of a micrometer.
2. To provide the student an opportunity to learn how to read the decimal values on a micrometer.
3. To provide the student an opportunity to learn the application of a micrometer.

TECHNICAL INFORMATION:

It should be noted that the micrometer readings are specified in terms of decimals. When working with tenths, hundredths, and thousandths of an inch in measurements, the results may be easily expressed using decimals. In the number .143, the 1 represents tenths of an inch, the 4 represents hundredths of an inch, and the 3 represents thousandths.



The major divisions on the sleeve of a micrometer are tenths of an inch. Thus, 3 on the sleeve represents $\frac{3"}{10}$ or .3". The smaller marks on the sleeve represent divisions of 25 thousandths of an inch or using decimals, .025". The thimble markings represent thousandths of an inch. Thus, 10 on the thimble represents 10 thousandths or .010.

Example 1. Read and record the micrometer reading in Figure 2.

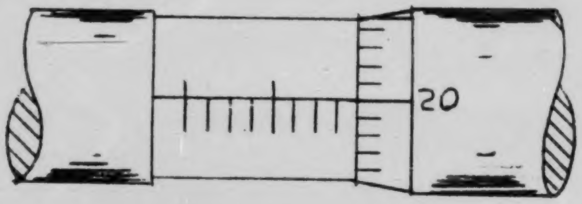


Figure 2

Step 1.	Read whole number on sleeve as 1.	.100
Step 2.	Read divisions of .025 as 3. 3(.025)	.075
Step 3.	Read thimble as 20.	.020
Step 4.	Add.	.195

Thus, the micrometer reading is .195".

Example 2. (Measuring the American National Form Thread with Micrometer and 3-Wire Method)

Find the measurement over the wires of a 1" - 8 NC Thread. See Figure 3.

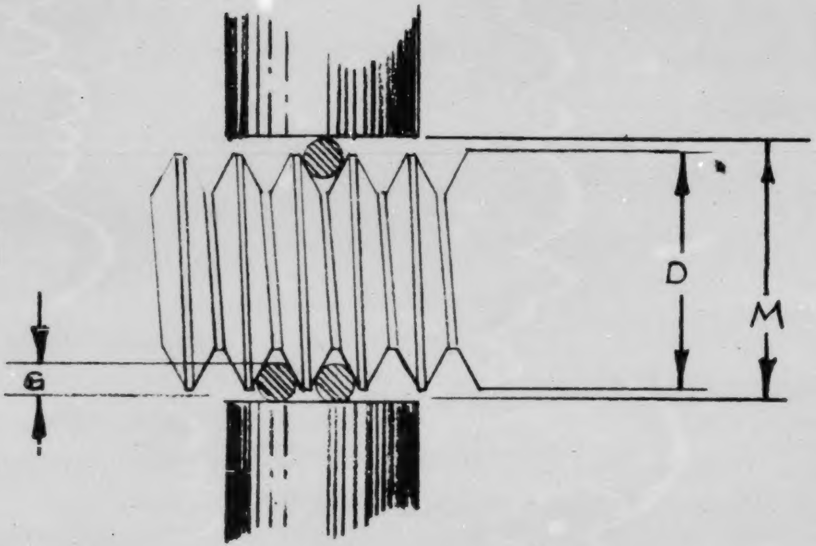


Figure 3

We use the following formulas:

$$M = D + 3G - \frac{1.5155}{N}$$

$$G = \frac{.57735}{N}$$

In these formulas:

G = Diameter of the wire
D = Major diameter of the screw (1" in this example)
M = Measurement over the wires
N = Number of threads per inch (8 in this example)

We first need to find the value for G.

$$G = \frac{.57735}{N}$$

$$= \frac{.57735}{8}$$

$$= .07217$$

Then, we may find M.

$$M = D + 3G - \frac{1.5155}{N}$$

$$= 1 + 3(.07217) - \frac{1.5155}{8}$$

$$= 1 + .2165 - .1894$$

$$= 1.0271 \text{ or } 1.027\text{"} \text{ (to the nearest thousandth)}$$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Calculations Involving Decimal Numbers

INTRODUCTION:

It is very important to know how to add, subtract, multiply, and divide with decimal numbers. This is especially true when working with micrometers or other precision measuring instruments and in computing dimensions from a print.

OBJECTIVE:

To learn how to add, subtract, multiply, and divide decimal numbers.

ASSIGNMENT:

Addition

1. 72.05 + 9.638 + 0.432 = ______
2. 0.0002 + 0.584 + 384.4832 = ______
3. 8.0002 + 6.44480 + .0000032 = ______
4. .358 + 4.235 + 7.534 + 6.281 = ______

Subtraction

5. 11.00421 - 7.02342 = ______
6. 1.0002 - 0.83245 = ______

Multiplication

7. 8.24 x 1.003 = ______
8. 6.725 x 3.117 = ______
9. 10.700 x 3.402 = ______

10. $1.045 \times 7.25 =$ ______

Division

11. $\frac{9.24}{.004} =$ ______

12. $\frac{44.7}{10.108} =$ ______

13. $\frac{.426}{2.05} =$ ______

14. $\frac{438}{.0001} =$ ______

15. In Figure 1, find:

    a. D
    b. E

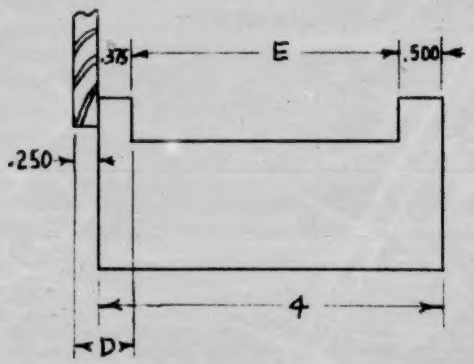

.375
E
.500
.250
4
D

Figure 1

16. In Figure 2, find:

    a. R
    b. C

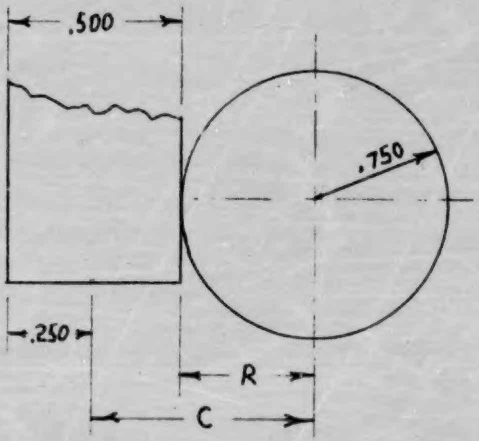

.500
.750
.250
R
C

Figure 2

ANSWERS

1. 82.120
2. 385.0674
3. 14.4450032
4. 18.408
5. 3.98079
6. .16775
7. 8.26472
8. 20.961825
9. 36.4014
10. 7.57625
11. 2310
12. 4.4222
13. .2078
14. 4,380,000
15. D = .625

    E = 3.125
16. R = .750

    C = 1.000

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: How to read a micrometer

COURSE UNIT TITLE: Precision Measurements--Micrometer

INTRODUCTION:

Micrometers are one of the most important instruments for measuring dimensions. A micrometer consists of a frame to which is fixed a barrel or sleeve. On the inside of the sleeve is a spindle, and outside of the sleeve is the thimble.

OBJECTIVE:

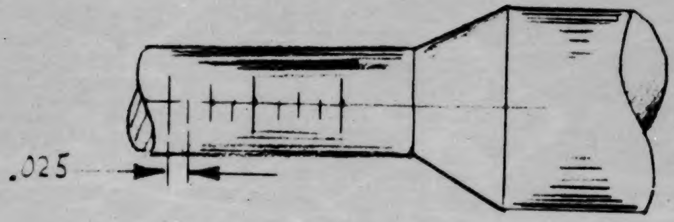
To provide the student an opportunity to learn how to read a micrometer.

TOOLS AND MATERIALS REQUIRED:

1" micrometer
Round steel less than an inch in diameter

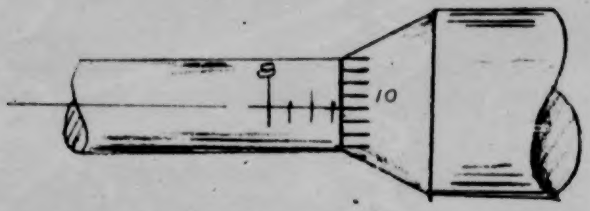
PROCEDURE:

(Operation)	(Related Information)
1. Record reading on sleeve.	1. Each interval is .025.



2. Record reading on thimble.

2. Each interval equals .001; one revolution of the thimble is equal to .025.



3. Add above results. (Refer to Technical Information Sheet)

3. Results equal the dimension.

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL INFORMATION TITLE: Reading and Calculations--Vernier Calipers

INTRODUCTION:

The verniers are used for measuring various dimensions such as outside and inside diameters, depths, and heights. They can be used to find the dimensions of centers of drilled holes when combined with a dial-indicator. Also, they can be used for layout work.

OBJECTIVES:

1. To provide the student an opportunity to learn how to read and interpret the values of the reading on a vernier.
2. To provide the student an opportunity to learn the various applications of vernier calipers.
3. To provide the student an opportunity to learn the terms and nomenclature associated with vernier calipers.

TECHNICAL INFORMATION:

There are three different divisions used on vernier calipers: one with 20 divisions, one with 25 divisions, and one with 50 divisions.

The vernier calipers normally used are those with 25 divisions. The bar on this tool is graduated in 40ths or .025 of an inch. Every fourth division, thus, represents a tenth of an inch, and is numbered. The vernier plate is divided into 25 divisions numbered: 0, 5, 10, 15, 20, and 25. The 25 divisions on the vernier plate occupy the same space as 24 divisions on the bar.

Since one division on the bar equals .025 inch, 24 divisions equal 24 x .025 inch or .600 inch, and 25 divisions on the vernier plate also equal .600 inch. Therefore, one division on the vernier plate equals $\frac{1}{25} \times .600$ inch or .024 inch. The difference between one bar division (.025 and one vernier plate division (.024) equals .025 inch minus .024 inch or .001 inch.

Example 1. Find the reading on the vernier in Figure 1.

Solution: The vernier plate has been moved to the right 1.000 plus .400 plus .025 which equals 1.425 inches as shown on the bar. The eleventh line on the vernier plate coincides with a line on the bar, as indicated by the arrow. Therefore, .011 inch is to be added to the reading on the bar. The total reading is 1.436 inches.

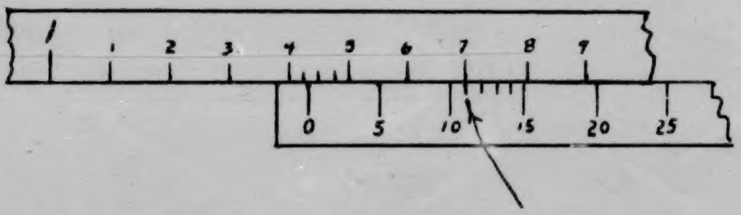


Figure 1

1. FINDING AN AVERAGE

Given two read numbers a and b (See Figure 2), the average of a and b is defined as $\frac{a + b}{2}$. Note that in Figure 2, $\frac{a + b}{2}$ is midway between a and b on the number line.

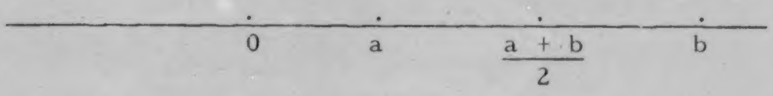


Figure 2

For three real numbers a, b, and c, the average is $\frac{a + b + c}{3}$.

The average of n numbers = $\frac{\text{sum of the n numbers}}{n}$.

Example 2. Find the average of the six readings of outside diameters in Figure 3.

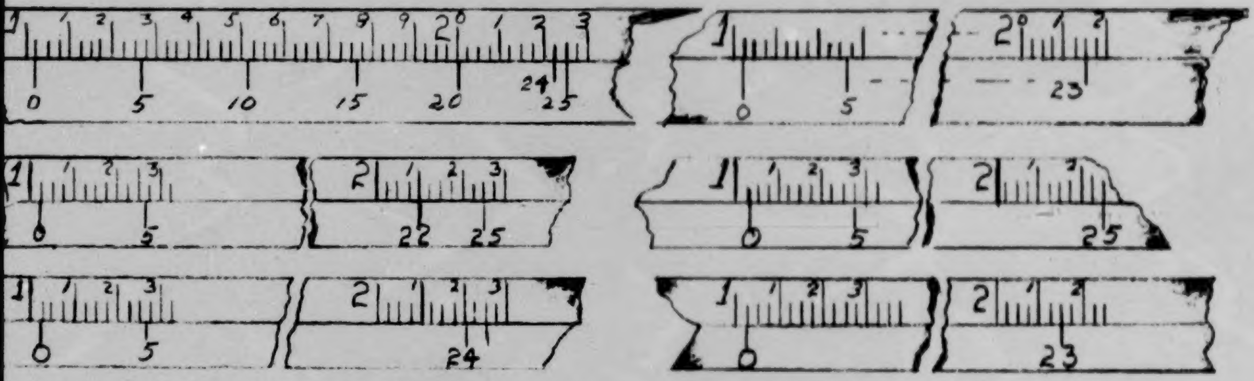


Figure 3

The six readings are: 1.024
1.023
1.022
1.025
1.024
1.023

$$\text{Average} = \frac{\text{sum of the 6 numbers}}{6}$$

$$= \frac{1.024 + 1.023 + 1.022 + 1.025 + 1.024 + 1.023}{6}$$

$$= \frac{6.141}{6}$$

= 1.0235 or 1.024 (Rounding to to the nearest thousandth)

II. MEASURING INSIDE DIAMETERS

If the specifications indicate a dimension of 1.003", and the actual caliper reading is 1.005", what is the difference? See Figure 4.

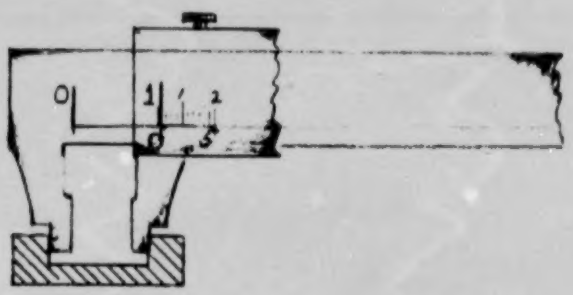


Figure 4

Difference = 1.005" - 1.003"

= .002" (2 thousandths of an inch)

III. MEASURING OUTSIDE DIAMETERS

To find the allowance to be allowed for grinding multiply the diameter by .004 . See Figure 5. If we wish to have a diameter of .875 for the outside diameter, what should be the allowance for grinding?

Allowance for grinding = (.875) x (.004)

= .0035" or .004"

Therefore,

D = .875 + .0035

= .875 + .004

= .879"

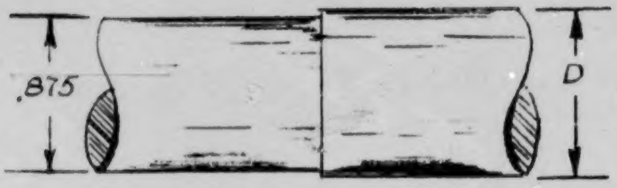


Figure 5

IV. MEASURING DISTANCES BETWEEN TWO HOLES OF A PLATE

In Figure 6, one hole has a dimater of 1.500" and the other has a diameter of .750". Find the value of dimension A. Use a vernier height gage and a dial indicator.

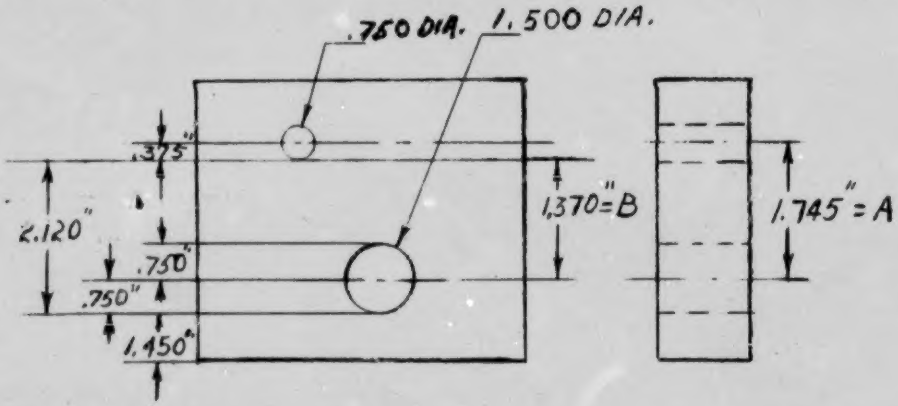

.750 DIA.
1.500 DIA.
.375"
2.120"
.750"
.750"
1.450"
1.370"=B
1.745"=A

Figure 6

First of all, find B

B = 2.120" - .750"

= 1.370"

Then, find A.

A = 1.370" + .375"

= 1.745"

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

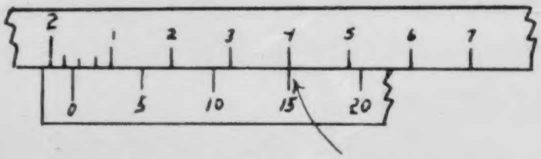
TECHNICAL ASSIGNMENT TITLE: Reading and Calculations--Vernier Calipers

INTRODUCTION:
Problems involving vernier calipers are similar to those involving micrometers. These problems involve aligning marks on the scales and the addition of decimals to determine the correct measurement.

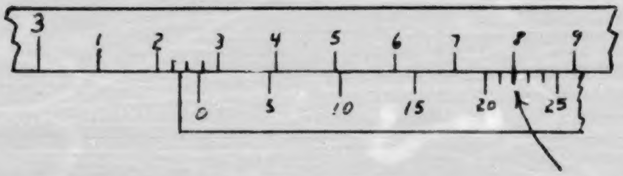
OBJECTIVES:
To provide the student practice in problems involving vernier calipers, including the addition of decimal numbers and the averaging of readings.

TECHNICAL ASSIGNMENT:

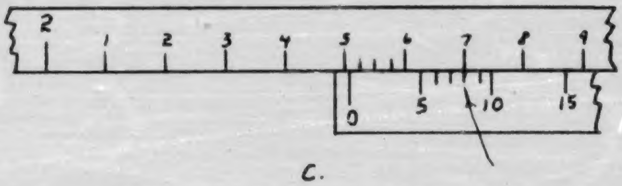
1. Find the following values:
    - a. 2.043 + 0.005 =
    - b. .483 + 6.002 + .002 =
    - c. 2.000 + .300 + .050 + .010 =
    - d. 5.000 + .400 + .075 + .012 =
    - e. 6.000 + .800 + .025 + .005 =
    - f. 9.000 + .002 =
2. Find the average of the following five readings: 2.325, 2.324, 2.326, 2.325, 2.324.
3. If we multiply the diameter by .004 to determine the allowance for grinding, and the final dimater is to be .925", what should be the diameter before grinding?
4. Determine the following vernier caliper readings:



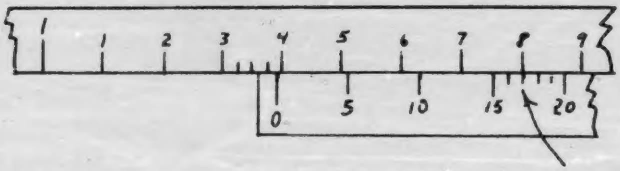
a.



b.



c.



d.

ANSWERS

1. a. 2.048
   b. 6.487
   c. 2.360
   d. 5.487
   e. 6.830
   f. 9.002

2. 2.325

3. .929"

4. a. 2.040"
   b. 3.272"
   c. 2.512"
   d. 1.392"

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Setting up and reading the vernier calipers

COURSE UNIT TITLE: Precision Measurements--Vernier Calipers

INTRODUCTION:

The vernier calipers are used both in the machine trade and in the diemaking trade where fine and exacting work is required. The advantage is that the range of length for one tool may be from 6" to 12" or even greater with extensions or rods of any kind. It may be applied for measuring O.D., I.D., and height.

OBJECTIVE:

To provide the student an opportunity to learn how to read and set up a vernier more accurately.

TOOLS AND MATERIALS REQUIRED:

Plate with holes
Vernier height gage
O.D. and I.D. vernier calipers

PROCEDURE:

(Operation)	(Related Information)
1. Record the reading on the main scale A that includes all the whole divisions up to the 0 of the vernier. See Figure 1.	1. Usually, the interval is .025 on the main scale A; however, on some verniers it may be .050 or .020.
2. Find the line for the vernier scale B in Figure 1 that coincides with a line of the main scale.	2. Choose the one that is the most coincidental.
3. Count the number of spaces on the vernier from this line to the zero line.	3. Multiply this number by .001 inch.
4. Add the results recorded in step 1 to the results recorded in step 3.	4. The sum of the results of step 1 and step 3 is the answer. See Technical Information Sheet.

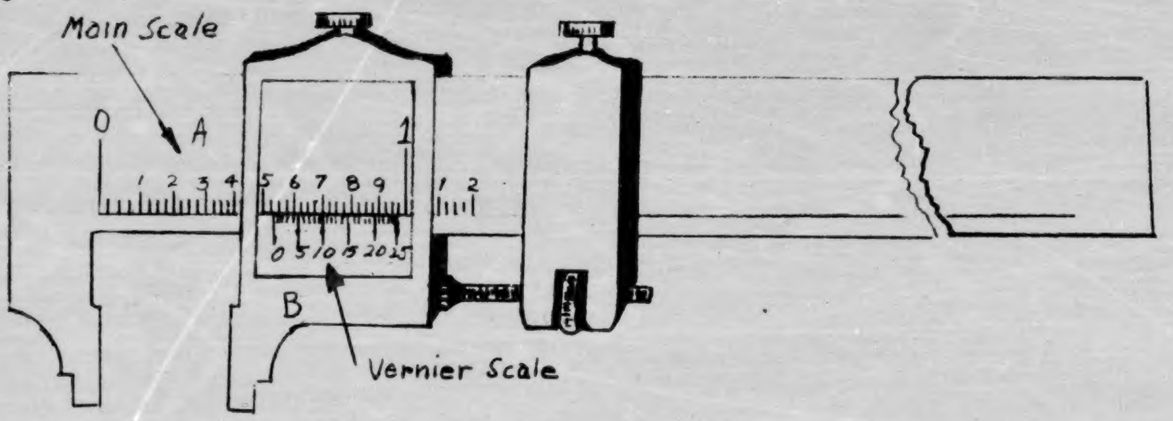

Main Scale
0
A
1
1 2 3 4 5 6 7 8 9 1 2
0 5 10 15 20 25
B
Vernier Scale

Figure 1

Step 1	.525	Main scale
Step 3	.010	Vernier scale
Step 4	.535	Answer

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL INFORMATION TITLE: Vernier Micrometers

INTRODUCTION:

On one-thousandth micrometers readings can be determined to the nearest thousandth of an inch. On vernier micrometers it is possible to approximate the readings to the nearest ten-thousandth of an inch.

OBJECTIVE:

To provide the student with information regarding the reading of a vernier micrometer.

TECHNICAL INFORMATION:

The vernier divisions on a vernier micrometer are located on the sleeve. In reading the ten-thousandth micrometer, we first determine the measurement to the nearest one-thousandth, as with a one-thousandth micrometer. Then we observe which of the lines on the vernier is aligned with a line on the thimble. The number of the vernier line which is aligned is the number of ten-thousandths in the measurement. If the 1 on the vernier is aligned, we add one ten-thousandth (0.0001) to the reading. If the 2 is aligned, we add 0.0002, etc.

Example 1. Find the reading on the micrometers in Figure 1.

$$.3 + .075 + .017 + .0006 = .3929$$

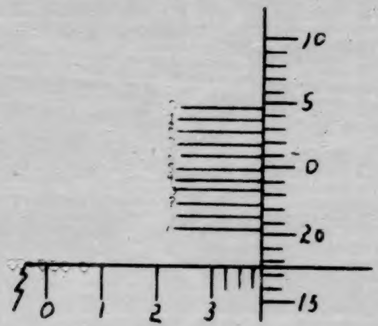
10
5
0
20
15
0
1
2
3

Figure 1

Example 2. Find the reading on the micrometer in Figure 2.

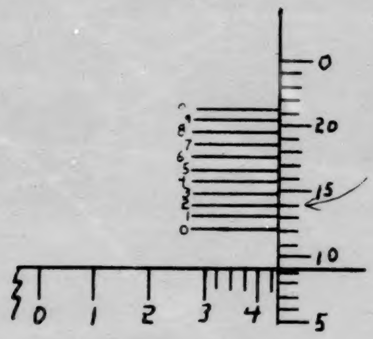
0
20
15
10
5
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4

Figure 2

.4 + .025 + .009 + .0002 = .4342

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

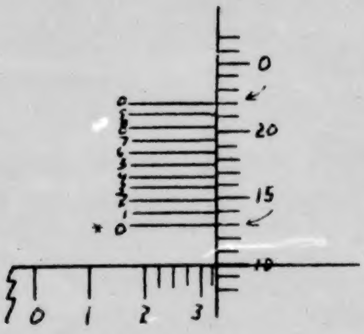
TECHNICAL ASSIGNMENT TITLE: Vernier Micrometers

INTRODUCTION:
On vernier micrometers it is possible to approximate readings to the nearest ten-thousandth of an inch. This allows very accurate readings of measurements.

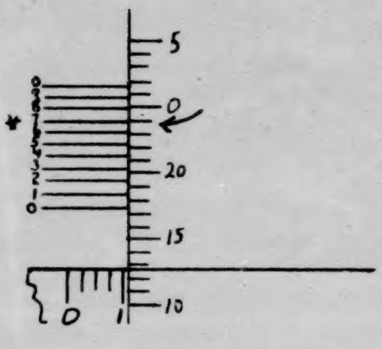
OBJECTIVE:
To provide the student practice in reading vernier micrometers.

TECHNICAL ASSIGNMENT:

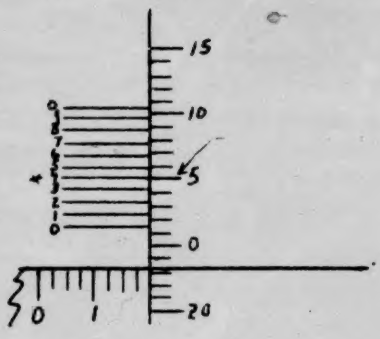
Determine the readings on the following micrometers.



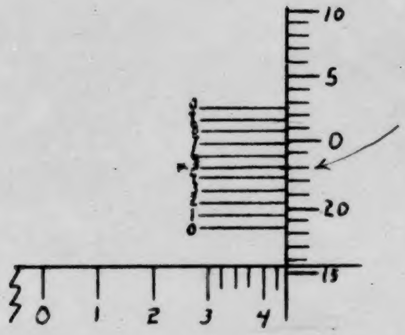
A


5
0
20
15
10
0
1

B


15
10
5
0
20
0
1

C


10
5
0
20
15
0
1
2
3
4

D

ANSWERS

A: .3350

B: .1127

C: .1984

D: .4405

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precisions Measurements

TECHNICAL INFORMATION TITLE: Calculations with Angles--Protractor

INTRODUCTION:

Interpreting the value of the reading of a protractor is very important. This involves an understanding of the units used in angular measure. Also, working with angles implies that the student should be familiar with the procedures used in addition, subtraction, and division with angles.

A toolmaker, machinist, machine operator, or an inspector must decide whether he is reading the value off the side of a workpiece or whether he is reading the value in terms of an included angle. See Figure I and Figure 2.

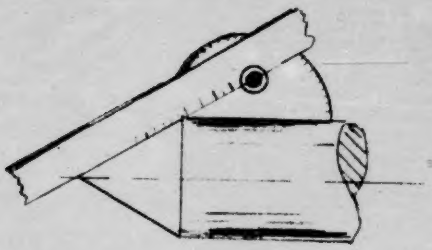


Figure I

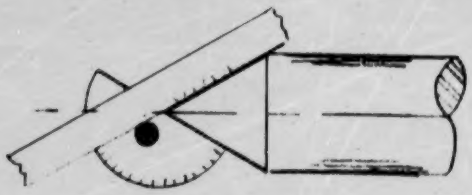


Figure 2

OBJECTIVES:

1. To provide the student a background in the basic concepts of angular measure. This is very important as a basis for the use of the protractor.

2. To provide the student a background in the techniques used in problems involving addition, subtraction, multiplication, and division with angles.

TECHNICAL INFORMATION:

Consider the circle in Figure 3. One method of finding the measure of an angle is with the use of degrees, minutes, and seconds. If the circle is divided into 360 angles with equal measures, then each of the angles will have a measure of 1°. Therefore, the measure of angle AOC (written as $m\angle AOC$) is 1/4 of 360° or 90°. We then write: $m\angle AOC = 90^\circ$. An angle having a measure of 90° is called a right angle. ($^\circ$ is the symbol for degrees)

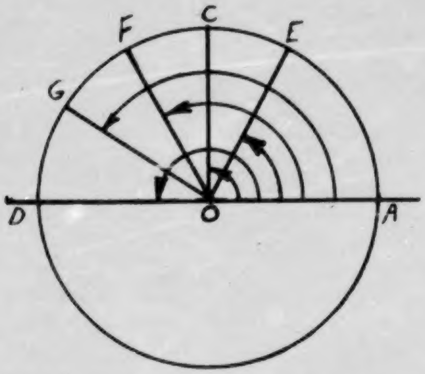


Figure 3

Again in Figure 3, $m\angle AOD = 1/2$ of 360° or 180°. An angle having a measure of 180° is called a straight angle. An angle with a measure which is less than 90° is called an acute angle. If the measure is larger than 90°, the angle is an obtuse angle. If $m\angle AOE = 60^\circ$, then $\angle AOE$ is an acute angle since 60° is less than 90°. If $m\angle AOF = 120^\circ$, then $\angle AOF$ is an obtuse angle.

Each degree can be divided into 60 equal parts of which each part is called a minute (denoted by the symbol '). Then, each minute can be divided into 60 equal parts of which each part is called a second (denoted by the symbol "). The measure of an angle can then be given in terms of degrees, minutes, and seconds. Remember that $1^\circ = 60'$, $1'' = 60''$. Therefore, $1^\circ = 360''$. For example, the measure of angle AOG may be written as $m\angle AOG = 150^\circ 12' 30''$ (read as: 150 degrees, 12 minutes, and 30 seconds).

In the following section, examples will illustrate the methods utilized in the addition, subtraction, multiplication, and division with angles.

APPLICATION OF THE RULE:

I. Addition of Angle Measures

Example 1. If $m\angle A = 18^\circ 42' 23''$, $m\angle B = 6^\circ 36' 4''$, and $m\angle C = 22^\circ 56' 42''$, find $m\angle A + m\angle B + m\angle C$.

$$\begin{array}{rrr} 18^\circ & 42' & 23'' \\ 6^\circ & 36' & 4'' \\ 22^\circ & 56' & 42'' \\ \hline & & 69'' \end{array}$$

First, add the last column to find that the total seconds is 69.

$$\begin{array}{rrr} & 1' & \\ 18^\circ & 42' & 23'' \\ 6^\circ & 36' & 4'' \\ 22^\circ & 56' & 42'' \\ \hline & 135' & 9'' \end{array}$$

Since $1' = 60''$, then $69'' = 1'9''$. Therefore, we must add 1' to the minutes column. Then, we find the total minutes to be 135.

$$\begin{array}{rrr} 2^\circ & & \\ 18^\circ & 42' & 23'' \\ 6^\circ & 36' & 4'' \\ 22^\circ & 56' & 42'' \\ \hline 48^\circ & 15' & 9'' \end{array}$$

Since $1^\circ = 60'$, then $135' = 2^\circ 15'$. Therefore, add 2° to the degrees column. Then, we find the total degrees to be 48.

Therefore:

$$m\angle A + m\angle B + m\angle C = 48^\circ 15' 9''$$

II. Subtraction of Angle Measures

Example 2. If $m\angle A = 13^\circ 8' 36''$ and $m\angle B = 86^\circ 32' 5''$, find $m\angle B - m\angle A$.

$$\begin{array}{rrrr} & 86^\circ & 32' & 5'' \\ - & 13^\circ & 8' & 36'' \\ \hline \end{array}$$

We first need to find the difference in the seconds column. However, 36 is larger than 5. We must, therefore, change 32' to 31' + 60". Then, $m\angle B$ can be rewritten as $86^\circ 31' 65''$.

$$\begin{array}{rrrr} & 86^\circ & 31' & 65'' \\ - & 13^\circ & 8' & 36'' \\ \hline & 73^\circ & 23' & 29'' \end{array}$$

Now, find the difference in the seconds column first, the minutes column next, and the degrees column last.

Therefore:

$$m\angle B - m\angle A = 73^\circ 23' 29''$$

III. Multiplication of Angle Measures by a Constant Number

Example 3. If $m\angle A = 13^\circ 8' 36''$, find 3 times $m\angle A$.

$$\begin{array}{rrr} 13^\circ & 8' & 36'' \\ & & \times\ 3 \\ \hline 39^\circ & 24' & 108'' \end{array}$$

Multiply the degrees, minutes, and seconds columns by 3.

Now:

$$39^\circ 24' 108'' = 39^\circ 25' 48''$$

Since 108" is 1' + 48".

Therefore:

$$3(13^\circ 8' 36'') = 39^\circ 25' 48''$$

Note that above we multiply each number by 3 since:

$$\begin{aligned} 3(13^\circ 8' 36'') &= 3(13^\circ + 8' + 36'') \\ &= 3\cdot 13^\circ + 3\cdot 8' + 3\cdot 36'' \qquad \text{(Distributive property)} \end{aligned}$$

This justifies the procedure which we used in simply multiplying each column by 3.

IV. Division of the Measure of an Angle by a Constant Number

Example 4. If $m\angle A = 22^\circ 16' 26''$, find $m\angle A/6$.

$$\begin{aligned} \frac{m\angle A}{6} &= \frac{22^\circ 16' 26''}{6} \\ &= \frac{22^\circ + 16' + 26''}{6} \\ &= \frac{22^\circ}{6} + \frac{16'}{6} + \frac{26''}{6} \end{aligned}$$

$$= 3\frac{2}{3}^{\circ} + 2\frac{2}{3}' + 4\frac{1}{3}''$$

$$= 3^{\circ} + 40' + 2' + 40'' + 4\frac{1}{3}''$$ (Since $\frac{2}{3}^{\circ} = \frac{2}{3} \cdot 60' = 40'$ and $\frac{2}{3}' = 40''$)

$$= 3^{\circ} + 42' + 44\frac{1}{3}''$$

$$= 3^{\circ}42'44''$$ (to the nearest second)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Angles Involving Protractors, Sine Bars, etc.

INTRODUCTION:

Interpreting the values of vernier protractors, bevel protractors, sine bars, comparators and other measuring instruments is very important if one is to machine parts accurately.

OBJECTIVES:

To provide the student an opportunity to solve problems involving measuring instruments and angles.

ASSIGNMENT:

1. $14^\circ 42'22'' + 5^\circ 30'28'' + 41^\circ 21'43'' =$ ____________

2. $48^\circ 13'43'' - 2^\circ 48'24'' =$ ____________

3. $5 \times (14^\circ 3'24'') =$ ____________

4. $\dfrac{43^\circ 18'22''}{2} =$ ____________

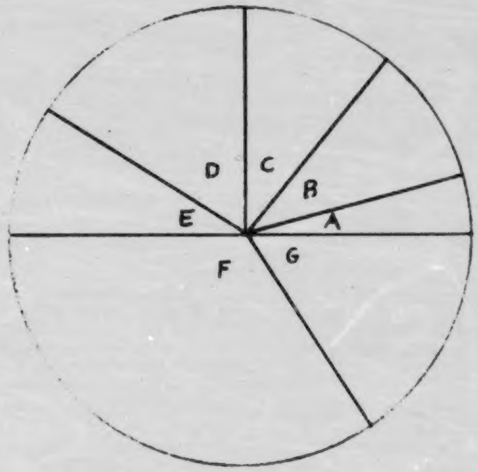
5. $\dfrac{55^\circ 28'4''}{4} =$ ____________

6. A machinist is required to lay out a bolt circle with a 3.5" diameter. He is to divide this circle into 15 angles having equal measures. What will be the measure for each of these angles?

7. A machinist must lay out holes on a bolt circle of a timing device. The bolt circle has a radius of 2 1/4 in.
   a. One fourth of the circle is to be divided into 4 angles having equal measures. What will be the measure of each angle? Find the answer to the nearest second.
   b. The second quarter of the circle is to be divided into 8 angles having equal measures. What will be the measure of each angle? Find the answer to the nearest second.
   c. The third quarter is to be divided into three times as many angles as in the first quarter, or 12. What will be the measure of each angle? Find the answer to the nearest second.
   d. The holes in the fourth quarter are to be laid in such a way

that the measure of each angle will be 1 1/3 times as great as that for each angle in the third quarter. What will be the measure for each angle in this portion of the circle? Find the answer to the nearest second.

8. In the figure below you are given the following information: $m\angle A = 20^\circ$, $m\angle B = 30^\circ$, $m\angle D = 65^\circ 22'$, $m\angle F = 118^\circ 40'$.
   a. What is the value of the sum of the measures of angles A and B?
   b. Find the value of the measure of angle C if the measures of angles A, B, and C total 90°.
   c. What is the sum of the measures of angles A, B, C, and D?
   d. Find the value of the measure of angle E.
   e. Find the sum of the measures of angles A, B, C, D, E, and F.
   f. Find the value of the measure of angle G.



ANSWERS

1. $61^\circ34'33''$
2. $45^\circ25'19''$
3. $70^\circ17'$
4. $21^\circ39'11''$
5. $13^\circ52'1''$
6. 24°
7. a. $22^\circ30'00''$
   b. $11^\circ15'00''$
   c. $7^\circ30'00''$
   d. $10^\circ00'00''$
8. a. 50°
   b. 40°
   c. $155^\circ22'$
   d. $24^\circ38'$
   e. $298^\circ40'$
   f. $61^\circ20'$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Rectangular Coordinate System

INTRODUCTION AND/OR OBJECTIVES:

The rectangular coordinate system is basic to both an understanding of the problems involving trigonometry and also an understanding of the concepts in numerical control. This section will be restricted to two dimensions, that is, points lying in the same plane.

TECHNICAL INFORMATION:

Consider two perpendicular lines (two lines which intersect so that the angles formed are right angles) as in Figure I. Call the vertical line

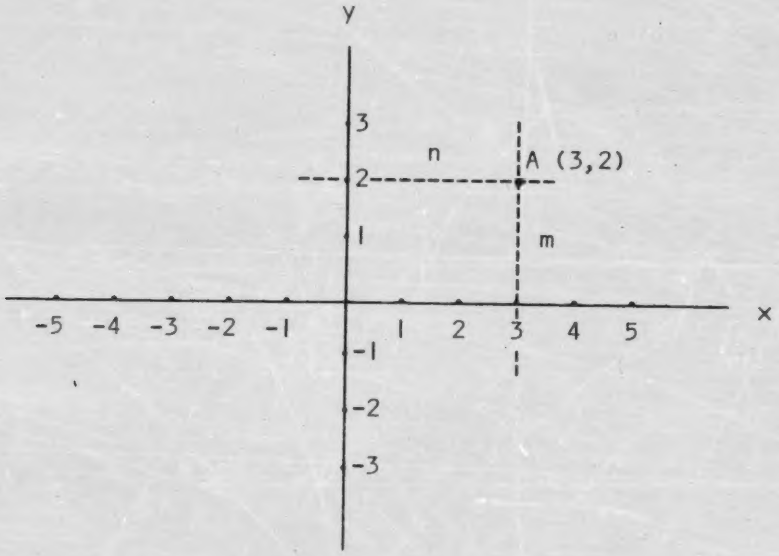


Figure I

the y-axis and the horizontal line the x-axis. Call the point of intersection, O, of the two lines the origin. Now, a scale is selected for both the x-axis and the y-axis. Normally, positive numbers are assigned to points on the right end of the x-axis and negative numbers to points on the left end of the x-axis. Similarly, positive numbers are associated with points on the upper end of the y-axis and negative numbers with points on the lower end of the y-axis. 0 is assigned to each axis at the origin.

To find the coordinates for point A (See Figure 1), first of all, consider the line m through A parallel to the y-axis. This line intersects the x-axis at 3. The first coordinate for A is defined to be 3. Then, consider the line n through A parallel to the x-axis. This line intersects the y-axis at 2. The second coordinate of point A is then defined to be 2. The coordinates for A are defined to be (3, 2). The first coordinate, 3,

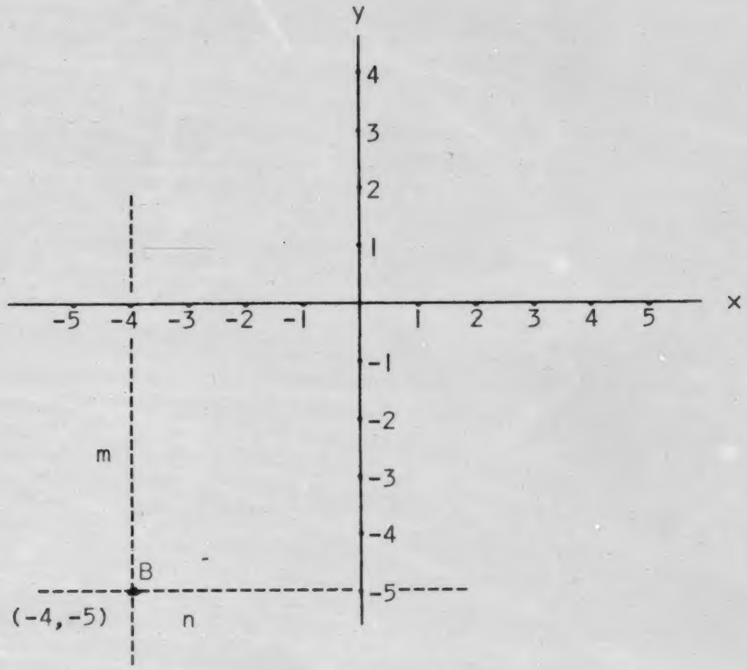


Figure 2

is called the abcissa. The second coordinate, 2, is called the ordinate.

In Figure 2, the line m through B parallel to the y-axis intersects the x-axis at -4. The line n through B parallel to the x-axis intersects the y-axis at -5. The coordinates for B are, therefore, (-4, -5).

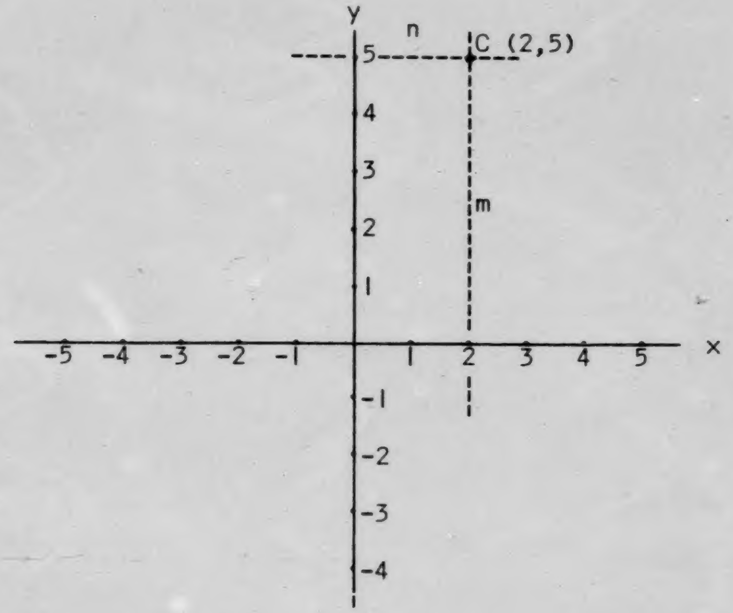


Figure 3

In Figure 3, to find the point with coordinates (2, 5) first of all draw the line m through 2 on the x-axis parallel to the y-axis. Next, draw the line n through 5 on the y-axis parallel to the x-axis. Lines m and n intersect at some point C. Point C has coordinates (2, 5).

Now, what about the coordinates for points on the x-axis and points on the y-axis? The point A (See Figure 4) at 3 on the x-axis is assigned the coordinates (3, 0). Likewise, point B at -2 on the x-axis is assigned the coordinates (-2, 0). The point C at 2 on the y-axis is assigned the coordinates (0, 2). Likewise, the point D at -4 on the y-axis is assigned the coordinates (0, -4). Similarly, other points on the x-axis and y-axis can be assigned coordinates. What about the coordinates for 0, the origin? Since the origin is at 0 on both axes, then its coordinates will be (0, 0).

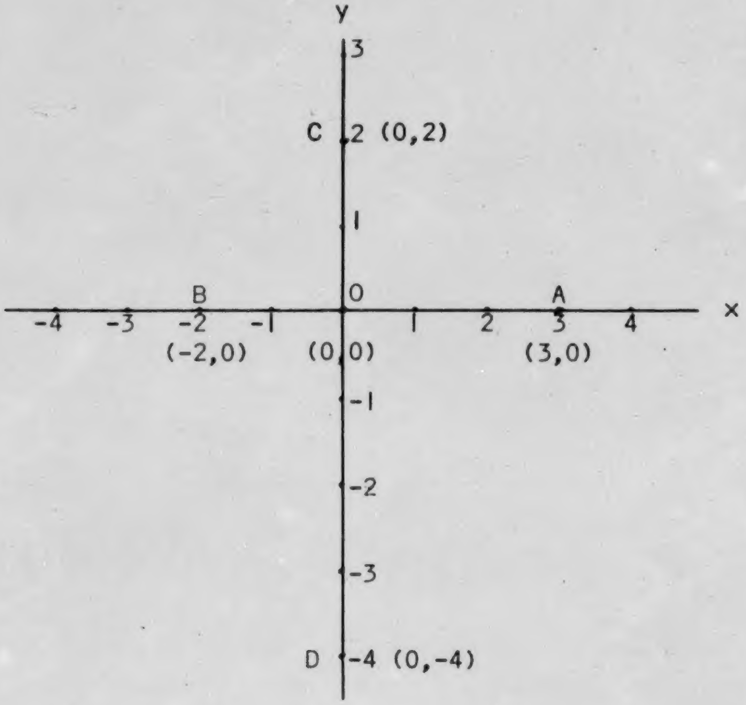
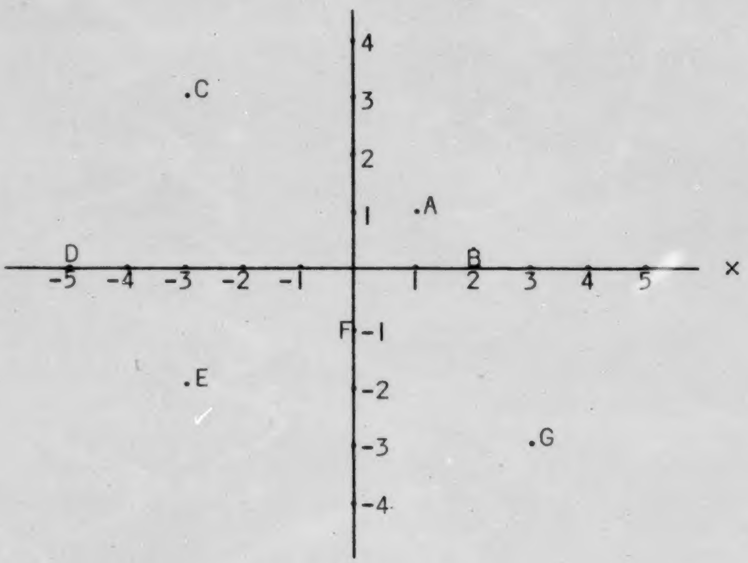


Figure 4

EXERCISES

1. Find the coordinates for points A, B, C, D, E, F, and G in the figure below.


4
3
2
1
-1
-2
-3
-4
-5
-4
-3
-2
-1
1
2
3
4
5
x
.A
B
.C
D
.E
F
.G

2. In the figure below, plot the points with the given coordinates.

A: (1, 2)	C: (0, -3)	E: (-5, 0)
B: (-3, 5)	D: (2, -1)	F: (3, -4)

ANSWERS

1. A: (1, 1)
   B: (0, 2)
   C: (-3, 3)
   D: (-5, 0)
   E: (-3, -2)
   F: (0, -1)
   G: (3, -3)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: The Trigonometric Functions

INTRODUCTION AND/OR OBJECTIVES:

Trigonometry must be considered one of the most important areas of applied mathematics in the machine trades. Trigonometry is a very valuable tool in being able to work with the measures of angles and the measures of sides of a triangle. By using trigonometry, the lengths of sides will determine the measures of angles in the triangle, and vice versa. This section will deal entirely with right triangles.

I. RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM

In Figure I consider the point B having coordinates (3, 4). The length of the segment directed from O to A (denoted by OA) is equal to 3. The length of the segment directed from A to B (denoted by AB) is equal

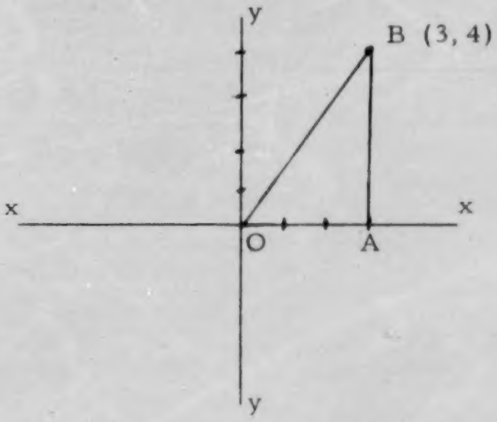


Figure I

to 4. How can we find the length of the segment from O to B (OB)?

The three points O, A, and B form a triangle which we may denote by $\triangle OAB$. This triangle is called a right triangle since one angle ($\angle OAB$) is a right angle. The segments $\overline{OA}$ and $\overline{AB}$ are called the legs of the triangle and $\overline{OB}$ is called the hypotenuse. (Note that $\overline{AB}$ denotes the segment whereas AB denotes the directed length of the segment.) $\overline{OA}$, $\overline{AB}$, and $\overline{OB}$ are all called sides of the triangle.

The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. That is: (In Figure 1)

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$(OB)^2 = (3)^2 + (4)^2$$

$$(OB)^2 = 3 \cdot 3 + 4 \cdot 4$$

$$(OB)^2 = 9 + 16$$

$$(OB)^2 = 25$$

Therefore:

$$OB = \sqrt{25}$$

$$OB = 5$$

Example 1. In triangle ABC in Figure 2 below, find the length of the hypotenuse (AC).

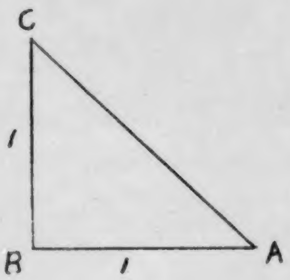


Figure 2

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (1)^2 + (1)^2$$

$$(AC)^2 = 1 + 1$$

$$(AC)^2 = 2$$

$AC = \sqrt{2}$ or approximately 1.414

II. THE TRIGONOMETRIC FUNCTIONS

Consider triangle ABC in Figure 3.

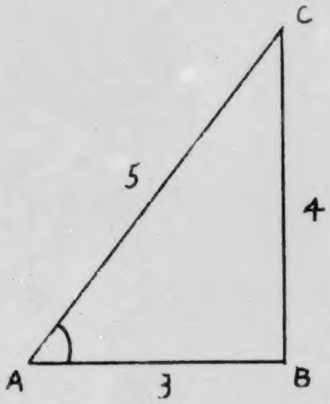


Figure 3

Note that AB = 3, BC = 4, and AC = 5. For ease in writing let us refer to ∠CAB as just ∠A. We can now set up ratios of the lengths of the legs and the length of the hypotenuse of △ABC. These various ratios are called trigonometric functions. These trigonometric functions allow us to find the lengths of various sides of the triangle if we know an angle or allow us to find the angle if we know lengths of the various sides.

The first trigonometric function that we will define is the sine of ∠A. The sine of ∠A or more simply sin ∠A is defined as the length of the side opposite $\angle A$ divided by the length of the hypotenuse.

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

In Figure 3, $\overline{BC}$ is the side opposite $\angle A$ and $\overline{AC}$ is the hypotenuse. Therefore:

$$\sin \angle A = \frac{BC}{AC}$$

$$= \frac{4}{5}$$

$$= .8000$$

A second trigonometric function of $\angle A$ is the cosine of $\angle A$ or more simply $\cos \angle A$. The $\cos \angle A$ is defined as the length of the side adjacent to $\angle A$ divided by the length of the hypotenuse.

$$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

In Figure 3, $\overline{AB}$ is the side adjacent to $\angle A$ and $\overline{AC}$ is the hypotenuse. Therefore:

$$\cos \angle A = \frac{AB}{AC}$$

$$= \frac{3}{5}$$

$$= .6000$$

A third trigonometric function of $\angle A$ is the tangent of $\angle A$. The tangent of $\angle A$ or more simply $\tan \angle A$ is defined as the length of the side opposite $\angle A$ divided by the side adjacent to $\angle A$.

$$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

In Figure 3, BC is the side opposite to ∠A and AB is the side adjacent to ∠A.

$$\tan \angle A = \frac{BC}{AB}$$

$$= \frac{4}{3}$$

$$= 1.3333$$

Note that the letters of the vertices can change in different problems. Therefore, check to see which side is the opposite side, which is the adjacent side, and which is the hypotenuse for the particular angle used in the problem.

Example 6. In Figure 4, find sin ∠B, cos ∠B, tan ∠B.

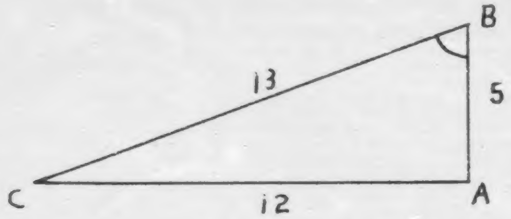


Figure 4

$$\sin \angle B = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{12}{13}$$

$$\cos \angle B = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$= \frac{AB}{BC}$$

$$= \frac{5}{13}$$

$$\tan \angle B = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{AC}{AB}$$

$$= \frac{12}{5}$$

There are three other trigonometric functions which are probably not used as frequently as the sine, cosine and tangent functions.

One of these is the cotangent of an angle. The cotangent of ∠A or more simply cot ∠A is defined as the length of the adjacent side divided by the length of the opposite side.

$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

In Figure 3, $\overline{AB}$ is the side adjacent ∠A and $\overline{BC}$ is the side opposite ∠A.

$$\cot \angle A = \frac{AB}{BC}$$

$$= \frac{3}{4}$$

$$= .750$$

The cosecant of ∠A abbreviated to csc ∠A is defined as the length of the hypotenuse divided by the length of the side opposite to ∠A.

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

In Figure 3, $\overline{AC}$ is the hypotenuse and $\overline{BC}$ is the side opposite $\angle A$.

$$\csc \angle A = \frac{AC}{BC}$$

$$= \frac{5}{4}$$

$$= 1.250$$

The secant of $\angle A$ abbreviated to sec $\angle A$ is defined as the length of the hypotenuse divided by the length of the adjacent side.

$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

In Figure 3, $\overline{AC}$ is the hypotenuse and $\overline{AB}$ is the side adjacent to $\angle A$.

$$\sec \angle A = \frac{AC}{AB}$$

$$= \frac{5}{3}$$

$$= 1.6667$$

Example 2. Find the cot $\angle B$, csc $\angle B$, and sec $\angle B$ in Figure 4.

$$\cot \angle B = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$= \frac{AB}{AC}$$

$$= \frac{5}{12}$$

____

$$\csc \angle B = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

$$= \frac{BC}{AC}$$

$$= \frac{13}{12}$$

____

$$\sec \angle B = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

$$= \frac{BC}{AB}$$

$$= \frac{13}{5}$$

Example 3. Find the values of sin ∠A, cos ∠A, tan ∠A, cot ∠A, csc ∠A, and sec ∠A in Figure 5.

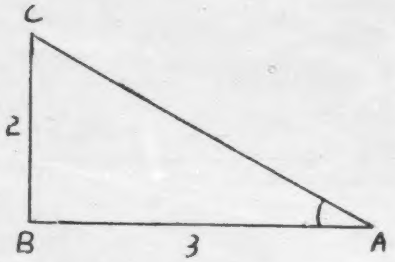


Figure 5

First of all, we need to find the value of AC.

From the Pythagorean Theorem:

$$
\begin{aligned}
(AC)^2 &= (AB)^2 + (BC)^2 \\
&= (3)^2 + (2)^2 \\
&= 9 + 4 \\
&= 13 \\
AC &= \sqrt{13} \\
&= 3.6056
\end{aligned}
$$

Then:

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$= \frac{BC}{AC}$$

$$= \frac{2}{\sqrt{13}}$$

$$= \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$$

(We multiply the numerator and the denominator by $\sqrt{13}$ so that we can change the denominator from a square root to a whole number. This will change the problem so that instead of dividing by a square root (which will be 3.6056) we will multiply by the square root.

$$= \frac{2 \cdot \sqrt{13}}{13} \qquad \left(\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}\right)$$

$$= \frac{2(3.6056)}{13}$$

$$= \frac{7.2112}{13}$$

$$= .5547$$

$$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$= \frac{AB}{AC}$$

$$= \frac{3}{\sqrt{13}}$$

$$= \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{3 \cdot \sqrt{13}}{13}$$

$$= \frac{3(3.6056)}{13}$$

$$= \frac{10.8168}{13}$$

$$= .8321$$

$$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{BC}{AB}$$

$$= \frac{2}{3}$$

$$= .6667$$

$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$= \frac{AB}{BC}$$

$$= \frac{3}{2}$$

$$= 1.5000$$

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

$$= \frac{AC}{BC}$$

$$= \frac{\sqrt{13}}{2}$$

$$= \frac{3.6056}{2}$$

$$= 1.8028$$

$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

$$= \frac{AC}{AB}$$

$$= \frac{\sqrt{13}}{3}$$

$$= \frac{3.6056}{3}$$

$$= 1.2019$$

SUMMARY

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

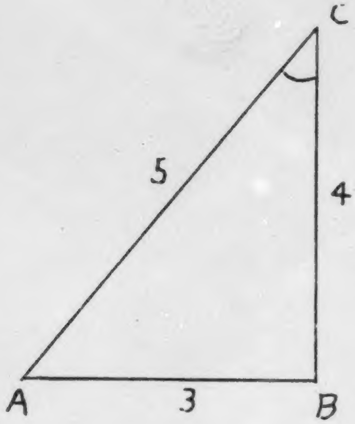
$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

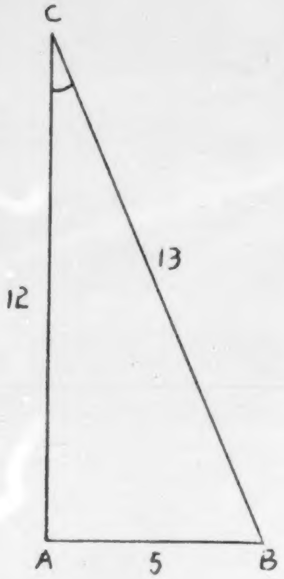
$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

EXERCISES

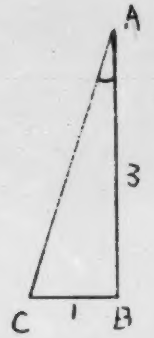
1. Find the values of the six trigonometric functions for ∠C in the triangle below.



2. Find the values of the six trigonometric functions for ∠C in the triangle below.



3. Find the values of the six trigonometric functions for ∠A in the triangle below.



ANSWERS

1. $\sin \angle C = \frac{3}{5} = .6000$

$\cos \angle C = \frac{4}{5} = .8000$

$\tan \angle C = \frac{3}{4} = .7500$

$\cot \angle C = \frac{4}{3} = 1.3333$

$\csc \angle C = \frac{5}{3} = 1.6667$

$\sec \angle C = \frac{5}{4} = 1.2500$

2. $\sin \angle C = \frac{5}{13} = .3846$

$\cos \angle C = \frac{12}{13} = .9231$

$\tan \angle C = \frac{5}{12} = .4167$

$\cot \angle C = \frac{12}{5} = 2.4000$

$\csc \angle C = \frac{13}{5} = 2.6000$

$\sec \angle C = \frac{13}{12} = 1.0833$

3. $\sin \angle A = \frac{1}{\sqrt{10}} = .3162$

$\cos \angle A = \frac{3}{\sqrt{10}} = .9487$

$\tan \angle A = \frac{1}{3} = .3333$

$\cot \angle A = \frac{3}{1} = 3.0000$

$\csc \angle A = \frac{\sqrt{10}}{1} = 3.1623$

$\sec \angle A = \frac{\sqrt{10}}{3} = 1.0541$

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Using Trigonometric Tables

INTRODUCTION AND/OR OBJECTIVES:

In the solution for measures of angles in a triangle or the lengths of sides of a triangle, the accurate use of trigonometric tables is a must. Very frequent practice with the use of tables including interpolation to determine values not in the tables will provide the user with confidence in his ability.

TECHNICAL INFORMATION:

For a given angle the value for each of the six trigonometric functions for that angle may be read directly from trigonometric tables or can be found by using a method called interpolation. It is also possible to find the angle if the value for any one of the six trigonometric functions is given. (For the following discussion and examples the student should refer to prepared trigonometric tables of the instructor's choice in which values are listed to the nearest minute for angles from 0° to 90°.)

In these tables the student should check the following readings:

$\sin 15^\circ 22' = .26499$ $\tan 64^\circ 38' = 2.1092$

$\cos 27^\circ 32' = .88674$ $\sec 85^\circ 45' = 13.494$

Now let us look at two examples in which the angle is to be found when a function value is known.

Example I. If $\cos A = .92421$, find A. By checking the cosine values in the tables, it is found that the angle A is $22^\circ 27'$.

Example 2. If in a right triangle, $\tan B = 1.2131$, then from the tables, $B = 50^\circ 30'$.

If angle measurements involve seconds, the table may still be used, but a process called interpolation must be utilized.

Example 3. Find the value of $\sin 37^\circ 23'20''$.

First, find the values for $\sin 37^\circ 23'$ and $\sin 37^\circ 24'$. See below.

Angles | Sine Values

$$60''\left[\begin{array}{l} 20''\left[\begin{array}{l}37^\circ 23' \\ 37^\circ 23'20''\end{array}\right. \\ 37^\circ 24'\end{array}\right. \qquad \sin 37^\circ 23'20'' = \left.\begin{array}{r}\left.\begin{array}{r}.60714\\ \underline{\qquad}\end{array}\right] x \\ .60737\end{array}\right] .00023$$

The difference between $37^\circ 23'$ and $37^\circ 24'$ is $1'$ or $60''$. The difference between $37^\circ 23'$ and $37^\circ 23'20''$ is $20''$. These differences are written by the brackets as above. The difference between .60714 and .60737 is .00023.

Now, x (the difference between .60714 and the number we are after) divided by .00023 should be in the same ratio as 20 divided by 60. That is:

$$\frac{x}{.00023} = \frac{20}{60}$$

$$\frac{x}{.00023} = \frac{1}{3}$$

$$3x = (.00023)1 \qquad \left(\frac{a}{b} = \frac{c}{d} \text{ implies that } a \cdot d = b \cdot c\right)$$

$$3x = .00023$$

$$\frac{1}{3}3x = \frac{1}{3}(.00023) \qquad \text{(Multiply both sides by } 1/3\text{)}$$

$$x = \frac{.00023}{3} \qquad \left(\text{Multiplicative inverse, } \frac{a}{b} \cdot c = \frac{a \cdot c}{b}\right)$$

$$x = .00008 \qquad \text{(to the nearest hundred thousandth)}$$

Therefore:

$$\sin 37^{\circ}23'20'' = .60714 + .00008$$

$$= .60722$$

Example 4. Find the value of $\cos 27^{\circ}32'15''$.

Angles | Cosine Values

$$60''\left[\begin{array}{l} 15''\left[\begin{array}{l} 27^{\circ}32' \\ 27^{\circ}32'15'' \end{array}\right. \\ \;27^{\circ}33' \end{array}\right. \qquad \cos 27^{\circ}32'15'' = \left.\begin{array}{r} \left.\begin{array}{r} .88674 \\ \underline{\qquad\quad} \end{array}\right] \times \\ .88661 \end{array}\right] .00013$$

Therefore:

$$\frac{x}{.00013} = \frac{15}{60}$$

$$\frac{x}{.00013} = \frac{1}{4}$$

$$4x = (.00013)1$$

$$\frac{1}{4}4x = \frac{1}{4}\cdot(.00013)$$

$$x = \frac{.00013}{4}$$

$x = .00003$ (to the nearest hundred thousandth)

Therefore:

$$\cos 27^{\circ}32'15'' = .88674 - .00003$$

$$= .88671$$

(Notice that we subtract .00003 since the bottom cosine value is less than the upper cosine value.)

Example 5. Find A if tan A = .66030.

$$\begin{array}{ccc} & \underline{\text{Angle}} & \qquad\qquad \underline{\text{Tangent Values}} \\ 60''\left[\begin{array}{l} x\left[\begin{array}{l}33^\circ26'\\ \underline{\qquad\quad} = A\end{array}\right.\\ 33^\circ27'\end{array}\right. & & \qquad\qquad \left.\begin{array}{l}\left.\begin{array}{l}.66021\\ .66030\end{array}\right].00009\\ .66063\end{array}\right].00042 \end{array}$$

Therefore:

$$\frac{x}{60} = \frac{.00009}{.00042}$$

$$\frac{x}{60}\cdot 60 = \frac{.00009}{.00042}\cdot 60$$ (Multiply both sides by 60)

$$x = \frac{(.00009)60}{.00042}$$ (Multiplicative inverse, $\frac{a}{b}\cdot c = \frac{a\cdot c}{b}$)

$$x = \frac{.00540}{.00042}$$

$$x = \frac{540}{42}$$

$$x = \frac{90\cdot 6}{7\cdot 6}$$

$$x = \frac{90}{7}$$

$x = 13$ (to the nearest whole number)

Therefore:

$$A = 33^\circ26' + 13''$$

$$= 33^\circ26'13''$$

EXERCISES

1. Find tan 12°15'
2. Find cos 6°49'
3. Find sin 78°2'
4. Find sec 38°16'
5. Find cot 2°49'
6. Find csc 87°12'
7. Find sin 31°12'15"
8. Find tan 68°17'34"
9. Find cos 42°48'30"
10. Find tan 17°32'45"

ANSWERS

1. .21712
2. .99293
3. .97827
4. 1.2737
5. 20.325
6. 1.0012
7. .51809
8. 2.5120
9. .73363
10. .31618

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL INFORMATION TITLE: Calculations--Gage Blocks

INTRODUCTION:

Gage blocks are very accurate and have many uses such as: checking the accuracy of a micrometer, setting up a sine bar for angular measurements, and adapting special accessories that can be used as inside and outside calipers. They can be used as layout instruments and to set a height gage.

OBJECTIVES:

1. To provide the student an opportunity to learn how to calculate the gage blocks needed for a linear dimension.
2. To provide the student an opportunity to learn how to calculate the gage blocks needed for the length of a side of an angle.

TECHNICAL INFORMATION:

Gage blocks are made from various types of steel. Today, gage blocks are being made of long wearing chrome carbide which is almost diamond hard, practically corrosion proof, fine grained, and with thermal variation similar to steel. Accuracy, which is maintained many times longer than in regular steel blocks, practically eliminates production inaccuracies and reduces gage block inspection. Measurements range up to 12 inches. Gage blocks are available in both rectangular and square sets, and are available in A+, AA, and Laboratory Master accuracies as well as in metric sizes. Gage blocks are available in 88, 85, 84, 81, 36, and 34 block sets.

Gage block accessories may include half round jaws and straight jaws, for calipers, scriber points, center points, eccentric points, quick acting clamps, base blocks, indicator accessory sets, and various rods and screws.

Angle gage block are also made for fast, simple, and extremely accurate measurements of any angle from zero to 99 degrees in steps of one second, one minute, or one degree. They are available in three accuracies: Laboratory Master grade with ±1/4 second accuracy, Inspection grade with ±1/2 second accuracy, and Tool Room grade with ±1 second accuracy.

1. GAGE BLOCKS

Example 1. Set up blocks for a dimension of 3.4817.

		Block	Dimension Needed
Step 1.	Find a block to give the 7 ten thousandths in 3.4817. The block chosen has a thickness of .1007. Subtract .1007 from 3.4817 to find that 3.3810 is the dimension still needed.	.1007	$\begin{array}{r} 3.4817 \\ -\ .1007 \\ \hline 3.3810 \end{array}$
Step 2.	Select a block to give the 1 thousandth in 3.3810. The block chosen has a thickness of .1310. Subtract the .1310 from 3.3810 to find that 3.2500 is the dimension still needed.	.1310	$\begin{array}{r} 3.3810 \\ -\ .1310 \\ \hline 3.2500 \end{array}$
Step 3.	Select a block to give the 5 hundredths in 3.2500. The block chosen has a thickness of .2500. Subtract .2500 from 3.2500 to find that 3.0000 is the dimension still needed.	.2500	$\begin{array}{r} 3.2500 \\ -\ .2500 \\ \hline 3.0000 \end{array}$
Step 4.	Select the block with a thickness of 3.0000 to finish.	3.0000	$\begin{array}{r} 3.0000 \\ -\ 3.0000 \\ \hline 0.0000 \end{array}$

Check by adding the dimensions of the blocks chosen to make sure that the total is 3.4817.

$$\begin{array}{r} .1007 \\ .1310 \\ .2500 \\ \underline{3.0000} \\ 3.4817 \end{array}$$

II. SINE BAR AND GAGE BLOCKS

Example I. The angle at which the 10" sine bar in Figure I is set has a measure of 34°40'. Find the dimension h. Then set up the appropriate gage blocks for h.

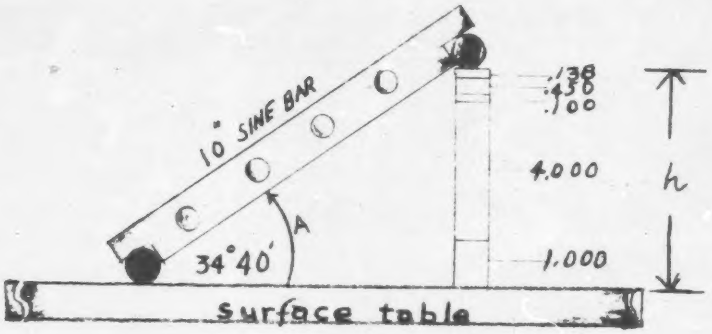


Figure I

Notice in Figure I that we know the measure of angle A is 34°40'. We also know the length of the hypotenuse, and we wish to find the length of the opposite side to the angle. Since we are concerned with the opposite side and the hypotenuse, we can use the sine function to find h.

$$\sin 34^\circ 40' = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\sin 34^\circ 40' = \frac{h}{10}$$

From the tables, $\sin 34^\circ 40' = .56880$. Therefore,

$$.56880 = \frac{h}{10}$$

$$.56880(10) = \frac{h}{10}(10) \qquad \text{(Multiply both sides by 10)}$$

$$5.6880 \quad = h$$

Next, find the gage blocks to give the dimension of 5.6880.

Block	Dimension Needed
	5.6880
.1380	5.6880 - .1380 5.5500
.4500	5.5500 - .4500 5.1000
.1000	5.1000 - .1000 5.0000
5.0000	5.0000 - 5.0000 0.0000

Finally, check.

.1380
.4500
.1000
5.0000
5.6880

Example 2. Find the measure of angle A in Figure 2 if h = 4.3200 and the sine bar is 10" long.

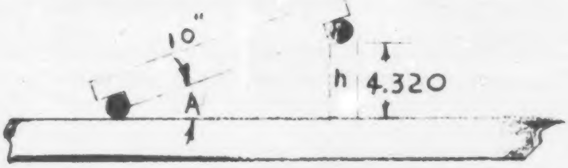


Figure 2

Since we know the length of the opposite side to angle A and the length of the hypotenuse, again we will use the sine function of angle A.

$$\sin A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\sin A = \frac{h}{10} = \frac{4.3200}{10}$$

$$\sin A = .43200$$

From the trigonometric tables, we find that:

$m\angle A = 25^\circ 35' 42''$ or $25^\circ 36'$ to the nearest minute

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL TITLE: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Gage Block Calculations

INTRODUCTION:

When accuracy is essential to within a millionth of an inch, then gage blocks become very important measuring tools. Addition of decimals, and calculation of sine functions are essential.

OBJECTIVE:

To learn how to resolve problems involving decimals and the sine function.

ASSIGNMENT:

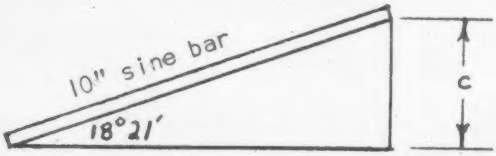
1. 5.400320 + 1.000400 = ________

2. 4.00065 + .00085 = ________

3. 4.8321 + .0300 = ________

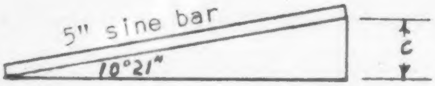
4. Find:

Side c = ______



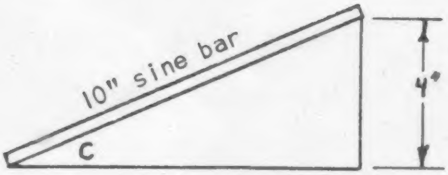
5. Find:

Side c = ______



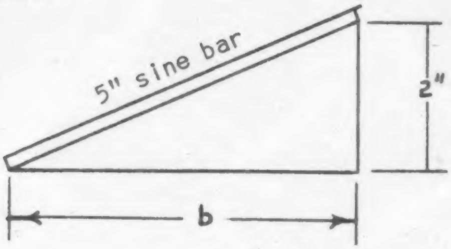
6. Find:

m∠C = ________


10" sine bar
C
4"

7. Find:

m∠C = ________


5" sine bar
2"
b

8. What gage blocks would be used for setting up a dimension of 1.3427 in.?

9. Set up the proper gage blocks for a dimension of 3.4817 in.

10. Set up blocks for a dimension of 1.6817 in.

ANSWERS

1. 6.400720
2. 4.00150
3. 4.8621
4. $c = 3.1482$
5. $c = .89830$
6. $m\angle C = 23^{\circ}34'42''$ or $23^{\circ}35'$ (to the nearest minute)
7. $m\angle C = 23^{\circ}34'42''$ or $23^{\circ}35'$ (to the nearest minute)
8. Gage blocks: .1007, .1120, .130, 1.0000
9. Gage blocks: .1007, .1310, .2500, 3.0000
10. Gage blocks: .1007, .1310, .4500, 1.0000

MODERN MATHEMATICS
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THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Gage Block Reading

COURSE UNIT TITLE: Precision Measurements

INTRODUCTION:

Gage blocks are used for setting up accurate dimensions, checking micrometers and verniers for accuracy, and for finding the measures of angles by using the sine bar.

OBJECTIVE:

To provide the student an opportunity to learn how to obtain various dimensions by using gage blocks.

TOOLS AND MATERIALS REQUIRED:

Gage blocks

PROCEDURE:

(Operation)	(Related Information)	
	Block	Dimension Needed
1. Set up gage blocks for a given dimension. For example, set up for 1.3427. Begin by writing the dimension. Find a block to give the 7 ten thousandths in 1.3427. The block chosen has a thickness of .1007. Subtract .1007 from 1.3427 to find that 1.2420 is the dimension still needed.	.1007	$\begin{array}{r} 1.3427 \\ -\ .1007 \\ \hline 1.2420 \end{array}$
2. Select a block to give the 2 thousandths in 1.2420. The block chosen has a thickness of .1120. Subtract the .1120 from 1.2420 to find that 1.1300 is the dimension still needed.	.1120	$\begin{array}{r} 1.2420 \\ -\ .1120 \\ \hline 1.1300 \end{array}$

3. Select a block to give the 3 hundredths in 1.1300. The block chosen has a thickness of .1300. Subtract .1300 from 1.1300 to find that 1.0000 is the dimension still needed.	.1300	$\begin{array}{r} 1.1300 \\ -\ .1300 \\ \hline 1.0000 \end{array}$
4. Select the block with a thickness of 1.0000 to finish.	1.0000	$\begin{array}{r} 1.0000 \\ -\ 1.0000 \\ \hline 0.0000 \end{array}$
5. Check by adding the dimensions of the blocks chosen to make sure that the total is 1.3427.	$\begin{array}{r} .1007 \\ .1120 \\ .1300 \\ 1.0000 \\ \hline 1.3427 \end{array}$	

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Layout Problems Involving the Right Triangle

TECHNICAL INFORMATION TITLE: Right Triangle Applications

INTRODUCTION:

A toolmaker and a machinist must be able to understand the right triangle and its applications. The right triangle is used extensively in layout and measurement of hole locations, angles, circles, irregular curves and/or in combination of any or all of these.

OBJECTIVE:

To provide the student an opportunity to work with layout problems involving right triangles.

TECHNICAL INFORMATION:

The right triangle is involved in a large number of different types of layout problems. A frequently used function is the sine function due to its use with the sine bar and gage blocks. The other trigonometric functions (tangent, cosine, secant, cosecant, and cotangent), however, are also extremely useful and should not be ignored. The trigonometric function to be used should be determined by the data available and the dimension or dimensions to be found.

Example I. In Figure I solve for the value of c.

Given: $m\angle C = 59^\circ$, $b = .750$

Find: c

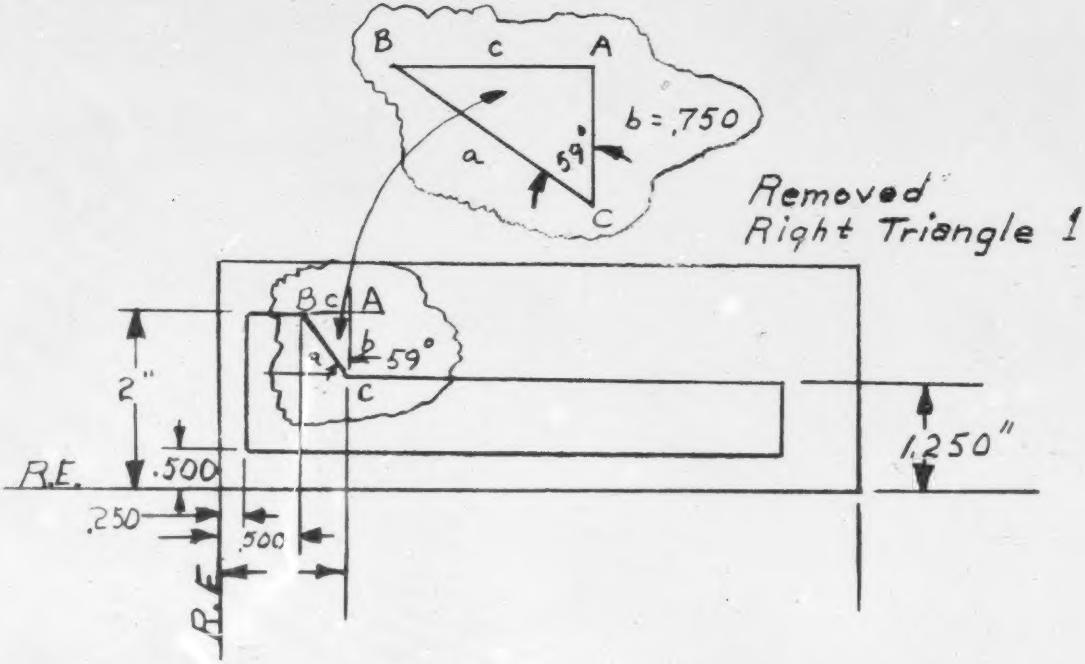

B
c
A
b = .750
a
59°
C
Removed Right Triangle 1
B c A
b
a
59°
C
2"
.500
R.E.
.250
.500
R.E.
1.250"

Figure 1

We know the measure of $\angle C$. Also, we know the length of the adjacent side (b) to angle C. We need to find the length of c (the opposite side to $\angle C$). This information suggests that we use the tangent function.

$$\tan \angle C = \frac{c}{b}$$

$$b \cdot \tan \angle C = b \cdot \frac{c}{b} \qquad \text{(Mult. both sides by b)}$$

$$b \cdot \tan \angle C = c \qquad \text{(Mult. inverse)}$$

Then:

$$c = (.750)(\tan 59^\circ)$$

$$= (.750)(1.6643)$$

$$= 1.248 \quad \text{(to the nearest thousandth)}$$

Example 2. Layout after solving for the measure of $\angle B$. See Figure 2.

Given: $b = 2.000$, $c = .250$

Find: $m\angle B$

We must first find $m\angle D$. Then we can find $m\angle B$.

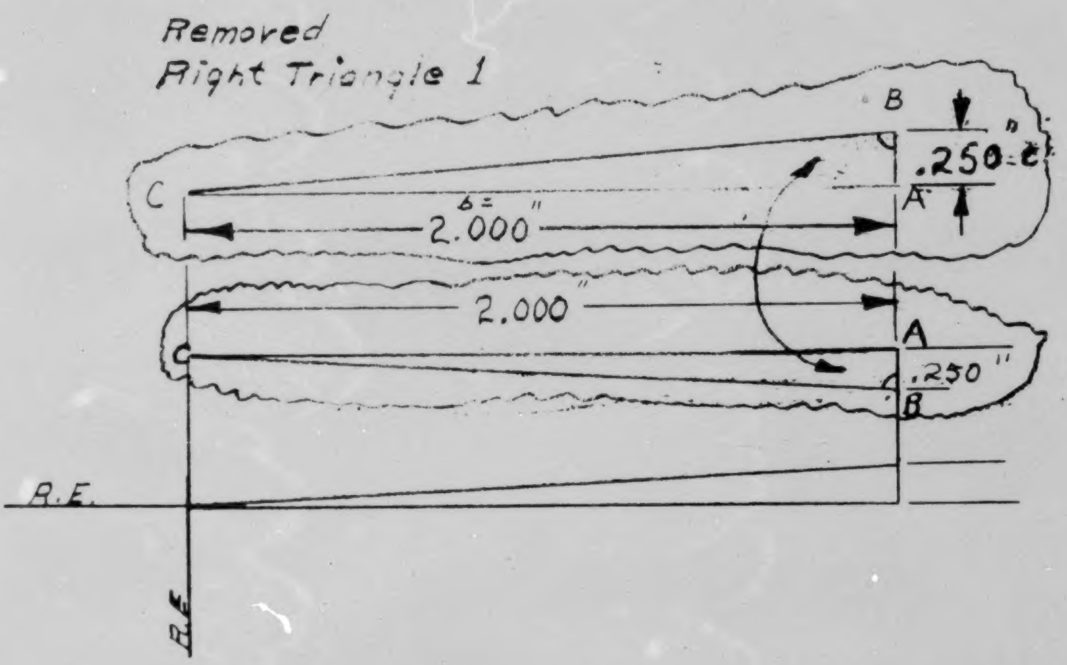

Removed
Right Triangle 1
B
.250 c
C
b = 2.000"
A
2.000"
A
C
.250"
B
R.E.
R.E.

Figure 2

We know the length of the opposite side to $\angle D$ and the length of the adjacent side to $\angle D$. This again suggests the use of the $\tan \angle D$, to find $m\angle D$.

$$\tan \angle D = \frac{b}{c}$$

$$= \frac{2.000}{.250}$$

$$= 8.000$$

Then, from the tables:

$m\angle D = 82^{\circ}52'$ (to the nearest minute)

Finally:

$$m\angle B = 180^\circ - 82^\circ 52'$$

$$= 179^\circ 60' - 82^\circ 52'$$

$$= 97^\circ 8'$$

Example 3. Drill nine holes in a plate of steel on a circumference of a circle having a diameter of 20 inches. What will be the distance between the centers of two adjacent holes? See Figure 3.

Since there are to be 9 holes, the measure of the angle between any two adjacent holes will be $\frac{360^\circ}{9} = 40^\circ$.

Given: $a = 10$, $m\angle C = 20^\circ$

Find: $d =$ (note that $d = 2c$)

Here, we know $m\angle C$. We know the length of the hypotenuse, and we wish to find the length of c. This given data suggests the use of the $\sin\angle C$.

$$\sin\angle C = \frac{c}{10}$$

$$10\cdot\sin\angle C = 10\cdot\frac{c}{10} \qquad \text{(Mult. both sides by 10)}$$

$$10\cdot\sin\angle C = c \qquad \text{(Mult. inverse)}$$

Then:

$$c = 10(\sin 20^\circ)$$

$$= 10(.34202)$$

$$= 3.420$$

Therefore:

$$d = 2c$$

$$= 2(3.420)$$

$$= 6.840$$

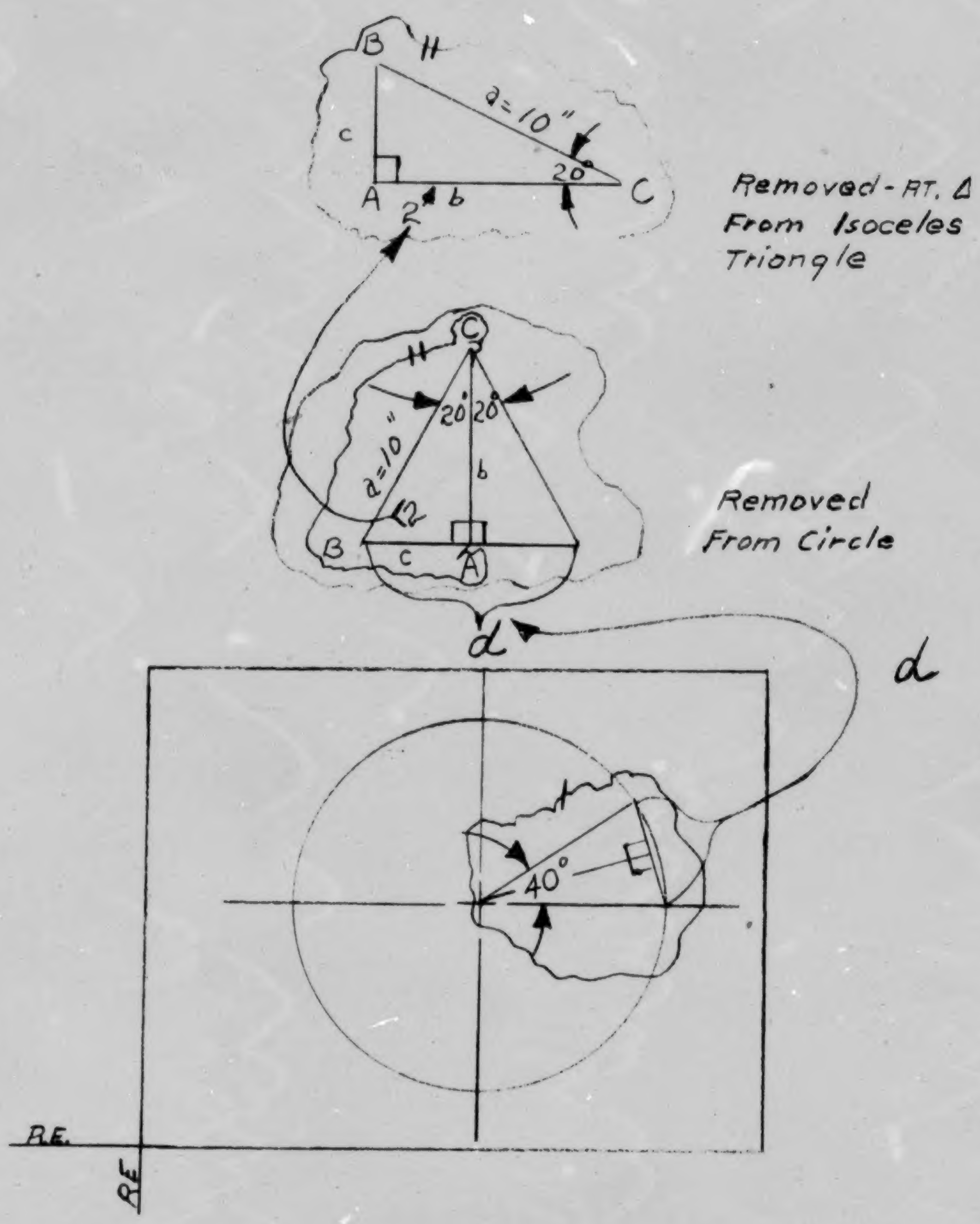
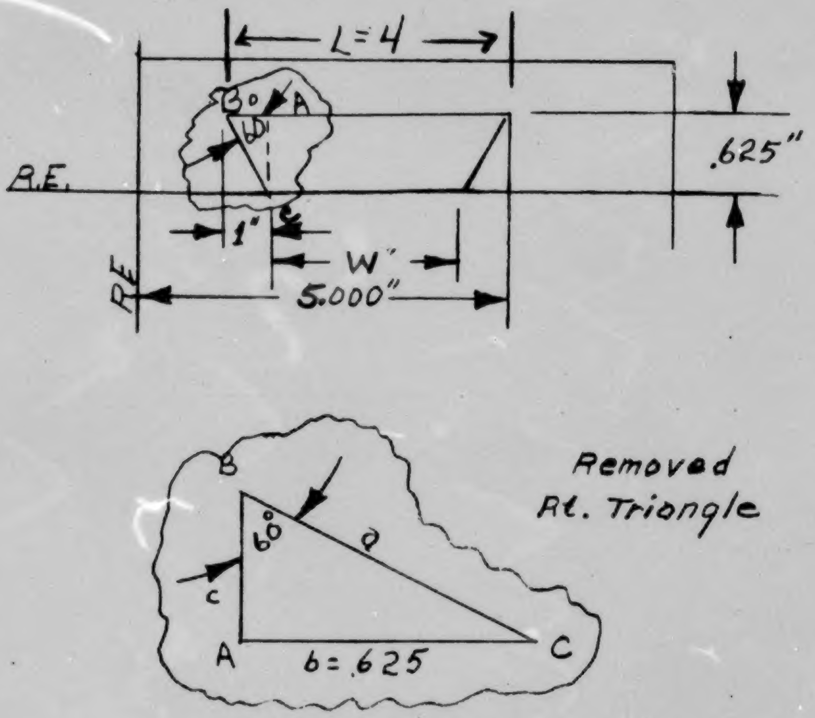

B
A
C
c
b
2
a = 10"
20°
Removed - RT. Δ
From Isoceles
Triangle
C
20° 20°
a = 10"
b
B
c
A
Removed
From Circle
d
d
40°
R.E.
R.E.

Figure 3

Example 4. Find the value for w in Figure 4.

Given: b = .625, L = 4.000, $m\angle B = 60^{\circ}$

Find: w



L=4
B
30°
A
60°
.625"
B.E.
1"
c
W"
R.E.
5.000"
Removed
Rt. Triangle
B
60°
a
c
A
b=.625
C

Figure 4

We, first of all, need to find the value for c. Then, w = 4.000 - 2c.

We know $m\angle B$. We know the length of the opposite side b, and we wish to find the length of the adjacent side c. This suggests the use of $\cot \angle B$.

$$\cot \angle B = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$= \frac{c}{b}$$

$$= \frac{c}{.625}$$

$(.625)\cot\angle B = .625\cdot\frac{c}{.625}$ (Mult. both sides by .625)

$(.625)\cot\angle B = c$ (Mult. inverse)

$(.625)\cot 60^\circ = c$

$(.625)(.57735) = c$

$.361 = c$ (to the nearest thousandth)

Then, $w = 4.000 - 2c$

$= 4.000 - 2(.361)$

$= 4.000 - .722$

$= 3.278$

Example 5. Find the center to center distance c on the plate in Figure 5.

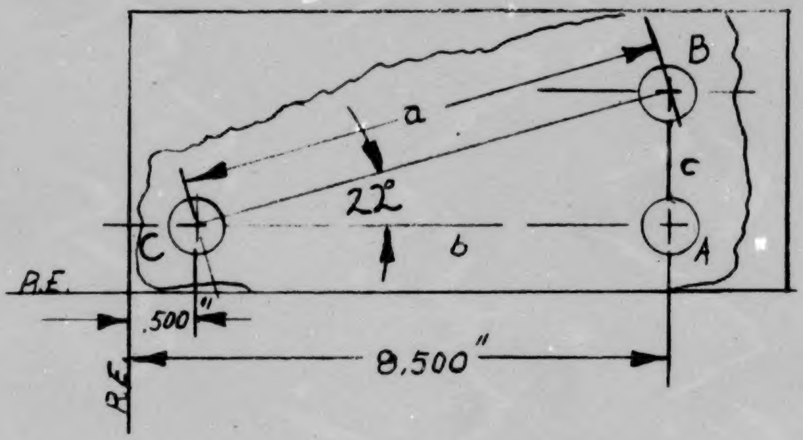


Figure 5

Given: $m\angle C = 22^\circ$, $b = 8.000$

Find: c

See the removed right triangle in Figure 6.

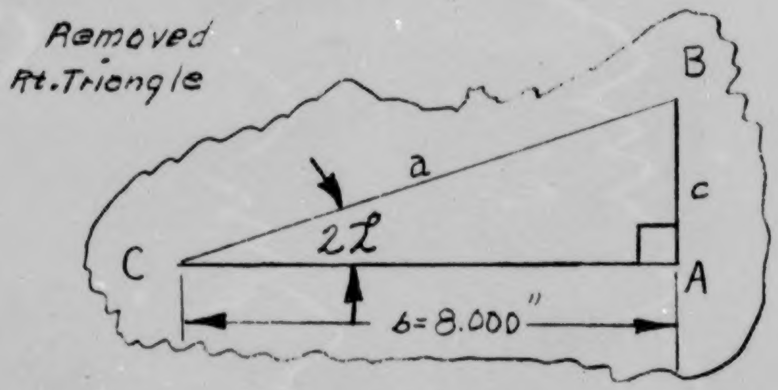


Figure 6

We know m∠C. We know the length of the adjacent side b, and we wish to find the length of the opposite side c. This suggests using tan C.

$$\tan \angle C = \frac{c}{b}$$

$$\tan \angle C = \frac{c}{8.000}$$

$$(8.000)\tan \angle C = 8.000 \cdot \frac{c}{8.000} \qquad \text{(Mult. both sides by 8.000)}$$

$$(8.000)\tan \angle C = c \qquad \text{(Mult. inverse)}$$

$$(8.000)\tan 22^\circ = c$$

$$(8.000)(.40403) = c$$

$$3.232 = c \quad \text{(to the nearest thousandth)}$$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: The Right Triangle

TECHNICAL ASSIGNMENT TITLE: Problems Involving the Right Triangle

INTRODUCTION:

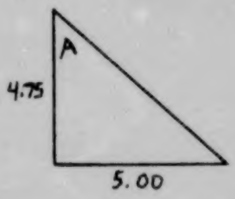
It is necessary to use calculations involving right triangles in most of the measuring, layout, and machining problems of the machine trades. The initial and sometimes the most difficult problem is to locate the triangle in the situation and then to determine the trigonometric function to use.

OBJECTIVE:

To provide practice in finding the values of various sides and angles in a right triangle.

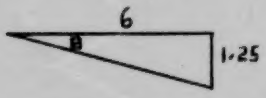
1. Find:

   a. The length of the side opposite ∠A = ______
   b. The length of the side adjacent to ∠A = ______
   c. tan∠A = ______



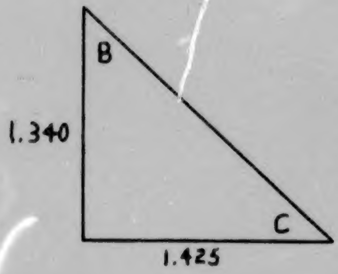
2. Find:

   a. The length of the side opposite ∠B = ______
   b. The length of the side adjacent to ∠B = ______
   c. tan∠B = ______



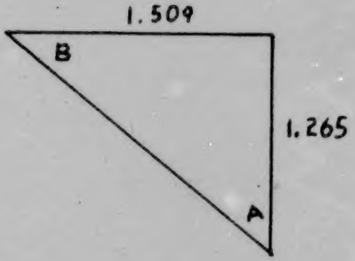
3. Find:

   a. The length of the side opposite ∠C = ______
   b. The length of the side adjacent to ∠C = ______
   c. tan ∠C = ______
   d. m∠C = ______
   e. m∠B = ______


B
1.340
C
1.425

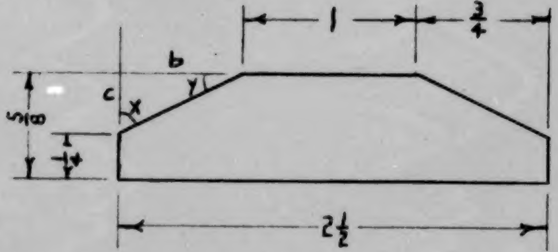
4. Find:

   a. The length of the side opposite ∠A = ______
   b. The length of the side opposite ∠B = ______
   c. tan ∠B = ______
   d. tan ∠A = ______

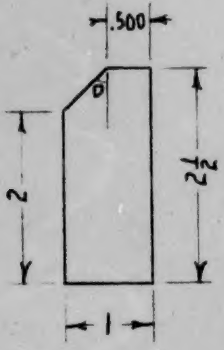

1.509
B
1.265
A

5. In the drawing of the depth gage base below, find:

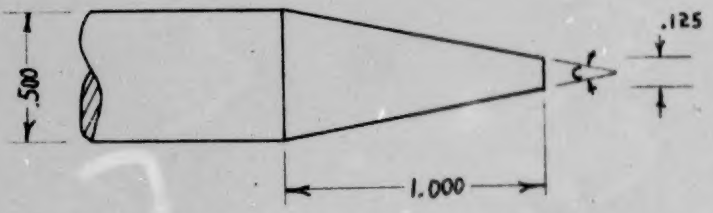
   a. The length of side c = ______
   b. The length of side b = ______
   c. m∠X = ______
   d. m∠Y = ______


1
3/4
b
Y
c
X
5/8
1/4
2 1/2

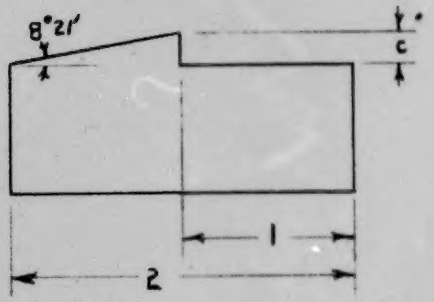
6. In the figure below, m∠D = ______.


.500
D
2
$2\frac{1}{2}$
1

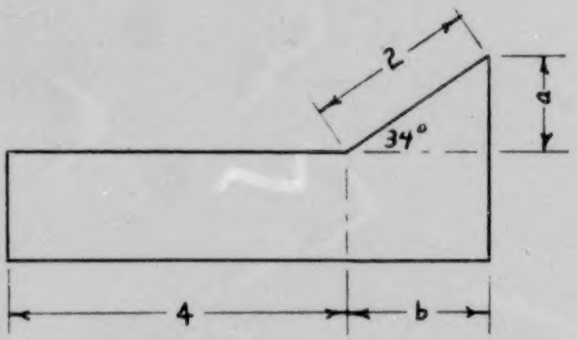
7. In the figure below, m∠C = ______.


.500
C
.125
1.000

8. In the figure below, c = ______.



9. In the figure below, a = ______.



10. In the figure in problem 9 above, b = ______.

ANSWERS

1.

   a. 5.00
   b. 4.75
   c. 1.0526

2.

   a. 1.25
   b. 6
   c. .20833

3.

   a. 1.340
   b. 1.425
   c. .94035
   d. 43°14'
   e. 46°46'

4.

   a. 1.509
   b. 1.265
   c. .83830
   d. 1.1929

5.

   a. .375
   b. .75
   c. 63°26'
   d. 26°34'

6. 45°

7. 21°14'

8. .147

9. 1.118

10. 1.658

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Problems Involving R.P.M. and Cutting Speed

INTRODUCTION:

R.P.M. and cutting speeds are vital factors when considering a set-up. Many factors should be considered. The cutting speeds for various metals, the size of the workpiece in terms of the diameter, the type of tool bit used, the power factor of the machine, the sturdiness of the machine, and the coolants are some of the factors.

OBJECTIVES:

To provide the student an opportunity to learn how to calculate the number of revolutions per minute and the cutting speed.

TECHNICAL INFORMATION:

In a circle the radius r is defined as the distance from the center of the circle to any point on the circle. The diameter d is twice the radius. That is, $d = 2r$.

The circumference, C, is the distance measured completely around the circle. A string could be placed around the circle. Then, it could be stretched out in a straight line and the length measured. This length would be the circumference of the circle. The circumference can be evaluated if the radius or the diameter is known by the use of the formulas:

$$C = 2\pi r$$

or since $d = 2r$, then

$C = \pi d$ (π is an irrational number which is approximately 3.142.)

Example 1. If the radius of a circle is 3", find the circumference. (See Figure 1)

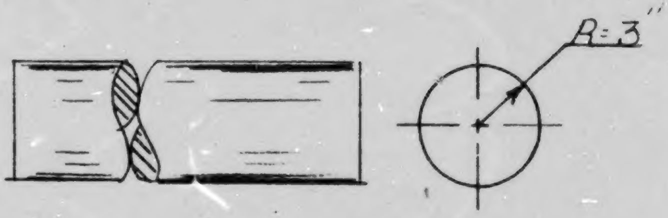

Figure 1

$$C = 2\pi r$$
$$= 2(3.142)3$$
$$= (6.284)3$$
$$= 18.852''$$

If on a lathe, a piece of round stock is being turned, then every time the stock makes one complete turn, we say it has completed 1 revolution. The cutting speed is the total distance around the stock which passes by the tool bit in a time interval of one minute. Thus, the cutting speed will be equal to the circumference of the piece of stock times the number of revolutions made in one minute. If N represents the number of revolutions per minute (or R.P.M.) then CS = C·N where CS represents the cutting speed, and C represents the circumference of the piece of stock.

Since $C = \pi d$, then the formula for cutting speed becomes, $CS = \pi dN$. If d is given in inches this formula will give us the cutting speed in inches per minute. Normally, we wish to express the cutting speed in feet per minute. Therefore, we need to divide by 12 so that the resulting cutting speed will be in feet per minute instead of inches per minute. Note that we divide by 12 since 1 foot = 12 inches. Therefore, the formula we will use is as follows:

$$CS = \frac{\pi dN}{12}$$

(where CS is the cutting speed in feet per minute, d is the diameter (expressed in inches), and N is the number of revolutions per minute)

Example 2. If the number of revolutions per minute is 125 and the diameter is 2", what is the cutting speed?

$$\begin{aligned} CS &= \frac{\pi dN}{12} \\ &= \frac{(3.142)(2)(125)}{12} \\ &= \frac{(3.142)(125)}{6} \\ &= \frac{392.750}{6} \\ &= 65.46 \text{ feet per minute} \quad \text{(to the nearest hundredth)} \end{aligned}$$

Example 3. A piece of steel 2.5 inches in diameter is to be turned in a lathe. What number of revolutions is necessary to give a cutting speed of 80 feet per minute?

Since in this example we wish to find N, we need to first of all solve the formula for CS in terms of N.

$$CS = \frac{\pi dN}{12}$$

$$12(CS) = 12\cdot\frac{\pi dN}{12} \qquad \text{(Mult. both sides by 12)}$$

$$12(CS) = \pi dN \qquad \text{(Mult. inverse)}$$

$$\frac{1}{\pi d}\cdot 12(CS) = \frac{1}{\pi d}\cdot dN \qquad \text{(Mult. both sides by 1/ d)}$$

$$\frac{12(CS)}{\pi d} = N \qquad \text{(Mult. inverse)}$$

Therefore, for Example 2:

$$N = \frac{12(CS)}{\pi d}$$

$$= \frac{12(80)}{3.142(2.5)}$$

$$= \frac{960}{7.855}$$

= 122.21 revolutions per minute (to the nearest hundredth)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: R.P.M. and Cutting Speed Lathe Problems

INTRODUCTION:

Setting up machine speeds is a very important part of machine set-up. The right speed is essential if efficiency is to be considered as an important factor.

OBJECTIVE:

To learn how to solve r.p.m. and cutting speed lathe problems.

ASSIGNMENT:

1. Find: R.P.M.
   Given: Cutting speed of metal = 40 feet per minute
   Diameter of workpiece = 1.125 in.

2. Find: R.P.M.
   Given: Cutting speed of metal = 90 feet per minute
   Diameter of drill bit = 1/2 in.

3. Find: Cutting speed
   Given: R.P.M. = 200
   Diameter = 1.500 in.

4. A job calls for an arbor to be made. The beginning stock size is 1 1/4 in. in diameter, is made of 1020 cold rolled steel and is being turned with a Momax high speed tool bit. What r.p.m. should be used if the cutting speed is 80 feet per minute?

5. A casting of soft grey iron is being milled on a milling machine. A 5 in. high speed milling cutter is to be used. At what r.p.m. should the milling machine be set if the cutting speed is 70 feet per minute?

6. A piece of 1140 steel is to be used for a job. The piece is being turned on a lathe, and a high speed steel bit is used. At what r.p.m. should the lathe be set if the cutting speed is 80 feet per minute and the diameter of the piece is 1 inch?

7. Find the cutting speed in feet per minute of a 3/4 in. piece of steel revolving at 240 revolutions per minute.

8. A turret lathe operator is turning out screws that are 1.25 in. in diameter. He finds that running his lathe at 260 revolutions per minute, he gets a fine finish on the screws. What is the cutting speed?

9. A piece-rate worker has a job of boring brass bushings with inside diameters of 1.75 in. How fast should he run his lathe so that the cutting speed of his tool will be 95 feet per minute?

10. Find the time required to take one cut over a piece of work 14 in. long and 1.5 in. in diameter. The cutting speed is 32 feet per minute, and the feed is 1/20 in. per revolution.

ANSWERS

1. 135.79 revolutions per minute
2. 687.46 revolutions per minute
3. 78.55 feet per minute
4. 244.43 revolutions per minute
5. 53.47 revolutions per minute
6. 305.54 revolutions per minute
7. 47.13 feet per minute
8. 85.10 feet per minute
9. 207.33 revolutions per minute
10. 3.44 min.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Problems Concerning the Jarno Taper

INTRODUCTION:
In the Jarno system, the taper is .6 inches per foot, and the number of the taper is the key by which all of the dimensions are immediately determined.

OBJECTIVES:
To provide the student an opportunity to learn how to calculate the taper in the Jarno system.

TECHNICAL INFORMATION:

The number of the taper is the number of tenths of an inch in diameter at the small end, the number of eights of an inch at the large end, and the number of halves of an inch in length or depth.

In the Jarno system:

Taper per foot = .6 inch

Taper per inch = .05 inch

$$\text{Diameter at Large End} = \frac{\text{No. of Taper}}{8}$$

$$\text{Diameter at Small End} = \frac{\text{No. of Taper}}{10}$$

$$\text{Length of Taper} = \frac{\text{No. of Taper}}{2}$$

APPLICATION OF THE RULE:

Example 1. Find the diameters and length for a number 6 Jarno Taper.

$$\text{Diameter at Large End} = \frac{\text{No. of Taper}}{8}$$

$$= \frac{6}{8} \text{ inch}$$

$$= \frac{2 \cdot 3}{2 \cdot 4} \text{ inch}$$

$$= \frac{3}{4} \text{ inch}$$

$$= .75 \text{ inch}$$

$$\text{Diameter at Small End} = \frac{\text{No. of Taper}}{10}$$

$$= \frac{6}{10} \text{ inch}$$

$$= \frac{2 \cdot 3}{2 \cdot 5} \text{ inch}$$

$$= \frac{3}{5} \text{ inch}$$

$$= .6 \text{ inch}$$

$$\text{Length of Taper} = \frac{\text{No. of Taper}}{2}$$

$$= \frac{6}{2} \text{ inches}$$

$$= 3 \text{ inches}$$

Example 2. Find the diameters and length for a number 7 Jarno Taper.

$$\text{Diameter at Large End} = \frac{7}{8} \text{ inch}$$

$$= .875 \text{ inch}$$

$$\text{Diameter at Small End} = \frac{7}{10} \text{ inch}$$

$$= .7 \text{ inch}$$

$$\text{Length of Taper} = \frac{7}{2} \text{ inches}$$

$$= 3.5 \text{ inches}$$

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Problems Concerning the Jarno Taper

INTRODUCTION:

In the Jarno system it is quite easy to determine the diameter at the large end, the diameter at the small end, and the length of the taper just given the number of the Jarno taper.

OBJECTIVE:

To provide the student practice in evaluating the diameters and the length of the taper for a Jarno taper.

ASSIGNMENT:

1. Find the diameters and length for a number 2 Jarno taper.
2. Find the diameters and length for a number 3 Jarno taper.
3. Find the diameters and length for a number 4 Jarno taper.
4. Find the diameters and length for a number 5 Jarno taper.
5. Find the diameters and length for a number 8 Jarno taper.
6. Find the diameters and length for a number 9 Jarno taper.
7. Find the diameters and length for a number 10 Jarno taper.
8. Find the diameters and length for a number 12 Jarno taper.
9. Find the diameters and length for a number 14 Jarno taper.
10. Find the diameters and length for a number 20 Jarno taper.

ANSWERS

	Diameter at large end	Diameter at small end	Length
1.	.250 in.	.200 in.	1.0 in.
2.	.375 in.	.300 in.	1.5 in.
3.	.500 in.	.400 in.	2.0 in.
4.	.625 in.	.500 in.	2.5 in.
5.	1.000 in.	.800 in.	4.0 in.
6.	1.125 in.	.900 in.	4.5 in.
7.	1.250 in.	1.000 in.	5.0 in.
8.	1.500 in.	1.200 in.	6.0 in.
9.	1.750 in.	1.400 in.	7.0 in.
10.	2.500 in.	2.000 in.	10.0 in.

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Calculations--Tapering by the Offset Tailstock Method

INTRODUCTION:

The offset method is generally used on long tapers when the taper attachment will not suffice for the desired length. It is more accurate than given credit, provided that the amount of offset is properly set up with a dial indicator, and the centers are properly center drilled. However, it is not used in industry as frequently as the compound rest and taper attachment methods.

OBJECTIVES:

1. To provide the student an opportunity to learn how to calculate the offset of a tailstock for a specified taper.
2. To provide the student an opportunity to learn how to work with formulas in order to find unknown dimensions.

TECHNICAL INFORMATION:

The amount of taper offset can be determined by using different formulas depending upon the information given on the drawing. Many times one is faced with a machining problem in which he knows the TPF and desires to find the values for specific dimensions. In this case he must be able to solve the given formula for the desired unknown.

Taper is defined as the change in the diameter (expressed in inches). The taper per inch is defined as the change in diameter (in inches) for each inch in length.

Therefore:

$$\text{Taper per inch} = \frac{\text{change in diameter (in inches)}}{\text{length (in inches)}}$$

$$\text{Taper per inch} = \frac{D - d}{L} \qquad \text{(See Figure 1)}$$

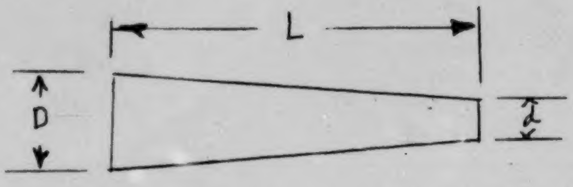


Figure 1

For example if: D = 2", d = 1", and L = 12", then:

$$\text{Taper per inch} = \frac{2 - 1}{12}$$

$$= \frac{1}{12} \text{ or } .08333$$

Thus, the diameter changes 1 inch for each 12 inches of length.

In most situations, it is desirable to express the ratio as taper per foot (TPF) instead of taper per inch. That is, the denominator (the length) is expressed in feet instead of inches. If L is given in inches, then $\frac{L}{12}$ will express the length in terms of feet (since 1 foot = 12 inches).

Therefore:

$$\text{Taper per foot} = \frac{D - d}{\frac{L}{12}} \qquad \text{(where D, d, and L are in inches)}$$

Then:

$$\text{TPF} = \frac{\frac{D - d}{1}}{\frac{L}{12}} \qquad \left(\frac{a}{1} = a\right)$$

$$= \frac{D - d}{1} \cdot \frac{12}{L} \qquad \left(\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}\right)$$

$$= \frac{(D - d)12}{L} \qquad \left(\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d},\ a \cdot 1 = a\right)$$

$$\text{TPF} = \frac{12(D - d)}{L} \qquad \text{(commutative property of multiplication)}$$

In addition to this formula, two other basic formulas are utilized:

$$\text{Tailstock set over} = \frac{(\text{total length})(D - d)}{2(\text{length of tapered portion})} \text{ inches}$$

$$\text{Tailstock set over} = \frac{(\text{total length})(\text{TPF})}{24} \text{ inches}$$

The use of these formulas depends on the particular problem.

APPLICATION OF THE RULE:

Example 1. (For problems when diameters at the taper ends are given) Compute the amount of tailstock offset needed for making a mandrel as the drawing in Figure 2 specifies.

We use the formula:

$$\text{Tailstock set over} = \frac{(\text{total length})(D - d)}{2(\text{length of tapered portion})} \text{ inches}$$

$$= \frac{(6)(1.000 - .995)}{2(4)}$$

$$= \frac{6(.005)}{2(4)}$$

$$= \frac{2(3)(.005)}{2(4)}$$

$$= \frac{3(.005)}{4}$$

$$= \frac{.015}{4}$$

Tailstock set over = .00375 inches

or .004 inches (to the nearest thousandth)

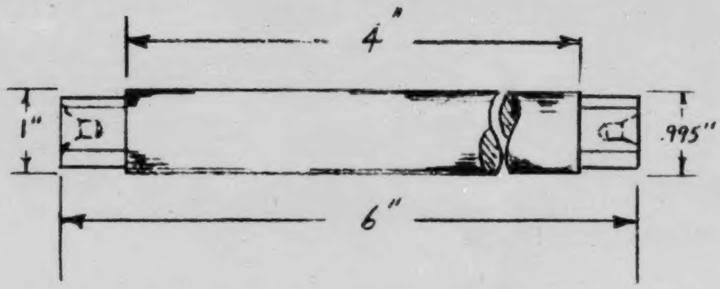


Figure 2

Example 2. (For problems when TPF is given)
Compute the amount of tailstock offset for the plug gage when the TPF is .050. See Figure 3.

We use the formula:

$$\text{Tailstock set over} = \frac{(\text{total length})(\text{TPF})}{24} \text{ inches}$$

$$= \frac{6(.050)}{24}$$

$$= \frac{6(.050)}{6 \cdot 4}$$

$$= \frac{.050}{4}$$

Tailstock set over = .0125 inches

or .013 inches (to the nearest thousandth)

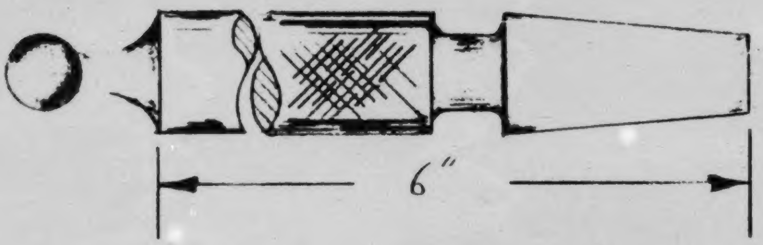


Figure 3

Example 3. (For problems when the diameter at the large end of the taper is to be determined) Suppose you were given a tapered shaft which had a standard Brown and Sharpe taper of .500" per foot; however, the large diameter end was damaged, and you had to find out its dimension in order to procure the proper diameter stock. How could you find the value of D? See Figure 4.

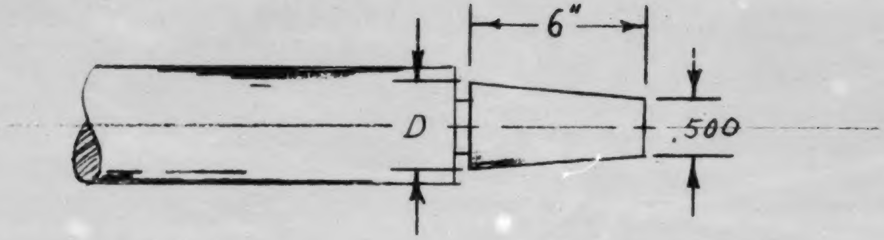


Figure 4

We need to solve the formula

$$TPF = \frac{12(D - d)}{L}$$

for D.

$$TPF = \frac{12(D - d)}{L}$$

$$(TPF)L = \frac{12(D - d)}{L} \cdot L \qquad \text{(Multiply both sides by L)}$$

$$(TPF)L = 12(D - d) \qquad \text{(Multiplicative inverse)}$$

$$(TPF)L = 12D - 12d \qquad \text{(Distributive property)}$$

$$(TPF)L + 12d = 12D - 12d + 12d \qquad \text{(Add 12d to both sides)}$$

$$(TPF)L + 12d = 12D \qquad \text{(Additive inverse)}$$

$$\frac{1}{12} \cdot (TPF)L + 12d) = \frac{1}{12} \cdot 12D \qquad \text{(Multiply both sides by 1/12)}$$

$$\frac{(TPF)L + 12d}{12} = D \qquad \text{(Multiplicative inverse)}$$

$$\text{or} \qquad D = \frac{(TPF)L + 12d}{12} \text{ inches}$$

Therefore, in our problem:

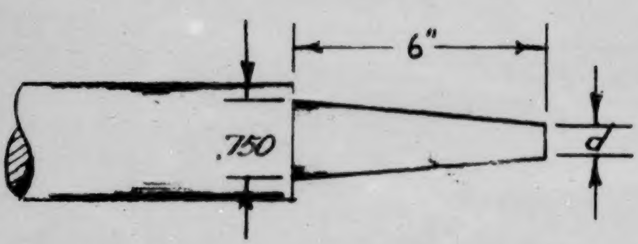
$$D = \frac{(.500)6 + 12(.500)}{12}$$

$$= \frac{3 + 6}{12}$$

$$= \frac{9}{12}$$

$$D = .750 \text{ in.}$$

Example 4. (For problems when the diameter at the small end of the taper is to be determined)
Find the value of d in Figure 5.



TPF = .500

Figure 5

We need to solve the formula

$$TPF = \frac{12(D - d)}{L}$$

for d.

Similar to the procedure in Example 3, we find that:

$$d = \frac{12D - (TPF)L}{12} \text{ inches}$$

Therefore in our problem:

$$d = \frac{12(.750) - (.500)6}{12}$$

$$= \frac{9 - 3}{12}$$

$$= \frac{6}{12}$$

$$d = .500 \text{ in.}$$

Example 5. (For problems where the length of the tapered section (L) is to be found)
Find the length of L in Figure 6 if TPF = .500.

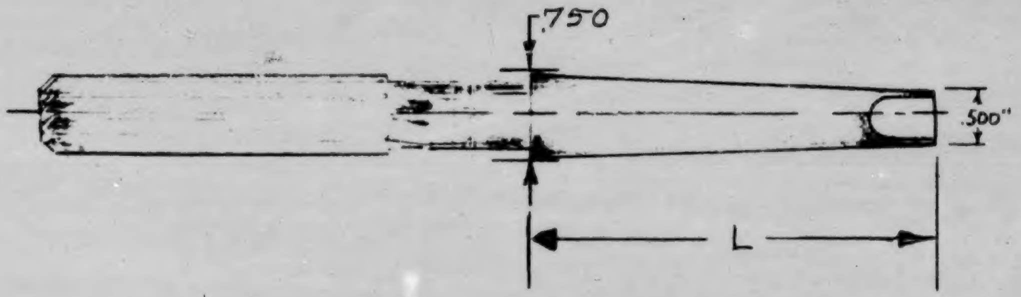


Figure 6

We need to solve the formula

$$\mathrm{TPF} = \frac{12(D - d)}{L}$$

for L.

$$\mathrm{TPF} = \frac{12(D - d)}{L}$$

$$(\mathrm{TPF})L = \frac{12(D - d)}{L} \cdot L \qquad \text{(Multiply both sides by L)}$$

$$(\mathrm{TPF})L = 12(D - d) \qquad \text{(Multiplicative inverse)}$$

$$\frac{1}{\mathrm{TPF}} \cdot (\mathrm{TPF})L = \frac{1}{\mathrm{TPF}} \cdot 12(D - d) \qquad \text{(Multiply both sides by 1/TPF)}$$

$$L = \frac{12(D - d)}{\mathrm{TPF}} \text{ inches} \qquad \text{(Multiplicative inverse)}$$

Therefore, in our problem:

$$L = \frac{12(.750 - .500)}{.500}$$

$$= \frac{12(.250)}{.500}$$

$$= \frac{3}{.500}$$

$$L = 6 \text{ in.}$$

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Calculations--Tapering by the Offset Tailstock Method

INTRODUCTION:

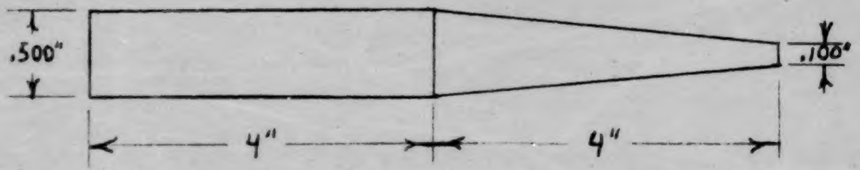
The offset method may not be used as often as it should for long tapers between centers. It can be a very accurate method when using the dial-indicator for measuring the proper offset. It has no longitudinal backlash as some taper attachments, unless equipped with tracer attachments controlled by hydraulics.

OBJECTIVE:

To learn how to solve problems involving tapers when using the offset tailstock method for machining a taper.

ASSIGNMENT:

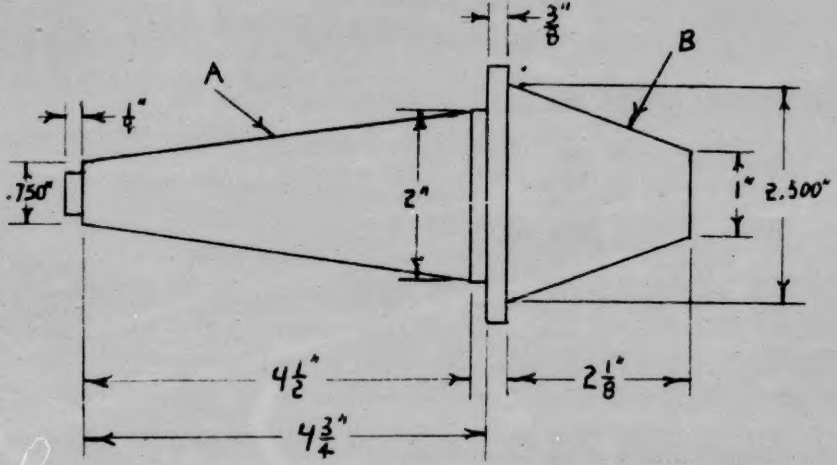
1. In the figure below find the amount of tailstock offset needed to machine the taper.



2. Find: amount of offset
   Given: total length = 8.000 in.
   TPF = .500 in.

3. Find the amount of offset needed for cutting the following tapers on pieces 1 inch long.
   a. A No. 1 Morse Taper
   b. A No. 5 Browne and Sharpe Taper
   c. A No. 7 Jarno Taper

4. In the figure below find the amount of tailstock set-over to cut
   a. A
   b. B



5. Find the amount of tailstock offset required to cut a taper of 1/4 in. per inch on a piece 8 inches long.

6. A No. 7 Morse taper plug has a taper of 0.052 in. per inch. The length of the plug is 16 5/8 in., and the small diameter is 2.75 in. What offset was necessary for this job?

7. An apprentice has an order to make 15 standard taper pins. The taper on these pins is 0.250 in. per foot. The stock is 3/4 in. long and is 1.233 in. in diameter at the large end. Find the tailstock offset.

8. Find the amount of tailstock offset in turning the tapered portion of a crosshead pin whose overall length is 8 1/2 in., the length of the tapered part is 6 1/2 in., and the large diameter is 4 1/8 in. and the small diameter is 3 11/16 inch.

Note: For this problem we can use the formula

$$\text{Offset} = \frac{D - d}{L_t} \times \frac{E.L.}{2}$$

where D = large diameter, d = small diameter, E.L. = entire length, L_t = length of tapered part.

ANSWERS

1. .400 in.

2. .167 in.

3. a. .025 in.
   b. .021 in.
   c. .025 in.

4. a. 1.042 in.
   b. 2.647 in.

5. 1.000 in.

6. .432 in.

7. .008 in.

8. .286 in.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Taper Cutting--Offset Tailstock Method

COURSE UNIT TITLE: Lathe

INTRODUCTION:

Tapers are used on various tools such as reamers, lathe centers, cotter pins, mandrels, punches, arbors, and shanks for chucks, etc. Brown and Sharpe, Morse, and Jarno are the most common tapers used.

Offsetting the tailstock is accurate because the offset can be set up with a dial-indicator very precisely. It is used only when the work is being turned between centers. This is good for long tapers. The only real disadvantage is that of wear on the centers.

TOOLS AND MATERIALS REQUIRED:

- Dial-indicator
- Micrometer
- Rule
- Round piece of steel stock
- Right hand tool-bit
- Center-drill
- Jacob chuck

PROCEDURE:

(Operation)	(Related Information)
1. Prepare stock by facing off the end. Then center-drill.	1. Do on both ends.
2. Calculate set-over of tailstock.	2. See Technical Information Sheet
3. Mount dial-indicator on the tool post. See Figure I.	3. See Figure I.
4. Feed the crossfeed in toward the tailstock spindle.	4. Lock the spindle
5. Set dial-indicator and the micrometer dial on the crossfeed on zero.	5. This must be set accurately.
6. Feed outward with the crossfeed for the exact amount of set-over plus at least one extra turn for	6. Do not forget the backlash.

backlash. Now turn in again to compensate for the backlash and stop when the indicator shows zero.

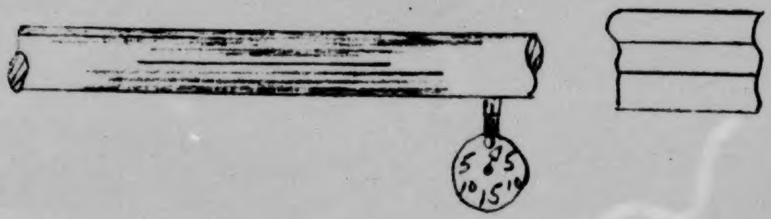


Figure I

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Calculations--Compound Rest Tapering

INTRODUCTION:

The lathe compound rest is used extensively, especially when the tapers are too large (see Figure 1) to turn by offsetting the tailstock or of a nature that the taper attachment cannot be used.

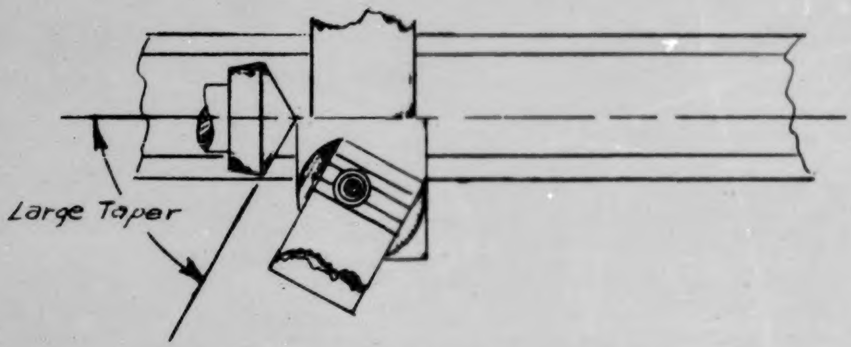

Figure 1

OBJECTIVES:

To provide the student the opportunity to learn how to calculate the proper angle from a given drawing in order to set a compound rest.

TECHNICAL INFORMATION:

There are two systems or ways that tapers are generally designated:

either by giving the degrees or by taper per foot (TPF). If the taper is given in terms of TPF (taper per foot) the angle at which the compound rest is to be set can be calculated by the following formula:

$\tan \angle A = \frac{TPF}{24}$ where $m\angle A$ is 1/2 the measure of the included angle

Then look up the resulting value of the quotient in the trigonometric tables and find the value for A in degrees.

APPLICATION OF THE RULE:

Example 1. Set the compound rest for a taper of 42° 22' starting from the longitudinal axis of the lathe. See Figure 2.

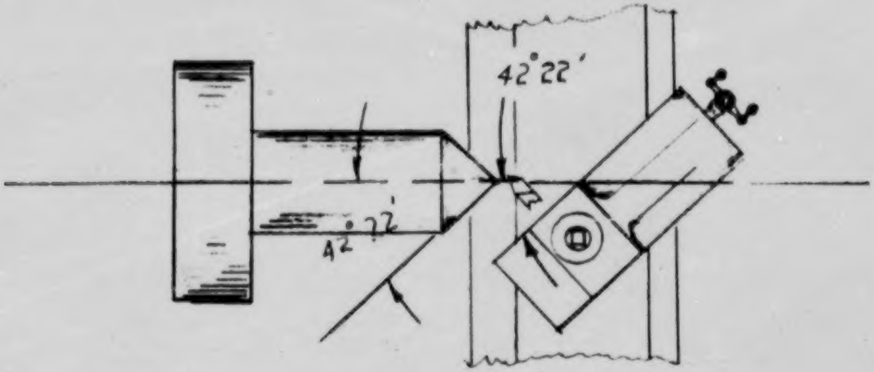

Figure 2

Example 2. Find the angle A for a 4" TPF.

Use the formula:

$\tan \angle A = \frac{TPF}{24}$

Then:

$$\tan \angle A = \frac{4}{24}$$

$$= \frac{4}{4 \cdot 6}$$

$$= \frac{1}{6}$$

$$= .16667$$

Now, we must find the value of A from the tables.

$A = 9^{\circ}\ 27'\ 46''$ or $9^{\circ}\ 28'$ to the nearest minute.

This answer of $9^{\circ}\ 28'$ indicates that the compound rest should be swiveled out of parallel with the axis of the work.

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Calculations--Compound Rest Tapering

INTRODUCTION:

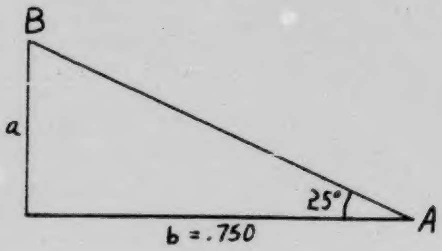
The compound rest is probably the most used accessory on a lathe. Knowing how to set it up for correct taper cutting is of utmost importance. Usually, short tapers are cut by the compound rest for exterior as well as interior angles.

OBJECTIVE:

To learn how to solve problems pertaining to the compound rest.

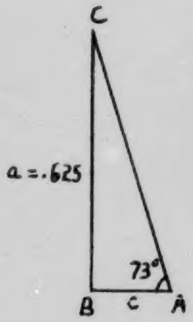
ASSIGNMENT:

1. What degree measure would you set the compound rest for a .750" TPF?
2. What degree measure would you set the compound rest for the following tapers?
   a. 2.000" TPF
   b. .250" TPF
   c. .625" TPF
3. For each of the following right triangles find the requested information.



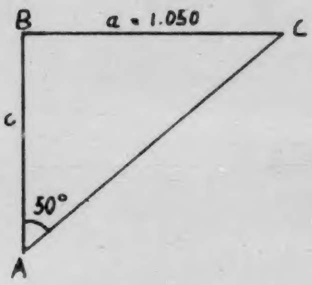
$m\angle B$ = ________

a = ________



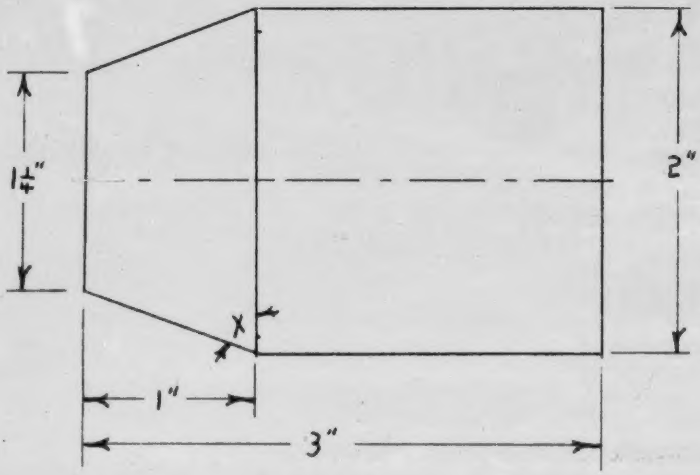
$m\angle C$ = ________

c = ________



m C = ______
c = ______

4. In the figure below, find the angle to set the compound rest for cutting angle X.

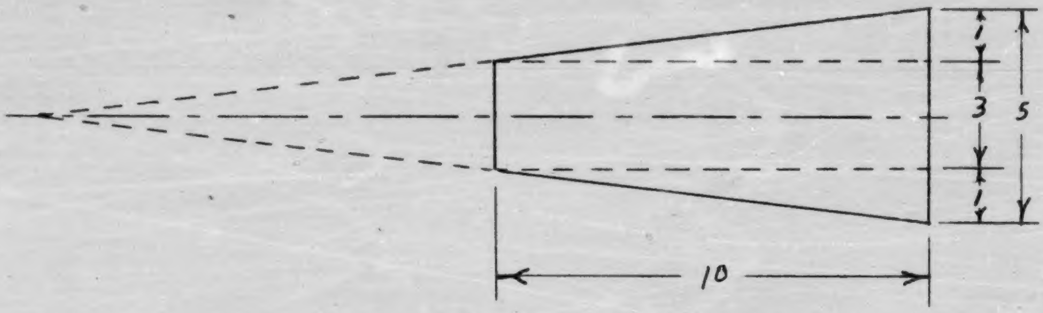


5. A milling machine face mill adapter is to be made to fit a milling machine having a National Standard milling machine taper. At what angle should the compound rest be set to cut this taper?

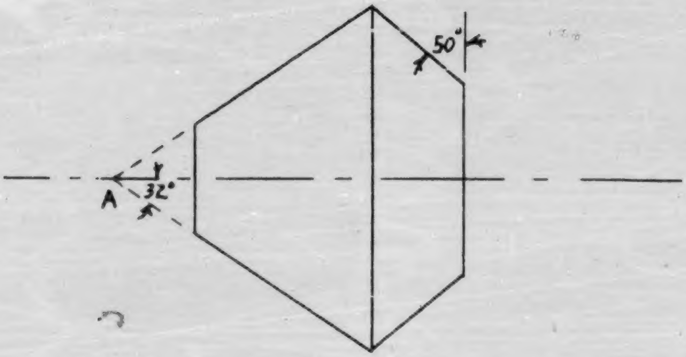
6. A countersink has an angle of 77° at its point. At what angle should the compound rest be set to cut the taper?

7. At what angle should the compound rest be set to cut the taper for a No. 7 Jarno taper?

8. Compute the setting for the compound rest for cutting the external taper on the piece shown below.



9. The compound rest is used to turn the angles on the bevel gear blank in the figure below. Find the degree measure to set the compound rest.



ANSWERS

1. $1^\circ 47'$ (or $88^\circ 13'$)

2. a. $4^\circ 46'$ (or $85^\circ 14'$)
   b. $0^\circ 36'$ (or $89^\circ 24'$)
   c. $1^\circ 30'$ (or $88^\circ 30'$)

3. $m\angle B = 65^\circ$
   $a = .350$ in.

   $m\angle C = 17^\circ$
   $c = .191$ in.

   $m\angle C = 40^\circ$
   $c = .881$ in.

4. $20^\circ 33'$ (or $69^\circ 27'$)

5. $8^\circ 18'$ (or $81^\circ 42'$)

6. $38^\circ 30'$ (or $51^\circ 30'$)

7. $1^\circ 26'$ (or $88^\circ 34'$)

8. $5^\circ 43'$ (or $84^\circ 17'$)

9. 32° (or 58°)
   40° (or 50°)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Taper--Compound Rest

COURSE UNIT TITLE: Lathe

INTRODUCTION:

One of the most used methods for turning short tapers is the compound rest method. It is used for turning centers, various tapers on punches, dies, tools, inserts, extruding dies, progressive dies, trim dies, and many more jobs which have both internal and external tapers.

OBJECTIVE:

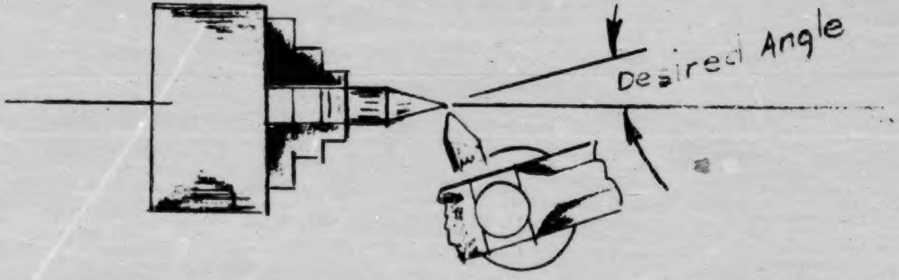
To provide an opportunity for the student to learn how to turn a taper with the use of a compound rest.

TOOLS AND MATERIALS REQUIRED:

Right hand tool bit
Protractor
Piece of round stock

PROCEDURE:

(Operation)	(Related Information)
1. Secure workpiece in chuck.	1. It can be any type chuck or collet.
2. Set the compound rest to the desired angle or taper.	2. See Technical Information Sheet for calculations.
3. Set tool-bit perpendicular (normal) to the cutting angle.	3. The easiest way to remember this is to set it perpendicular to the compound rest.
4. Set the tool-bit on the center of the longitudinal axis.	4. The tool-bit must be on center or the taper will not be exact.
5. Feed compound rest toward chuck.	5. Feed evenly for smooth unit.


Desired Angle

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Calculations--Taper Attachment Method

INTRODUCTION:

Lathes are generally equipped with a taper attachment; however, it is an option. Most tapers that are long (within the limits of the taper attachment bar) are turned with the taper attachment, especially if the workpiece is held in a chuck. The accuracy depends on the rigidity of the taper attachment as well as the set-up itself. Some taper attachments do not work freely while others are too loose and move under force. This must be accounted for when turning a taper.

OBJECTIVE:

To provide a student an opportunity to learn how to calculate the taper per foot and/or the angle of a taper.

TECHNICAL INFORMATION:

The taper attachments are generally equipped and calibrated on both ends of the taper attachment bar. On one end it is calibrated with TPF settings, and on the other end it is calibrated in degrees for angular settings.

APPLICATION OF THE RULE:

Example I. Compute the setting in taper per foot (TPF) for the taper attachment for the plug gage in Figure I.

$$TPF = \frac{12(D - d)}{\text{Length of Taper Portion}}$$

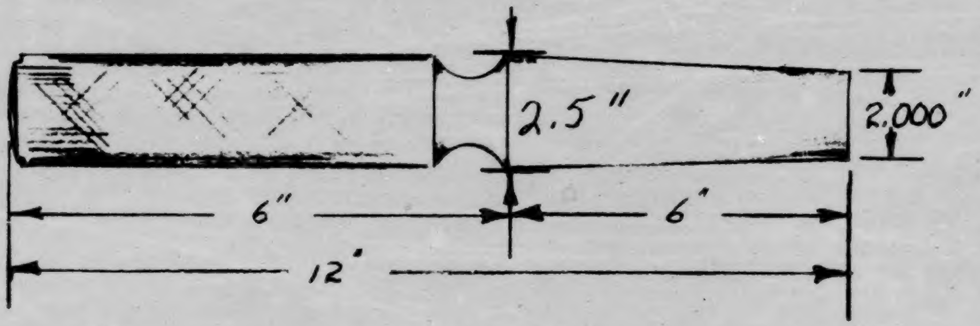


Figure I

$$\text{TPF} = \frac{12(2.500 - 2.000)}{6}$$

$$= \frac{12(.500)}{6}$$

$$= \frac{6}{6}$$

$$= 1.000$$

Note that in the given formula, D is the large diameter at one end of the taper, and d is the small diameter at the other end of the taper.

Example 2. Find the angle of the taper in Example I. See Figure 2.

Since in reference to angle A, we know the length of the opposite side is .250 and the length of the adjacent side is 6, the best trigonometric functions to use would be the tangent or cotangent of angle A. Let us use the tangent function.

$$\tan A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{.250}{6}$$

$$= .04167$$

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Calculations--Taper Attachment Method

INTRODUCTION:

Most lathes are equipped with a taper attachment. Tapers are set up by knowing the amount of degree measure of the taper in inches per foot.

OBJECTIVE:

To learn how to solve problems for setting up a taper attachment.

ASSIGNMENT:

1. Find: TPF
   Given: Large diameter = .500 in.
   Small diameter = .100 in.
   Length of tapered portion = 4.000 in.

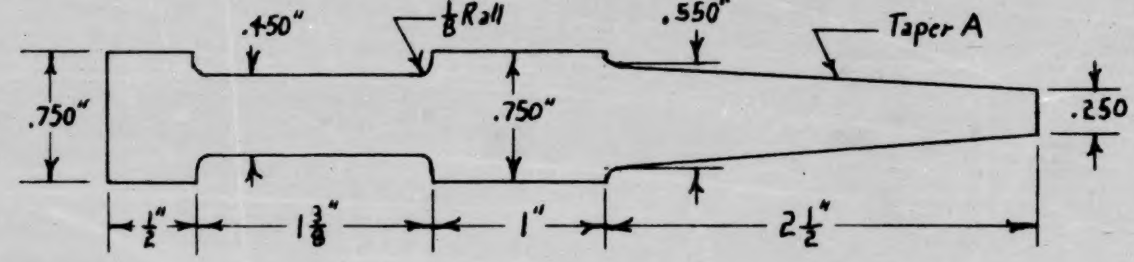
2. Find: Angle for the taper
   Given: Length of tapered portion = 6.000 in.
   Large diameter = .500 in.
   Small diameter = .100 in.

3. Find the amount of taper per foot in each of the following in order to set the taper attachment.
   a. No. 9 Jarno Taper
   b. No. 50 National Milling Machine Taper
   c. No. 4 Morse Taper
   d. No. 5 Browne and Sharpe Taper

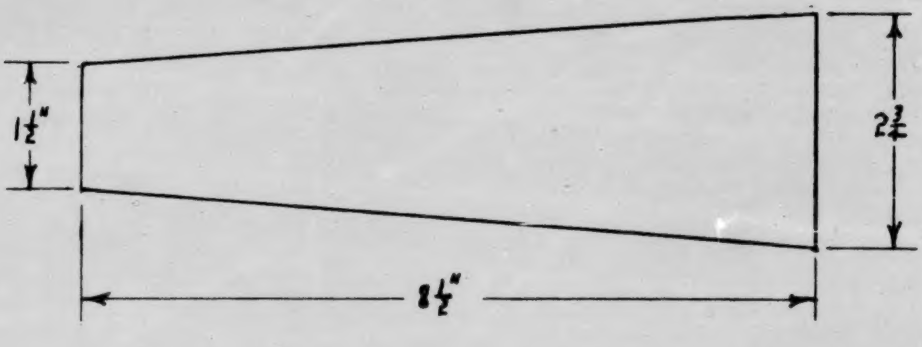
4. Using the figure for exercise 4 on the previous Technical Assignment Sheet, "Calculations--Tapering by the Offset Tailstock Method," find:
   a. TPF for angle A
   b. TPF for angle B

5. A piece is 10 inches overall, and the tapered portion is 5 inches long. The small diameter is 2 inches, and the large diameter is 3 inches. What is the angle for the taper?

6. Find the amount of taper per foot in taper A in the figure below.


.450"
1/8 R all
.550"
Taper A
.750"
.750"
.250
1/2"
1 3/8"
1"
2 1/2"

7. Find the Angle for the taper shown in the figure below.



ANSWERS

1. 1.200 in.

2. $1^{\circ}55'$ (to the nearest minute)

3. a. .600 in.
   b. 3.5 in.
   c. .6233 in.
   d. .500 in.

4. a. 3.333 in.
   b. 8.471 in.

5. $5^{\circ}43'$ (to the nearest minute)

6. 1.440 in. (approximately)

7. $4^{\circ}12'$ (to the nearest minute)

MODERN MATHEMATICS
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OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Taper--Taper Attachment Method

COURSE UNIT TITLE: Lathe

INTRODUCTION:

Using the taper attachment method has certain advantages: the centers are always in line; a taper of a given amount may be cut independent of length; and the method can be used either for external or internal surfaces.

OBJECTIVE:

To provide the student an opportunity to set up and develop manipulative skills in turning a taper using the taper attachment method.

TOOLS AND MATERIALS REQUIRED:

Micrometer
Rule
One piece of round stock

PROCEDURE:

(Operation)	(Related Information)
1. Secure workpiece between centers.	1. If work is such that it requires centers.
2. Disengage the crossfeed screw.	2. Remove the screw that holds the crossfeed control nut on the saddle.
3. Attach the connecting slide arm to the crossfeed. (See Figure 1)	3. Tighten handle.
4. Engage the taper attachment on the ways and fasten it with the setscrew. (See Figure 2)	4. Tighten securely.
5. Set the taper slide bar at the angle or taper in inches per	5. Be careful with the angle.

foot desired, and clamp it in position with the set-screws. (See Technical Information Sheet)

6. Make cut.

6. Start back enough to compensate for the backlash.

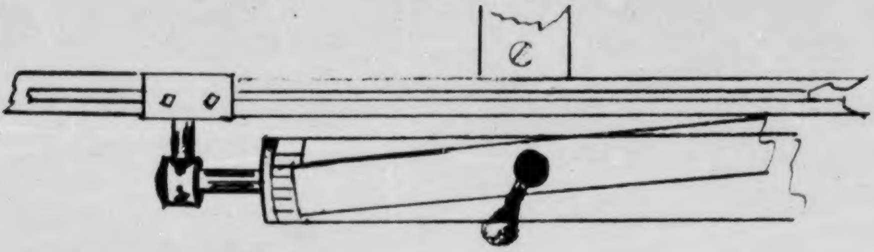

Figure I

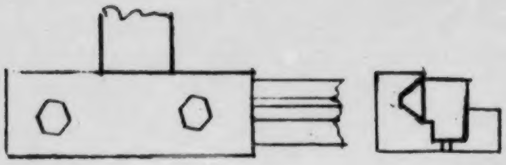

Figure 2

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INTRODUCTION TO SCREW THREADS

The screw thread is one of the mechanical devices which has been used by man and/or industry for many centuries. The screw thread as we know it today has gone through many stages of development before reaching the position it has today.

To illustrate the early beginnings of the use of the screw thread, the ancient Egyptians invented a water screw to raise water from wells for irrigating their fields. The screw thread was used in Gutenberg's early printing presses. Leonardo DaVinci, the inventor of the square and buttress thread, made many sketches of machines using screw threads of various types.

Even though the screw thread had its start and was used in various forms many years ago, each thread was developed to meet the needs of the particular situation. For this reason there was no standardization or interchangeability of screw threads for many years.

Sir Joseph Whitworth devised the Whitworth Thread system in the middle 1800's. It did appear as if this might become the standardized system. However, shortly after this system was devised, a special committee of the Franklin Institute investigated the screw thread system and adopted the William Sellers thread system. This system later became known as the United States Standard Thread.

However, this system had its range limitations, and in order to overcome these limitations three distinct thread systems were developed to supplement the United States Standard Thread system. These were the S.A.E. (Society of Automotive Engineers), the A.S.M.E. (American Society of Automotive Engineers), and the U.S.S. (United States Standard) thread series. All of these systems used the 60° included angle, but each system had its own range of thread sizes, pitch, pitch diameters, and depths. The form of these threads was the sharp 60° V form. (See Figure I - A)

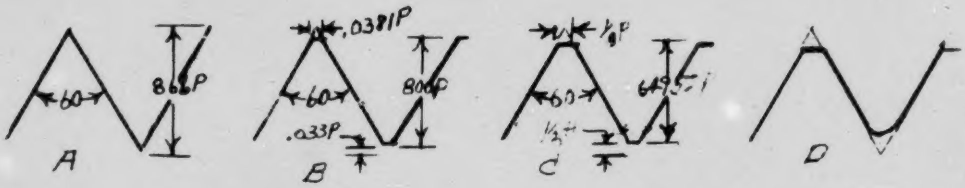

Figure I

In the early 1900's the name was changed to "American National Form" of thread. The S.A.E., A.S.M.E. and the U.S.S. were incorporated into this form. The present general purpose standard threads for the United States are as follows:

1. American National Coarse Thread series
2. American National Fine Thread series
3. American National Extra-Fine Thread series
4. American National 8-pitch Thread series
5. American National 12-pitch Thread series
6. American National 16-pitch Thread series
7. American National Pipe Thread series
8. American National Acme Thread series

Special fields of industry and business established standards in their own special fields. These were as follows:

1. American National hose coupling series
2. American National fire hose coupling thread series

3. American Petroleum Institute of oil-well drilling equipment thread series

4. American National rolled thread series for electric sockets and lamp bases

These standards all became a part of the national standardization of screw threads.

In the middle 1900's the United States, Canada, and Great Britain made a major step in industrial cooperation by agreeing on a standardization of screw threads. Prior to this agreement the screw threads were not interchangeable; that is, U. S. nuts would not fit British bolts, etc. This new standardization effort resulted in the Unified Thread system. Contrary to the belief of many people, this system does not replace the American National Form Thread system; to the contrary, it incorporates and compliments this system. Instead of a thread notation being written as 1/2 - 13 - N.C. - 2 it would be written 1/2 - 13 - U.N.C. - 2B.

To reach the Unified Thread system the 60° V thread (Figure 1 - A) underwent many progressive changes. The sharp crest of the sharp V was unsatisfactory many times in actual use. The sharp crest would peel off causing the threads to become rough and unsatisfactory. This weakness resulted in putting a slight flat on the nose of the threading tool and leaving a flat crest on the thread as shown in Figure 1 - B. This worked much better. Therefore, the present American National Form as shown in Figure 1 - C was adopted.

Because sharp edges are hard to hold on production tools, the Unified Thread system shown in Figure 1 - D was accepted. Really, the radius of the round section at the root represents the cut made by a worn tool. The tolerances on thread fits were made more liberal in the Unified system. This made mass production much easier and even allowed for interchangeability of fits on mating parts; for example, a 2B screw may be used in a 1A nut, etc.

In our vocational machine trade shops we are usually concerned with the following:

1. The Unified and/or American National form of thread in both the coarse and fine series.
2. The 29° General Purpose Acme Thread
3. The Square Thread
4. The 10-degree Modified Square Thread
5. The American Standard Buttress Thread

In reference to numbers 3 and 4 directly above, the Square Thread was one of our earliest thread forms and was satisfactory as long as the craftsman cut individual square threads only. However, the form did not lend itself to mass production of these threads. For this reason the 10° Modified Square Thread form was recently introduced.

It is of utmost importance that teachers and students in the machine trade area keep abreast of the developments in screw threads. It has become a very important part of our industrial production as well as a major factor in the design and development of new machines and tools.

The obsolete thread forms and systems should be de-emphasized and more attention be given to the newly developed thread forms and systems.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Thread Terminology

INTRODUCTION:
Regardless of the type of thread being cut the student should be familiar with the terminology regarding threads. In order to evaluate any of the dimensions associated with threads it is mandatory that the terminology be thoroughly understood in order to choose the appropriate formula.

OBJECTIVE:
To provide an understanding of the terminology associated with threads.

TECHNICAL INFORMATION:

The following are frequently used terms associated with threads:

Screw thread: A cut ridge in the form of a helix on an external or internal surface.

External and internal threads: An external thread is a thread on the outside of the piece. An example is a threaded plug. An internal thread is a thread on the inside of the piece. An example is a threaded hole.

Major diameter: (also known as the outside diameter) This is the largest diameter of the thread (external or internal).

Minor diameter: (also known as the core or root diameter) This is the smallest diameter of the thread (external or internal).

Pitch diameter: The pitch diameter is the mathematical average of the major and minor diameters. Thus, it is a value which is halfway between the major and minor diameter. This is the diameter which is measured by thread micrometers.

Pitch: This is the distance from one point on a screw thread to the corresponding point on the next thread measured on a line parallel to the axis of the screw. The pitch in inches is equal to 1 divided by the number of threads per inch.

Lead: The distance a screw thread advances along the axis of the screw in one turn. On a single thread the lead is equal to the pitch. On a double thread the lead is twice the pitch, and on a triple thread the lead is three times the pitch.

Angle of thread: The angle included between the sides of the thread measured in any plane passing through the axis of the screw. For the American National form thread the angle is 60°. For the Unified Thread form it is 60°, and for the American General Purpose Acme form the included angle is 29°.

Half angle of thread: As implied it is equal to one half of the included angle.

Crest: The top surface of the thread (from which the major diameter is found).

Root: The bottom surface of the thread (from which the minor diameter is found).

Side or flank: The surface of the thread between the crest and the root.

Axis of the screw: The longitudinal line through the center of the screw.

Base of thread: The bottom portion of the thread which lies between two adjacent roots.

Depth of thread: The distance between the crest and the base of the thread measured perpendicular to the axis of the thread.

Number of threads: This refers to the number of threads per inch in length.

Length of engagement: The actual length of contact per thread between the mating parts measured parallel to the axis of the thread.

Depth of engagement: The actual depth of thread contact of the mating parts measured perpendicular to the axis of the thread.

Pitch line: An imaginary line parallel to the screw axis and passing midway between the crests and roots of the thread.

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Calculations--Sharp 60° V-Thread

INTRODUCTION:

When cutting the V Form Threads on a lathe, it is necessary to be able to calculate various dimensions concerning a thread. An understanding of the thread terminology, dimensions, and principles involved in machining are very essential for thread cutting. The Unified Thread is an adaptation of the 60° V-Thread.

OBJECTIVE:

To provide the student an opportunity to learn how to calculate various dimensions of a Sharp 60° V-Thread.

TECHNICAL INFORMATION:

It is necessary to understand the terminology of the screw thread. The two important factors associated with screws are the pitch and the lead of the screw.

The pitch of a screw thread is the distance from any point on a screw thread to a corresponding point on the next thread measured parallel to the axis of the screw. It is measured from a point of one thread to the corresponding point of the next thread.

The lead of a screw is the distance the screw progresses into the work as the screw is given one complete turn. The top or outside edge of the thread is the crest. The major diameter is the diameter measured on the crest.

The inside or bottom of the thread is called the root of the thread.

The root diameter is the diameter measured across the root of the thread.

The depth of the thread is the perpendicular distance from the crest to the root of the thread. The double depth is twice this depth. See Figure 1.

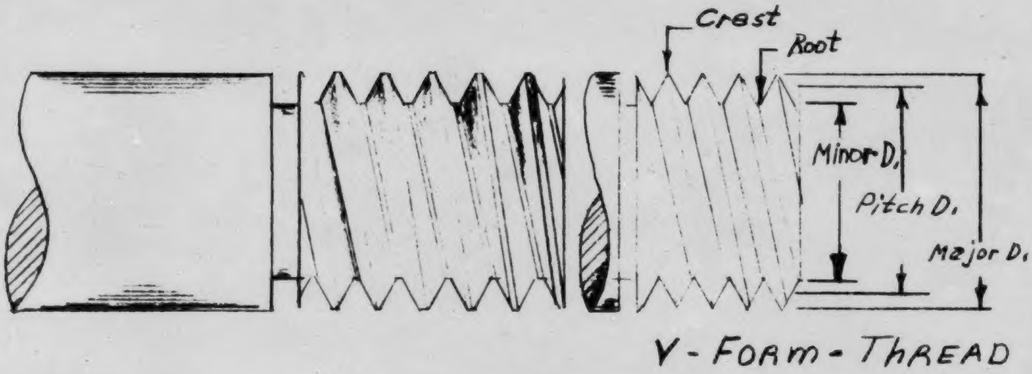

Figure 1

In a V-thread, if a line $\overline{AB}$ is drawn in Figure 2, the triangle formed is an equilateral triangle (a triangle with all three sides having the same length). The pitch is the length of each side. An altitude of a triangle is the perpendicular distance from a given side to the opposite vertex. In Figure 2 it can be seen that the depth H is an altitude of the triangle. The length of this altitude H is .866 times the length of a side. That is:

$$H = .866p$$

Now, if there are n threads per inch, then $n \cdot p = 1$.

Then:

$$n \cdot p = 1$$

$$\frac{1}{n} \cdot n \cdot p = \frac{1}{n} \cdot 1 \qquad \text{(Multiply both sides by 1/n)}$$

$p = \frac{1}{n}$ (Multiplicative inverse, 1 is the multiplicative identity)

Therefore, using the formula that H = .866p together with this formula for p, we find that:

$$H = .866 \cdot \frac{1}{n}$$

$$H = \frac{.866}{n} \qquad (a \cdot \frac{b}{c} = \frac{a \cdot b}{c})$$

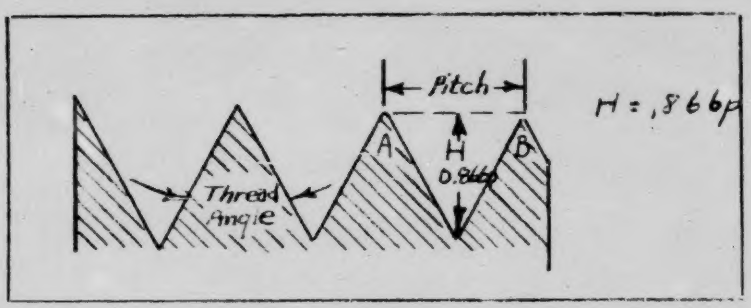

Figure 2

From Figure 2, it can be seen that:

$$H = \frac{\text{Major diameter} - \text{Root diameter}}{2}$$

Using this formula for H together with the previous formula for H, it can be found that:

$d = D - \frac{1.732}{n}$ (where d is the root diameter, D is the major diameter, and n is the number of threads per inch)

APPLICATION OF THE RULE:

Example 1. Calculate the root diameter of a thread whose major diameter is .750" and has 16 threads per inch.

Use the formula:

$$d = D - \frac{1.732}{n}$$

Then:

$$d = .750 - \frac{1.732}{16}$$

$$= .750 - .109$$

$$= .641''$$ (to the nearest thousandth)

Example 2. What is the pitch of a thread having 16 threads per inch?

Use the formula:

$$p = \frac{1}{n}$$

Then:

$$p = \frac{1}{16}$$

$$= .0625'' \text{ or } .063''$$ (to the nearest thousandth)

Example 3. Find the depth H of a V-thread having 32 threads per inch.

Use the formula:

$$H = \frac{.866}{n}$$

$$= \frac{.866}{32}$$

$$= .027''$$ (to the nearest thousandth)

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Calculations--Sharp 60° V-Thread

INTRODUCTION:
Very often you are required to cut a thread on a lathe. Certain calculations for a thread including major diameter, root diameter, depth, and pitch are involved.

OBJECTIVE:
To provide practice in the calculation of certain dimensions involved in sharp V-threads.

TECHNICAL ASSIGNMENT:

1. Find the root diameter of a 1/4" - 20 sharp V-thread.
2. Find the pitch of a sharp V-thread having 18 threads per inch.
3. Find the pitch of a thread having 40 threads per inch.
4. Find the root diameter of a 1" - 8 sharp V-thread.
5. Find the root diameter of a 1/2" - 20 sharp V thread.
6. Find the number of threads if the pitch is .10".
7. Find the number of threads if the pitch is .0312.
8. Find the depth H of a thread if the major diameter is 3/4", and the root diameter is .641".
9. Find the depth H of a 1" - 14 sharp V-thread.
10. Find the depth H of a 3" - 4 sharp V-thread.

ANSWERS

1. .163" (to the nearest thousandth)
2. .056" (to the nearest thousandth)
3. .025" (to the nearest thousandth)
4. .783" (to the nearest thousandth)
5. .413" (to the nearest thousandth)
6. 10 threads per inch
7. 32 threads per inch
8. .055" (to the nearest thousandth)
9. .062" (to the nearest thousandth)
10. .217" (to the nearest thousandth)

MODERN MATHEMATICS
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OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Cutting a 60° V-Form Thread

COURSE UNIT TITLE: Lathe

INTRODUCTION:

A 60° V-Form Thread is the most common type of thread used for fasteners. It is cut with either a lathe using a single point tool, a die, or it can be rolled when produced in quantities.

OBJECTIVE:

To provide the student an opportunity to learn how to turn a 60° V-Thread on a lathe.

TOOLS AND MATERIALS REQUIRED:

Lathe
V-Form Thread
Turning tool
Micrometer
Round steel
3-wire gage
Thread micrometer

PROCEDURE:

(Operation)	(Related Information)
1. Secure stock in lathe.	1. Either in chuck or between lathe centers
2. Turn outer diameter.	2. See Technical Information Sheet.
3. Turn root diameter.	3. Calculate. See Technical Information Sheet.
4. Set machine for proper number of threads per inch.	4. See Technical Information Sheet.
5. Set compound rest. See Figure I.	5. 29° set by swinging compound to the right.
6. Set thread tool.	6. Set on center of axis and perpendicular to the axis using center gages.
7. Set threading dial.	7. Check each respective lathe

			for even, odd and half thread.
8.	Set feed directional lever on neutral.	8.	
9.	Touch outer diameter.	9.	Tightly

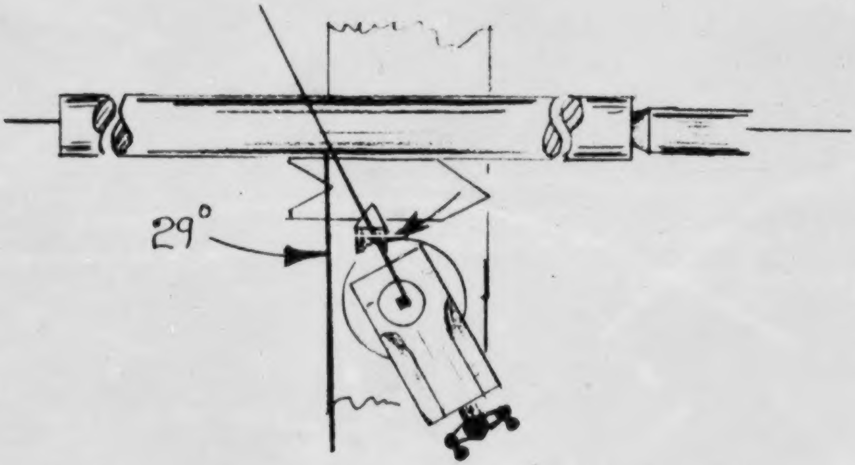

Figure I

10.	Set compound rest and cross-feed micrometer dials on zero.	10.	Set compound rest and cross feed after moving toward axis to eliminate backlash.
11.	Take .003 trial cut by engaging half-nut lever.	11.	Use compound rest and half-nut lever.
12.	Stop thread cutting.	12.	Withdraw thread tool using crossfeed and disengage half-nut lever (all at the same time).
13.	Check pitch. (See Technical Information Sheet)	13.	Number of threads per inch.
14.	Bring tool back to position.	14.	By using the crossfeed and bringing it to original zero.
15.	Take a .002, .003, or .004 cut, depending on the size of the thread, O.D., and machine.	15.	These successive cuts are taken by using the compound rest. Always leave at the last position and turn in from it until you nearly reach the root diameter.
16.	Finish cutting thread.	16.	Use the crossfeed for the final 2 or 3 cuts.
17.	Check the fit.	17.	Use either a thread ring gage, thread micrometer, comparator, or 3-wire method. See Technical Information Sheet for 3-wire method.

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Unified and American (National) Thread Forms

INTRODUCTION:

There have been many changes since the old Sharp 60° V Thread form was used. Several years ago the American Standard Thread form series replaced the Sharp 60° V form. Then just a few years ago a move was made to standardize even more the thread system. This effort resulted in the Unified Thread System.

OBJECTIVES:

1. To provide the student an opportunity to become acquainted with the Unified Thread System.
2. To provide the student an opportunity to learn how to calculate the proper dimensions for cutting the American National Thread form and the Unified Thread form.

TECHNICAL INFORMATION:

The first change from the Sharp 60° V Thread was very slight. A flat equal to .0381P, where P is the pitch, was made on the crest and the root of the thread. A few years later the size of this flat was increased to .125P. Since this thread had the flat on the crest and on the root, the depth was not as great as it was on the Sharp 60° V Thread form. The included angle of 60° remained the same. (See Figure 1)

The Unified Thread System uses the American Standard form of thread. The basic differences lie in the contour of the root of the thread and the size of the flat at the crest.

The rounded roots (and crests in England) do not have a specific radius. It really represents the form produced by a worn tool. (See Figure I-D)

The Unified tolerances are a refinement of the previous American practices. The Unified Thread standards involve three classes of tolerances for the external thread, 1A, 2A, and 3A, and three classes for the internal thread, 1B, 2B, and 3B.

Classes 1A and 1B are usable for ordnance and various parts where quick and easy assembly is desired. Classes 2A and 2B are designed for a wide range of applications and are generally the recognized standards. Class 2A which provides for moderate allowance decreases the possibility of galling and seizing in assembly and use. It also allows a moderate clearance for plating and other similar coatings.

Both class 2A and 2B are perfect for production line assembly where speed wrenches are used. The great bulk of screw thread work falls in these two classes.

Classes 3A and 3B are useful where tolerances closer than 2A and 2B are desired. Class 4 has been discontinued for the Unified and American Standard Thread forms.

Under the Unified system tolerance varies directly with the diameter. In other words, as the diameter increases the tolerances increase. Previously for the coarse and fine series a formula was used which disregarded the size of the diameter.

A class 3 screw does not have to be mated with a class 3 nut. The type of assembly and the requirements of the fastener should determine the classes to be used. Any combination of classes for the screw and the nut may now be used.

The following formulas should be utilized in determining various values for the American Standard Thread system and the Unified Thread system.

I. The American Standard Thread System

P (pitch) = $\frac{1}{N}$ (where N represents the number of threads per inch)

h (depth) = .64952P (where P represents the pitch)

f (width of flat) = .125P (where P represents the pitch)

Minor diameter = major diameter - 2h (where h is the depth)

Pitch diameter = major diameter - h (where h is the depth)

II. The Unified Thread System

P (pitch) = $\frac{1}{N}$ (where N represents the number of threads per inch)

h (depth) = .61343P (for external threads) (where P represents the pitch)

h (depth) = .54127P (for internal threads) (where P represents the pitch)

f (width of flat at crest) = .125P (for external threads) (where P represents the pitch)

f (width of flat at crest) = .25P (for internal threads) (where P represents the pitch)

f (width of flat at root) = .125P (for internal threads) (where P represents the pitch)

Minor diameter = major diameter - 2h (where h is the external depth)

Pitch diameter = major diameter - .64952P (where P is the pitch)

Measure of included angle = 60°

Root of thread - Rounded instead of flat

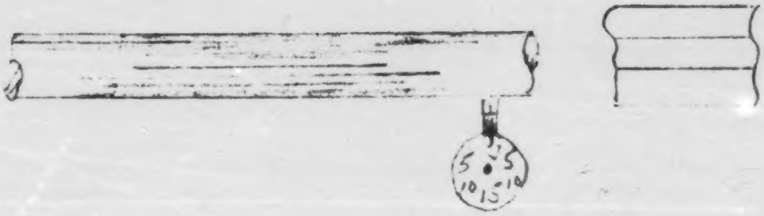

Figure 1

APPLICATION OF THE RULE:

Example 1. Find the pitch, depth, and width of flat for an American National Thread having 8 threads per inch.

$$P = \frac{1}{N}$$

$$= \frac{1}{8}$$

$$= .125 \text{ in.}$$

$$h = .64952P$$

$$= (.64952)(.125)$$

$$= .081 \text{ in.} \quad \text{(to the nearest thousandth)}$$

$$f = .125P$$

$$= (.125)(.125)$$

$$= .016 \text{ in.} \quad \text{(to the nearest thousandth)}$$

Example 2. Find the pitch, the minor diameter, and the pitch diameter for a Unified Thread having 5 threads per inch. The thread is 1 3/4 in. and external.

$$P = \frac{1}{N}$$

$$= \frac{1}{5}$$

$$= .200 \text{ in.}$$

h = .61343P

= (.61343)(.200)

= .123 in. (to the nearest thousandth)

Minor diameter = major diameter - 2h

= 1.750 - 2(.123)

= 1.750 - .246

= 1.504 in.

Pitch diameter = major diameter - .64952P

= 1.750 - (.64952)(.200)

= 1.750 - .130

= 1.620 in.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Unified and American (National) Thread Forms

INTRODUCTION:

In order to cut and work with American National Threads and Unified Threads the student needs to be able to compute the various dimensions including pitch, depth, width of flat, minor diameter, and pitch diameter.

OBJECTIVE:

To provide the student practice in computing various dimensions regarding American National Threads and Unified Threads.

ASSIGNMENT:

1. What is the depth of an American National Thread having 10 threads per inch?
2. What is the width of the flat for a 10 thread per inch American National bolt?
3. The width of the flat on an internal Unified thread ring gage is 1/4 times the pitch. What would be the width of the flat on a ring gage to check 12 threads per inch?
4. What would the minor diameter be for the following external thread: 3/4 - 10 UNC - 2A?
5. Find the single depth for the following American National screw: 3/8 - 16 NC.
6. Thread micrometers measure the pitch diameter of a thread. What should be the correct reading when the following external threads are finished: 1 - 8 UNC - 2B?
7. A recess is to be cut to the minor diameter size at the end of the following threads: 7/8 - 14 UNF - 3B. What should be the diameter of the recess? (The threads are external.)

8. Find the following for the following external thread: 5/8 - 18 UNF - 2B:
    a. Pitch
    b. Single depth
    c. Pitch diameter
    d. Minor diameter

ANSWERS

1. .065 in.
2. .013 in.
3. .021 in.
4. .628 in.
5. .041 in.
6. .919 in.
7. .787 in.
8. a. .056 in.
   b. .034 in.
   c. .589 in.
   d. .557 in.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Change Gears--Simple Gearing

INTRODUCTION:
There are times when a thread with a special lead and pitch are needed for a specific machine, and it becomes necessary to change gears. Usually the machine is provided with a chart showing the set of gears for each respective thread; however, there may be a time when this is not the case. In this situation it is necessary to calculate the gears needed.

OBJECTIVE:
To provide the student an opportunity to learn how to find the proper set of gears by calculations and to calculate the speed of gears.

TECHNICAL INFORMATION:

Most lathes have quick change gear boxes already designed on the lathe; however, in making a special thread or cutting a metric thread, it requires changing gears in order to cut the special pitch and lead.

The following formula is to be used:

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$ (also, see page

APPLICATION OF THE RULE:

Example. Select from the following set of gears those available for cutting 16 threads per inch with a screw constant of 4.

Gears available: 24, 30, 36, 42, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

$$\frac{16}{4} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

$$\frac{4}{1} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

Therefore, the ratio of the lead screw gear to the spindle stud gear should be 4 to 1.

If the spindle stud gear is chosen to have 24 teeth, then:

$$4 = \frac{\text{Lead screw gear}}{24}$$

$$24 \cdot 4 = 24 \cdot \frac{\text{Lead screw gear}}{24} \qquad \text{(Mult. both sides by 24)}$$

$$96 = \text{Lead screw gear} \qquad \text{(Mult. inverse)}$$

If the spindle stud gear would have been chosen, for example, to have 30 teeth, then:

$$4 = \frac{\text{Lead screw gear}}{30}$$

$$30 \cdot 4 = 30 \cdot \frac{\text{Lead screw gear}}{30} \qquad \text{(Mult. both sides by 30)}$$

$$120 = \text{Lead screw gear}$$

However, this choice is not possible since a gear with 120 teeth was not available in the set of gears given.

Therefore, let us use a stud gear with 24 teeth and the lead screw gear with 96 teeth. See the illustration in Figure 1.

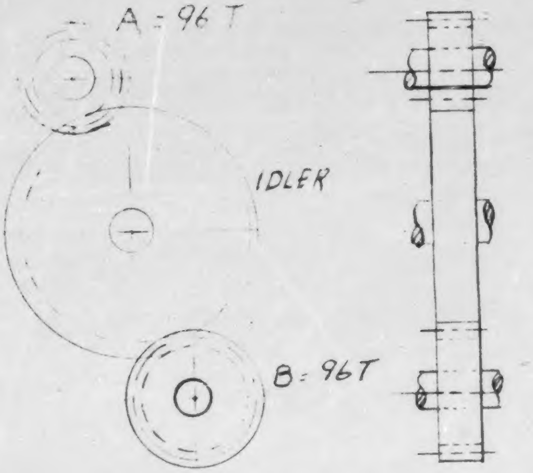
A = 96 T
IDLER
B = 96T

Figure 1

Now suppose we know the speed of gear A in Figure 2 is 100 revolutions per minute. Also, we know that gear A has 40 teeth and gear B has 20 teeth. Now, what is the speed in revolutions per minute of gear B?

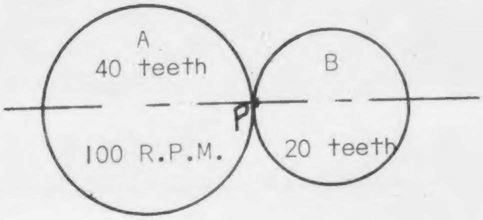
A
40 teeth
B
P
100 R.P.M.
20 teeth

Figure 2

Consider the fixed point P in Figure 2 on the axis through the centers of the two gears. Let us attempt to determine how many teeth of gear A will pass this point in one minute. We know that gear A makes 100 revolutions in one minute. Also, we know that for each revolution by gear A, the 40 teeth of gear A will pass point P. Therefore, to find the total number of teeth on gear A that will pass point P in one minute, all we need to do is multiply 100 and 40. That is, the number of teeth passing point P in one minute will be 100 x 40 or the number of revolutions per minute times the number of teeth on the gear.

Likewise, for gear B the number of teeth passing point P in one minute will be the number of revolutions per minute of gear B times 20 (the number of teeth on gear B).

Thus:

(for gear A)		(for gear B)	
$100 \cdot 40$	=	$20 \cdot s$	where s is the speed in revolutions per minute for gear B.
$\frac{1}{20} \cdot 100 \cdot 40$	=	$\frac{1}{20} \cdot 20 \cdot s$	(mult. both sides by 1/20)
$\frac{100 \cdot 40}{20}$	=	s	
200	=	s	

Thus, the speed of gear B is 200 revolutions per minute.

We can see from this illustration that for two gears A and B:

(for gear A)		(for gear B)	
$T \cdot S$	=	$t \cdot s$	where T is the number of teeth of gear A, S is the speed of gear A, t is the number of teeth of gear B, and s is the speed of gear B.

Example: In Figure 3 find the speed of gear C.

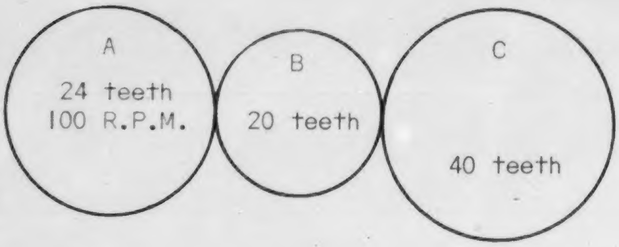

Figure 3

First, let us find the speed of gear B.

(for gear A) (for gear B)

$$T \cdot S = t \cdot s$$

$$24 \cdot 100 = 20 \cdot s$$

$$\frac{1}{20} \cdot 24 \cdot 100 = \frac{1}{20} \cdot 20 \cdot s \qquad \text{(mult. both sides by 1/20)}$$

$$\frac{24 \cdot 100}{20} = s$$

$$120 = s$$

Thus, gear B revolves at 120 revolutions per minute.

Now, let us find the speed of gear C.

(for gear B) (for gear C)

$$T \cdot S = t \cdot s$$

$$20 \cdot 120 = 40 \cdot s$$

$$\frac{1}{40} \cdot 20 \cdot 120 = \frac{1}{40} \cdot 40 \cdot s \qquad \text{(mult. both sides by 1/40)}$$

$$\frac{20 \cdot 120}{40} = s$$

$$60 = s$$

Therefore, gear C revolves at 60 revolutions per minute.

Note, that we could have found the speed for gear C without first of all finding the speed for gear B as follows:

(for gear A) (for gear C)

$$T \cdot S = t \cdot s$$

$$24 \cdot 100 = 40 \cdot s$$

$$\frac{1}{40} \cdot 24 \cdot 100 = \frac{1}{40} \cdot 40 \cdot s \qquad \text{(mult. both sides by 1/40)}$$

$$\frac{24 \cdot 100}{40} = s$$

$$60 = s$$

This illustrates that although the presence of gear B will affect the direction of gear C, the presence or absence of gear will not alter the speed of gear C.

Now let us take a look at the formula:

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

listed on the first page to see if we can understand its validity.

In Figure 4 if the spindle gear and the lead screw gear have the same number of teeth, then, obviously, the lead screw and the spindle (thus, the work being turned) will revolve at the same speed. Thus, if threads are being cut on the lathe, the number of threads cut will be the same as the number of threads per inch on the lead screw.

However, if the number of threads on the spindle gear is twice the number of threads on the lead screw gear then the spindle will turn only 1/2 as fast as the lead screw. Thus, the number of threads per inch being cut will be only 1/2 of the number of threads per inch on the lead screw.

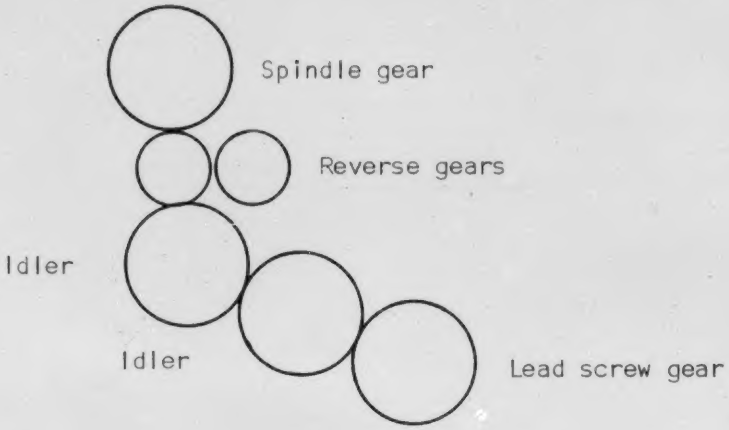

Figure 4

If the number of threads on the spindle gear is three times the number of threads on the lead screw gear, then the spindle will turn only 1/3 as fast as the lead screw. Thus, the number of threads per inch being cut will be only 1/3 of the number of threads per inch on the lead screw. This suggests that the ratio of the number of threads being cut to the number of threads on the lead screw is equal to the ratio of the speed of the spindle gear to the speed of the lead screw gear.

That is:

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{S\ \text{(speed of spindle stud gear)}}{s\ \text{(speed of lead screw gear)}}$$

However,

$T \cdot s = t \cdot s$ where
T is the number of threads per inch of the spindle stud gear,
S is the speed of the spindle stud gear,
t is the number of threads per inch of the lead screw gear and
s is the speed of the lead screw gear.

$$T \cdot S = t \cdot s$$

$$\frac{1}{T} \cdot T \cdot S = \frac{1}{T} \cdot t \cdot s \qquad \text{(multiply both sides by } 1/T\text{)}$$

$$S = \frac{t \cdot s}{T} \qquad \text{(mult. inverse)}$$

$$S \cdot \frac{1}{s} = \frac{t \cdot s}{T} \cdot \frac{1}{s} \qquad \text{(multiply both sides by } 1/s\text{)}$$

$$\frac{S}{s} = \frac{t}{T} \qquad \text{(mult. inverse)}$$

Thus, since

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{S}{s}$$

and

$$\frac{S}{s} = \frac{t}{T}$$

then,

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{t}{T}$$

or

$$\frac{\text{Threads per inch}}{\text{Lathe screw constant}} = \frac{\text{Lead screw gear (number of teeth)}}{\text{Spindle stud gear (number of teeth)}}$$

The student should note that this was the formula given on the first page.

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Change Gears (Simple Gearing)

INTRODUCTION:

At times it is necessary to set up for special threads or to work on small engine lathes that have no quick change gear box; thus, you are required to calculate for the use of simple gearing.

OBJECTIVE:

To learn how to calculate the simple gearing desired for a particular thread having a special pitch or lead, and, also, to learn how to calculate speeds of gears.

ASSIGNMENT:

1. Find the proper gears for cutting a 10 thread per inch screw with a lathe having a screw constant of 8.
2. Find the proper gears for threading a 20 threads per inch screw with a lathe having a screw constant of 8.
3. Find the proper gears for cutting a 16 threads per inch screw with a lathe having a screw constant of 8.
4. Four threads per inch are to be cut on a lathe having a screw constant of 5. What change gears are needed?
5. What are the proper gears for cutting 15 threads per inch if the lathe screw constant is 6?
6. Find the speed of gear A in Figure I on the next page. In what direction will gear A rotate if the first gear (B) moves in a clockwise direction?
7. If gear B in Figure I is running at 500 revolutions per minute, what will be the speed of gear A?

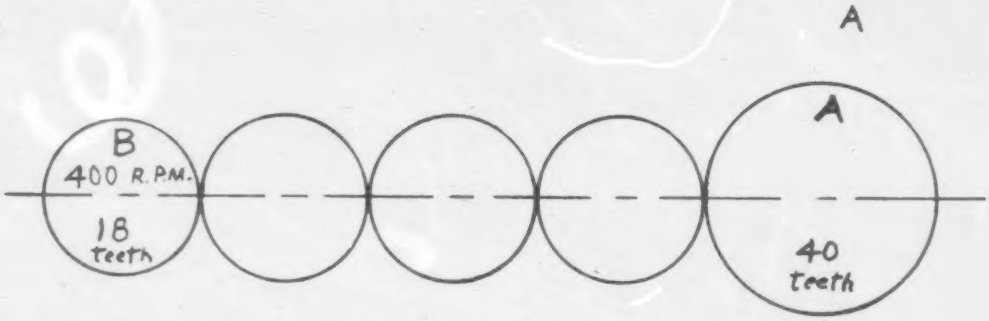

Figure 1

8. Two gears are connected by two idler gears each having 24 teeth. If the first gear has 35 teeth and is running at 500 revolutions per minute, what is the speed of the last gear if it has 50 teeth? In what direction will the last gear rotate if the first gear moves in a counterclockwise direction?

ANSWERS

1. Lead screw gear: 30
   Spindle stud gear: 24
2. Lead screw gear: 60
   Spindle stud gear: 24
3. Lead screw gear: 60
   Spindle stud gear: 30
4. Lead screw gear: 24
   Spindle stud gear: 30
5. Lead screw gear: 60
   Spindle stud gear: 24
6. 180 revolutions per minute, clockwise
7. 225 revolutions per minute
8. 350 revolutions per minute, clockwise

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL INFORMATION TITLE: Calculations--RPM and Feed Rate

INTRODUCTION:

It is a very difficult job to actually determine the right RPM (revolutions per minute) for a milling machine job, because of so many variables. However, it is safer for a machinist if he calculates the RPM and uses it as an approximation. The variables consist of such factors as: (a) machine rigidity, (b) size of the job, (c) height of the job, (d) shape of the job, (e) shape of the cut, (f) type of metal, (g) type of cutter, (h) type of coolant, and many others.

OBJECTIVES:

1. To provide the student an opportunity to learn how to calculate the RPM for a milling machine.
2. To provide the student an opportunity to learn how to judge what RPM should be used as he gains experience on the job.

TECHNICAL INFORMATION:

Following is a table for cutting speeds for milling roughing cuts with high speed cutters:

Material	Cutting Speed Range in sfpm
Low carbon steel	60-80
Medium carbon steel, annealed	60-80
High carbon steel, annealed	50-70
Tool steel, annealed	50-70
Stainless steel	50-80
Gray cast steel, soft	50-80
Malleable iron	80-100
Aluminum and its alloys	400-1000
Brass	200-300
Bronze	100-200

In one revolution, the distance traveled by a fixed point on the outside of the cutter will be equal to the circumference of the cutter. The circumference C is equal to πd. $C = \pi d$. The cutting speed is the distance traveled per minute by a fixed point on the outside of the cutter. Therefore, the cutting speed is equal to the number of revolutions per minute times πd. That is, $CS = \pi dN$. In the formula if the cutting speed is to result in feet per minute, then the diameter must be in feet. However, it is more convenient to measure the diameter in inches. Therefore, if d is measured in inces, the formula becomes: (since 1 foot = 12 inches)

$CS = \frac{\pi dN}{12}$ where CS is the cutting speed (in feet per minute), d is the diameter (in inches), and N is the number of revolutions per minute.

APPLICATION OF THE RULE:

Example 1. Calculate the cutting speed for a .750" diameter cutter operating at 306 revolutions per minute.

$$\begin{aligned} CS &= \frac{\pi dN}{12} \\ &= \frac{(3.142)(.750)(306)}{12} \\ &= \frac{(2.357)(306)}{12} \\ &= \frac{721.242}{12} \\ &= 60.104 \text{ feet per minute} \end{aligned}$$

Example 2. Calculate the number of revolutions per minute of a 3" diameter if the cutting speed is to be 80 feet per minute.

To find the number of revolutions per minute, we need to solve the formula for N.

$$CS = \frac{\pi dN}{12}$$

$$12(CS) = 12 \cdot \frac{\pi dN}{12} \qquad \text{(Mult. both sides by 12)}$$

$$12(CS) = \pi dN \qquad \text{(Mult. inverse)}$$

$$\frac{1}{\pi d} \cdot 12(CS) = \frac{1}{\pi d} \cdot \pi dN \qquad \text{(Mult. both sides by } \frac{1}{\pi d}\text{)}$$

$$\frac{12(CS)}{\pi d} = N \qquad \text{(Mult. inverse)}$$

Therefore, we have the formula:

$$N = \frac{12(CS)}{\pi d}$$

In the example:

$$N = \frac{12(80)}{(3.142)3}$$

$$= \frac{960}{9.426}$$

= 101.85 or 102 revolutions per minute (rounded to the nearest whole number)

Example 3. Determine the feed rate in inches per minute for machining low carbon steel at 80 feet per minute and 122 R.P.M. using a heavy duty plain milling cutter which is 2 1/2" in diameter, with 8 teeth, and having .005" feed per tooth.

If we multiply the amount of feed per tooth times the number of teeth, the result will be the total feed for each revolution. Then, if we multiply the result by N (the number of revolutions per minute), the answer should be the feed rate (per minute).

Thus:

$F = (RT)N = RTN$ where F is the feed rate in inches per minute, R is the feed per tooth (in inches), T is the number of teeth, and N is the number of revolutions per minute

In this example:

$$F = RTN$$

$$= (.005)(8)(122)$$

$= (.040)(122)$

$= 4.880''$ per minute

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL ASSIGNMENT TITLE: Calculations--R.P.M. and Feed Rate

INTRODUCTION:

The number of revolutions per minute is very important in milling. This is especially true because of the expense of the cutters. Yet, it is very difficult using mathematical formulas to determine the precise proper speed. The best we can do is to calculate an approximate value and then through experience achieve the most effective and efficient results.

OBJECTIVE:

To learn how to calculate the speed for a cutter on a milling machine.

ASSIGNMENT:

1. Find the r.p.m. for a 4.000 in. diameter cutter milling a workpiece if the cutting speed rating is 60-80 sfpm.

2. Find the cutting speed if the milling cutter has a 2 in. diameter. The number of revolutions per minute is 125.

3. Find the feed rate for machining steel at 70-80 sfpm and 180 r.p.m. The cutter has a 4 in. diameter, 24 teeth, and .005 feed per tooth.

4. Find the cutting speed of a milling cutter 3 1/2 in. in diameter and turning at 80 r.p.m.

5. Calculate the size of a milling cutter which has a cutting speed of 30 ft./min. at 50 r.p.m.

6. Compute the cutting speed of the drill being used in a milling machine. The drill is 2 in. in diameter and rotates at 96 r.p.m.

7. If a 3 in. diameter milling cutter rotates at 50 r.p.m., calculate the cutting speed.

8. A machinist is drilling on a milling machine. At what r.p.m. must a drill turn if the cutting speed is 35 ft./min., and the diameter of the drill is 1 in.?

9. Given two milling cutters of the same diameter rotating at 50 r.p.m. If one cutter is then made to rotate at 100 r.p.m., compare the cutting speeds of the two cutters.

10. Find the number of revolutions per minute of a milling cutter 4 1/2 in. in diameter if the cutting speed is 65 ft./min.

ANSWERS

1. 57.29 to 76.38 revolutions per minute
2. 65.46 feet per minute
3. 21.60 inches per minute
4. 73.31 feet per minute
5. 2.292 inches
6. 50.27 feet per minute
7. 39.28 feet per minute
8. 133.7 revolutions per minute
9. The cutting speed is doubled.
10. 55.17 revolutions per minute

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL INFORMATION TITLE: Calculations--Direct Indexing Systems

INTRODUCTION:
The Direct Indexing System is very advantageous for squares, hexagons, and such operations as fluting taps. It is very quickly set up and used extensively.

OBJECTIVE:
To provide the student an opportunity to learn how to calculate the spaces for direct indexing for respective divisions.

TECHNICAL INFORMATION:

Various hole circle plates are made such as: 24-hole circle, 30-hole circle, and 36-hole circle. They are exchangeable and provide the necessary spaces for specific divisions.

APPLICATION OF THE RULE:

Example. Determine the indexing movement that is required to mill an octagon by using a 24-hole rapid indexing circle or plate.

The formula used to determine the spacing is:

$$\text{Spaces} = \frac{24}{N} \quad \text{(where N is the number of divisions)}$$

Since there are 8 sides on an octagon, the number of divisions should be 8.

Therefore, in the given example:

$$\text{Spaces} = \frac{24}{8}$$

$$\text{Spaces} = 3$$

Therefore, a plunger pin should be placed in every third hole on the direct-indexing plate for a 24-hole circle.

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL ASSIGNMENT TITLE: Calculations--Direct Indexing

INTRODUCTION:
For simple indexing it is very easy to use the direct indexing system.

OBJECTIVE:
To learn how to calculate the spaces for direct indexing required for specific divisions.

ASSIGNMENT:

1. Determine the number of spaces required for each of the following divisions. Assume that a 24-hole rapid indexing circle or plate is being used.
    a. 2 divisions
    b. 3 divisions
    c. 4 divisions
    d. 6 divisions
    e. 8 divisions
    f. 12 divisions
2. State the direct indexing (number of holes to move) to mill a square with a 24-hole circle.
3. State the direct indexing to mill a square with a 36-hole circle.
4. Give the direct indexing (the number of holes on a certain circle) to cut 36 teeth on a gear blank using a direct indexing plate which has three circles with 24, 30, and 36 holes.
5. If four flutes are milled on a reamer, state the direct indexing required. The direct indexing plate contains 24 holes.
6. If the indexing head has a direct indexing plate containing 24 holes, state the direct indexing required to mill a hexagon.

7. Is it possible to mill a square with the 30-hole circle? (Yes or No) Which circles could be used on the direct indexing plate?

8. State the direct indexing required to make five equal divisions on the work when the direct indexing plate has three circles containing 24, 30, and 36 holes.

9. You are to cut 8 equally spaced splines on the end of a large shaft. If a direct indexing plate contains 24, 30, and 36 hole circles, what indexing would you use?

10. Is it possible to use direct indexing to cut 32 teeth on a gear blank if the indexing plate has three circles containing 24, 30, and 36 holes? (Yes or No) State the indexing required if you answer "yes."

ANSWERS

1. Spaces
   - a. 12
   - b. 8
   - c. 6
   - d. 4
   - e. 3
   - f. 2
2. Spaces = 6 (every 6th hole)
3. Spaces = 9 (every 9th hole)
4. Every hole on the 36 hole circle.
5. Spaces = 6 (every 6th hole)
6. Spaces = 4 (every 4th hole)
7. No, 24, 36
8. Every 6th hole on the 30 hole circle
9. Every 3rd hole on the 24 hole circle
10. No

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL INFORMATION TITLE: Calculations--Simple Indexing

INTRODUCTION:

The use of simple indexing is far reaching in terms of being able to equally divide spaces for a variety of divisions. It is used extensively for gear cutting.

OBJECTIVE:

To provide the student an opportunity to learn how to calculate data for specified divisions.

TECHNICAL INFORMATION:

The simple indexing head is designed for obtaining many divisions. The data usually needed is as follows: (a) divisions, (b) turns, (c) circle, and (d) spaces.

The formula for indexing is given as follows:

$$T = \frac{40}{N}$$ (where T is the number of turns and N is the number of divisions)

APPLICATION OF THE RULE:

Example 1. Index for 15 divisions.

$$T = \frac{40}{N}$$

$$= \frac{40}{15}$$

$$= \frac{8 \cdot 5}{3 \cdot 5}$$

$$= \frac{8}{3}$$

$$= 2\frac{2}{3}$$

The fractional part of the turn (2/3) indicates that an index plate and a sector will have to be utilized. The index plate may have these holes on each circle such as follows:

Plate 1: 15, 16, 17, 18, 19, 20
Plate 2: 21, 23, 27, 29, 31, 33
Plate 3: 37, 39, 41, 43, 47, 49

Any number on any plate may be used if that number times 2/3 is a whole number. That is, any circle of holes can then be used if the number of holes times 2/3 is a whole number.

Let us examine each plate for possible answers.

On plate 1:

15 will work since $15 \cdot \frac{2}{3} = \frac{15 \cdot 2}{3} = \frac{30}{3} = 10$ (a whole number)

18 will work since $18 \cdot \frac{2}{3} = \frac{18 \cdot 2}{3} = \frac{36}{3} = 12$ (a whole number)

On plate 2:

21 will work since $21 \cdot \frac{2}{3} = \frac{21 \cdot 2}{3} = \frac{42}{3} = 14$ (a whole number)

27 will work since $27 \cdot \frac{2}{3} = \frac{27 \cdot 2}{3} = \frac{54}{3} = 18$ (a whole number)

33 will work since $33 \cdot \frac{2}{3} = \frac{33 \cdot 2}{3} = \frac{66}{3} = 22$ (a whole number)

On plate 3:

39 will work since $39 \cdot \frac{2}{3} = \frac{39 \cdot 2}{3} = \frac{78}{3} = 26$ (a whole number)

Therefore, any of the above circle of holes can be used if the number of holes can be divided by three and the result is a whole number.

From above we, therefore, need 2 complete turns. Then we may use any one of the following:

The 10th hole on the 15-hole circle
or
The 12th hole on the 18-hole circle
or
The 14th hole on the 21-hole circle
or
The 18th hole on the 27-hole circle
or
The 22nd hole on the 33-hole circle
or
The 26th hole on the 39-hole circle

Example 2. Index for 42 divisions. Select from the plates in Example 1.

$$T = \frac{40}{N}$$

$$= \frac{40}{42}$$

$$= \frac{2 \cdot 20}{2 \cdot 21}$$

$$= \frac{20}{21}$$

The fractional value for T again involves the use of an index plate and a sector.

As in the first example, this requires a circle of holes such that if the number of holes is multiplied by $\frac{20}{21}$, the result will be a whole number. This means that the number of holes can be divided by 21 and the result will be a whole number. Of the numbers on Plates 1, 2, and 3, the only number divisible by 21 on the plates is 21 on Plate 2.

Then:

$$21 \cdot \frac{20}{21} = 20$$

Therefore, we use the 20th hole on the 21-hole circle.

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THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL ASSIGNMENT TITLE: Calculations--Simple Indexing

INTRODUCTION:

Simple indexing is very useful in dividing a workpiece such as a gear into a certain number of divisions.

OBJECTIVE:

To learn how to solve problems involving divisions using simple indexing.

ASSIGNMENT:

1. Find the indexing required for each of the following numbers of divisions. List the circle, number of turns, and number of spaces. Use simple indexing in all cases.

    a. 13 divisions: Circle____, No. of turns____, No. of spaces____
    b. 18 divisions: Circle____, No. of turns____, No. of spaces____
    c. 21 divisions: Circle____, No. of turns____, No. of spaces____
    d. 25 divisions: Circle____, No. of turns____, No. of spaces____
    e. 33 divisions: Circle____, No. of turns____, No. of spaces____
    f. 39 divisions: Circle____, No. of turns____, No. of spaces____
    g. 45 divisions: Circle____, No. of turns____, No. of spaces____

2. State the indexing required for the following numbers of divisions. List the circle, number of turns, and number of spaces. Use simple indexing in all cases.

    a. 4 divisions: Circle____, No. of turns____, No. of spaces____
    b. 10 divisions: Circle____, No. of turns____, No. of spaces____
    c. 60 divisions: Circle____, No. of turns____, No. of spaces____
    d. 72 divisions: Circle____, No. of turns____, No. of spaces____
    e. 115 divisions: Circle____, No. of turns____, No. of spaces____

ANSWERS

1.		Circle	No. of Turns	No. of Spaces
	a.	39	3	3
	b.	18	2	4
	c.	21	1	19
	d.	15	1	9
	e.	33	1	7
	f.	39	1	1
	g.	27	0	24
2.				
	a.		10	
	b.		4	
	c.	15	0	10
	d.	27	0	15
	e.	23	0	8

MODERN MATHEMATICS
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THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Setting Up Simple Indexing. (Sometimes called Plain Indexing)

COURSE UNIT TITLE: Milling Machines

INTRODUCTION:

When it is necessary to have gears cut or to index larger numbers than what is possible by using the direct index method, then we use the simple indexing method. Also, uneven numbers can be indexed.

OBJECTIVE:

To provide the student an opportunity to learn how to set up a simple index head.

TOOLS AND MATERIALS REQUIRED:

Milling machine
Workpiece (gear block)
Index head

PROCEDURE:

(Operation)	(Related Information)
1. Secure workpiece.	1. Chuck or between centers.
2. Determine the number of divisions you desire. (6)	2. Count number of teeth on gear, or whatever else it may be.
3. Calculate the number of turns, circles, and spaces. (6 turns, 21 circles, and 14 spaces)	3. See Technical Information Sheet
4. Adjust the index crank length for the circle chosen.	4. Fit index pin into the hole of the plate.
5. Set the sector (radial arms).	5. Count the proper holes in the circle selected. (Do not count the hole that the index pin is in.)
6. Turn the index crank for the complete number of turns that	6. If no complete turn is necessary, then turn within

	is necessary.		the sector arms.
7.	Move the index crank up to the second sector arm. See Figure 1.	7.	Do not move sector arm.

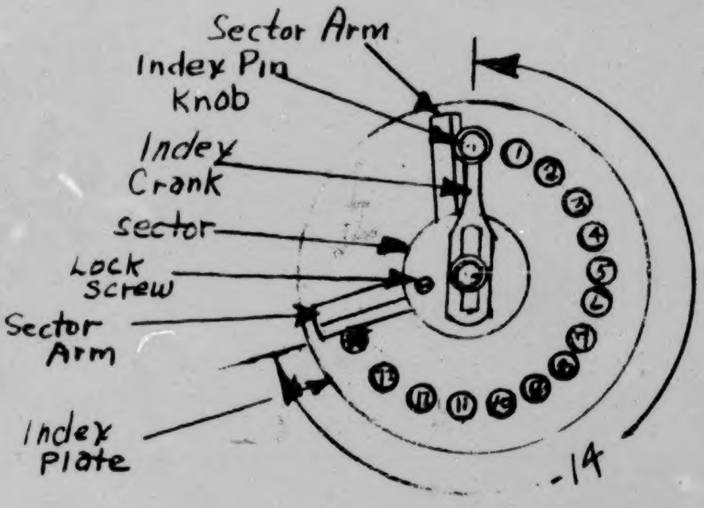
Sector Arm
Index Pin Knob
Index Crank
sector
Lock screw
Sector Arm
Index Plate
-14

Figure 1

8.	Set index crank pin into hole.	8.	Do not move sector arm.
9.	Revolve the sector arms until the follower (or first) arm is against the index crank pin. See Figure 2.	9.	Push the first sector arm. Don't pull; you may spread the sectors by hitting the pin.

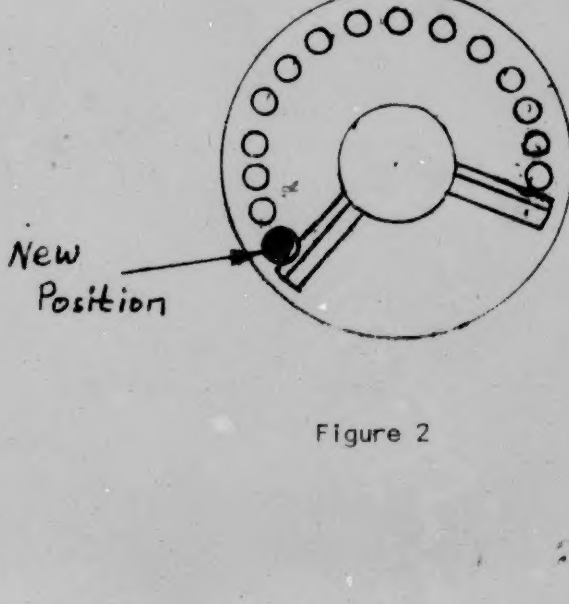
New Position

Figure 2

10. Repeat process until you have completed all the divisions and you are back to the original starting point.	10. If everything was done correctly, this will serve as a check.

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL INFORMATION TITLE: Spur Gear Formulas

INTRODUCTION:

In order to accurately cut spur gears, one needs to be able to work with the formulas pertaining to the spur gears. The information is essential in order to set up the indexing head and the milling machine including cutter selection, etc.

OBJECTIVE:

To provide the student an opportunity to learn how to calculate various dimensions concerning spur gears.

TECHNICAL INFORMATION:

Spur gears are used extensively and are an important part of many machines. Accuracy is vitally important if the gearing is to perform

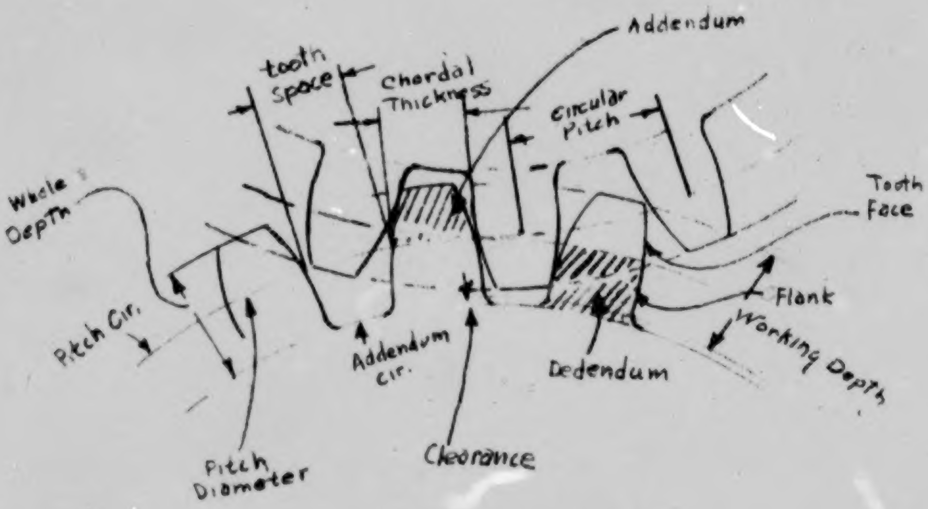
Addendum
tooth space
Chordal Thickness
Circular Pitch
Whole Depth
Tooth Face
Flank
Working Depth
Pitch Cir.
Addendum Cir.
Dedendum
Clearance
Pitch Diameter

Figure I

efficiently and smoothly. See Figure I for identification of the various spur gear parts.

There are several formulas which can be used to find various unknowns with regard to spur gears. The examples in the following section will illustrate the use of these various formulas.

APPLICATION OF THE RULE:

Example I. Find: Diametral Pitch
Given: Circular Pitch = .3927

Formula: $DP = \frac{3.1416}{CP}$

$$DP = \frac{3.1416}{.3927}$$

$$= 8.000$$

Example 2. Find: Diametral Pitch
Given: Number of Teeth = 38
Pitch Diameter = 4.750

Formula: $DP = \frac{N}{PD}$

$$DP = \frac{38}{4.750}$$

$$= 8.000$$

Example 3. Find: Diametral Pitch
Given: Number of Teeth = 38
Outside Diameter = 5

Formula: $DP = \frac{N + 2}{OD}$

$$= \frac{38 + 2}{5}$$

$$= \frac{40}{5}$$

$= 8$

Example 4. Find: Circular Pitch
Given: Diametral Pitch = 8.000

Notice that this problem involves the use of the same information as that in Example I except that here we are trying to find CP instead of DP. Therefore, let us solve for CP in the formula used in Example I.

$$DP = \frac{3.1416}{CP}$$

$$(DP)(CP) = \frac{3.1416}{CP}\cdot CP \qquad \text{(Multiply both sides by CP)}$$

$$(DP)(CP) = 3.1416 \qquad \text{(Multiplicative inverse)}$$

$$\frac{1}{DP}\cdot(DP)(CP) = \frac{1}{DP}\cdot 3.1416 \qquad \text{(Multiply both sides by 1/DP)}$$

$$CP = \frac{3.1416}{DP} \qquad \text{(Multiplicative inverse, } \frac{a}{b}\cdot c = \frac{a\cdot c}{b}\text{)}$$

Therefore, we have the new formula:

$$CP = \frac{3.1416}{DP}$$

In our problem:

$$CP = \frac{3.1416}{8.000}$$

$$= .393 \quad \text{(correct to three decimal places)}$$

Example 5. Find: Circular Pitch
Given: Pitch Diameter = 4.750
Number of Teeth = 38

Formula: $CP = \frac{3.1416(PD)}{N}$

$$= \frac{3.1416(4.750)}{38}$$

$$= \frac{14.9226}{38}$$

$$= .393 \quad \text{(correct to three decimal places)}$$

Example 6. Find: Number of Teeth
Given: Pitch Diameter = 4.750
Diametral Pitch = 8.000

Notice that the same information is involved as that in Example 2, except that here we are trying to find N. Therefore, let us solve the formula in Example 2 for N.

$$DP = \frac{N}{PD}$$

$$DP \cdot PD = \frac{N}{PD} \cdot PD \qquad \text{(Multiply both sides by PD)}$$

$$DP \cdot PD = N \qquad \text{(Multiplicative inverse)}$$

$$\text{or} \qquad N = DP \cdot PD$$

Therefore, we have the new formula:

$$N = DP \cdot PD$$

In our problem:

$$N = (8.000)(4.750)$$

$$= 38$$

Example 7. Find: Number of Teeth
Given: Outside Diameter = 5.000
Diametral Pitch = 8.000

Notice that the same information is involved as that in Example 3. Therefore, let us solve the formula in Example 3 for N.

$$DP = \frac{N + 2}{OD}$$

$$DP \cdot OD = \frac{N + 2}{OD} \cdot OD \qquad \text{(Multiply both sides by OD)}$$

$$DP \cdot OD = N + 2 \qquad \text{(Multiplicative inverse)}$$

$$DP \cdot OD - 2 = N + 2 - 2 \qquad \text{(Subtract 2 from both sides)}$$

$$DP \cdot OD - 2 = N \qquad \text{(Additive inverse)}$$

or $N = DP \cdot OD - 2$

Therefore, we have the new formula:

$$N = DP \cdot OD - 2$$

In our problem:

$$N = (8.000)(5.000) - 2$$

$$= 40.000 - 2$$

$$= 38.000$$

Example 8. Find: Pitch Diameter
Given: Number of Teeth = 38.000
Diametral Pitch = 8.000

Notice that the same information as in Example 2 is involved. Therefore, solve the formula in Example 2 for PD.

$$DP = \frac{N}{PD}$$

$$DP \cdot PD = \frac{N}{PD} \cdot PD \qquad \text{(Multiply both sides by PD)}$$

$$DP \cdot PD = N \qquad \text{(Multiplicative inverse)}$$

$$\frac{1}{DP} \cdot DP \cdot PD = \frac{1}{DP} \cdot N \qquad \text{(Multiply both sides by 1/DP)}$$

$$PD = \frac{N}{DP} \qquad \text{(Multiplicative inverse, } \frac{a}{b} \cdot c = \frac{a \cdot c}{b}\text{)}$$

Therefore, we have the new formula:

$$PD = \frac{N}{DP}$$

In our problem:

$$PD = \frac{38.000}{8.000}$$

$$= 4.750$$

Example 9. Find: Pitch Diameter
Given: Outside Diameter = 5.000
Addendum = .125

Use the formula:

$$PD = OD - 2(AD)$$

In our problem:

$$\begin{aligned} PD &= 5.000 - 2(.125) \\ &= 5.000 - .250 \\ &= 4.750 \end{aligned}$$

Example 10. Find: Outside Diameter
Given: Number of Teeth = 38.000
Diametral Pitch = 8.000

Notice that the same information is involved as in Example 3. Therefore, solve the formula in Example 3 for OD.

$$DP = \frac{N + 2}{OD}$$

$$DP \cdot OD = \frac{N + 2}{OD} \cdot OD \qquad \text{(Multiply both sides by OD)}$$

$$DP \cdot OD = N + 2 \qquad \text{(Multiplicative inverse)}$$

$$\frac{1}{DP} \cdot DP \cdot OD = \frac{1}{DP} \cdot (N + 2) \qquad \text{(Multiply both sides by 1/DP)}$$

$$OD = \frac{N + 2}{DP} \qquad \text{(Multiplicative inverse, } \frac{a}{b} \cdot c = \frac{a \cdot c}{b}\text{)}$$

Therefore, we have the new formula:

$$OD = \frac{N + 2}{DP}$$

In our problem:

$$OD = \frac{38.000 + 2}{8.000}$$

$$= \frac{40.000}{8.000}$$

$$= 5.000$$

Example 11. Find: Outside Diameter
Given: Number of Teeth = 38.000
Pitch Diameter = 4.750

From Example 10:

$$OD = \frac{N + 2}{DP}$$

From Example 2:

$$DP = \frac{N}{PD}$$

Now, let us put this value for DP into the formula from Example 10. Then, we have a new formula:

$$OD = \frac{N + 2}{\frac{N}{PD}}$$

In our problem:

$$OD = \frac{38.000 + 2}{\frac{38.000}{4.750}}$$

$$= \frac{40.000}{8.000}$$

$$= 5.000$$

Example 12. Find: Outside Diameter
Given: Pitch Diameter = 4.750
Diametral Pitch = 8.000

From Example 10:

$$OD = \frac{N + 2}{DP}$$

$$= \frac{N}{DP} + \frac{2}{DP} \qquad \left(\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}\right)$$

From Example 8:

$$PD = \frac{N}{DP}$$

Therefore, we have the new formula:

$$OD = PD + \frac{2}{DP}$$

In our problem:

$$OD = 4.750 + \frac{2}{8.000}$$

$$= 4.750 + .250$$

$$= 5.000$$

Example 13. Find: Thickness of Tooth
Given: Diametral Pitch = 8.000

Use the formula:

$$T = \frac{1.5708}{DP}$$

In our problem:

$$T = \frac{1.5708}{8.000}$$

$$= .1964$$

Example 14. Find: Clearance
Given: Diametral Pitch = 8.000

Use the formula:

$$C = \frac{0.157}{DP}$$

In our problem:

$$C = \frac{0.157}{8.000}$$

$= .020$

Example 15. Find: Whole Depth of Tooth or Tooth Space
Given: Diametral Pitch = 8.000

Use the formula:

$$WD = \frac{2.157}{DP}$$

In our problem:

$$WD = \frac{2.157}{8.000}$$

$$= .270$$

Example 16. Find: Center Distance
Given: Pitch Diameters of Two Gears = 4.750 and 4.000

Use the formula:

$$CD_i = \frac{PD + pd}{2}$$

In our problem:

$$CD_i = \frac{4.750 + 4.000}{2}$$

$$= \frac{8.750}{2}$$

$$= 4.375$$

Example 17. Find: Center Distance
Given: Number of Teeth = 38.000 and 32.000
Diametral Pitch = 8.000

Use the formula:

$$CD_i = \frac{N + n}{2(DP)}$$

In our problem:

$$CD_i = \frac{38.000 + 32.000}{2(8.000)}$$

$$= \frac{70.000}{16.000}$$

$$= \frac{70}{16}$$

$$= \frac{35 \cdot 2}{8 \cdot 2}$$

$$= \frac{35}{8}$$

$$= 4.375$$

SOME USEFUL FORMULAS FOR SPUR GEARS

$$DP = \frac{3.1416}{CP}$$

$$DP = \frac{N}{PD}$$

$$DP = \frac{N + 2}{OD}$$

$$CP = \frac{3.1416}{DP}$$

$$CP = \frac{3.1416(PD)}{N}$$

$$N = DP \cdot PD$$

$$N = DP \cdot OD - 2$$

$$PD = \frac{N}{DP}$$

$$PD = OD - 2(AD)$$

$$OD = \frac{N + 2}{DP}$$

$$OD = \frac{N + 2}{\frac{N}{PD}}$$

$$OD = PD + \frac{2}{DP}$$

$$T = \frac{1.5708}{DP}$$

$$C = \frac{0.157}{DP}$$

$$WD = \frac{2.157}{DP}$$

$$CD_i = \frac{PD + pd}{2}$$

$$CD_i = \frac{N + n}{2(DP)}$$

Others that may be used are as follows:

$$PD = \frac{N \cdot CP}{3.1416}$$

$$AD = \frac{1}{DP}$$

$$AD = \frac{CP}{3.1416}$$

$$C = \frac{CP}{20}$$

$$WD = .6866(CP)$$

$$T = \frac{CP}{2}$$

$$OD = \frac{(N + 2)CP}{3.1416}$$

$$N = \frac{3.1416(PD)}{CP}$$

$$OD = PD + 2(AD)$$

$$PD = OD - \frac{2}{DP}$$

$$PD = \frac{OD \cdot N}{N + 2}$$

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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL ASSIGNMENT TITLE: Calculations--Spur Gears

INTRODUCTION:

Often one needs to either repair a spur gear with a broken tooth or even make a new gear. This requires calculating all or some of the dimensions of the gear.

OBJECTIVE:

To learn how to calculate various dimensions of a spur gear.

ASSIGNMENT:

1. Calculate the following values for a spur gear which has a diametral pitch of 12 and a diameter of 4 inches.
    a. Circular pitch:
    b. Number of teeth:
    c. Pitch diameter
    d. Thickness of tooth
    e. Clearance
    f. Whole depth of tooth

2. If an 8 pitch gear has 32 teeth, find the pitch diameter.

3. A 5 pitch gear has 63 teeth. What is the pitch diameter?

4. If a gear blank is turned to an outside diameter of 6.5 in., find the pitch diameter. The gear is 8 pitch.

5. Find the pitch diameter of a gear with 24 teeth if the addendum equals 1/8 in.

6. If a 1.5 in. circular pitch gear has an addendum of 0.4775 in., find the pitch diameter. The gear has 20 teeth.

7. Find the pitch diameter of a gear with an addendum equal to 0.0909 in. and a circular pitch of 2/7 in. The gear has 24 teeth.

8. The pitch diameter of a 14 pitch gear is 2.429 in. What is the outside diameter?

9. The limits for the outside diameter of an 8 pitch gear are +0.000 and -0.007 in. The plus sign indicates the dimension over, and the minus sign the dimension under the nominal outside diameter. If a blank is turned 4.755 inch in diameter for an 8 pitch 36 tooth gear, is the blank too large or too small? How much? Could the mistake be remedied if detected before the teeth are cut? What about after the teeth are cut?

10. Find the center to center distance of a 12 pitch, 24 tooth gear, and a 12 pitch, 48 tooth gear.

ANSWERS

1. a. .262
   b. 46
   c. 3.833
   d. .131
   e. .013
   f. .180

2. 4

3. 12.600

4. 6.250

5. 3.000

6. 9.550

7. 2.183

8. 2.572

9. No. Too large by .005 in. Yes. No.

10. 3.000

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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL INFORMATION TITLE: Calculation--Indexing Degrees

INTRODUCTION:

Simple indexing can be used for moving the spindle a specified number of degrees. Also, the index head can be used for inspection.

OBJECTIVE:

To provide the student an opportunity to learn how to use the dividing head in terms of indexing degrees rather than numerical divisions.

TECHNICAL INFORMATION:

The dividing head is used primarily for indexing specific divisions, whether it be for so many teeth in a gear, a sprocket, or some other type of part having equally spaced divisions.

However, there are occasions which arise when specific divisions given in degrees are required to be indexed. The dividing head can be used for indexing degrees. There are 360° in a circle, and 40 turns will give one complete revolution. Therefore, one turn of the index crank will revolve the spindle $\frac{1}{40}$ of 360° or $\frac{1}{40} \cdot 360^\circ = 9^\circ$. Since 1 turn is equivalent to 9°, then $\frac{1}{9}$ turn is equivalent to 1°.

Example 1. Index for 72 degrees.

Since 1 turn is equivalent to 9°, then to determine the number of turns, all we have to do is divide 72° by 9°.

$$\text{No. of turns} = \frac{72^\circ}{9^\circ}$$

$$= 8$$

In general:

$$\text{No. of turns} = \frac{\text{Degree measure desired}}{9^\circ}$$

Example 2. Index for $5\frac{1}{2}^\circ$

$$\text{No. of turns} = \frac{\text{Degree measure desired}}{9^\circ}$$

$$= \frac{5\frac{1}{2}^\circ}{9^\circ}$$

$$= \frac{\frac{11}{2}^\circ}{9^\circ}$$

$$= \frac{\frac{11}{2}^\circ}{9^\circ} \cdot \frac{2}{2} \qquad \left(\text{since } \frac{2}{2} = 1 \text{ or in general, } \frac{a}{a} = 1\right)$$

(We multiply the numerator and denominator by 2 in order to eventually change the numerator from a fraction to a whole number.)

$$= \frac{\frac{11}{2}^\circ \cdot 2}{9^\circ \cdot 2} \qquad \left(\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}\right)$$

$$= \frac{11^\circ}{18^\circ} \qquad \text{(mult. inverse)}$$

$$= \frac{11}{18} \text{ turn}$$

Therefore, we use the 11th hole of an 18-hole circle.

Example 3. Index for $6\frac{2}{3}^\circ$

$$\text{No. of turns} = \frac{\text{Degree measure desired}}{9^\circ}$$

$$= \frac{6\frac{2}{3}^\circ}{9^\circ}$$

$$= \frac{\frac{20}{3}^\circ}{9^\circ}$$

$$= \frac{\frac{20}{3}^\circ}{9^\circ} \cdot \frac{3}{3} \qquad (\text{since } \frac{3}{3} = 1)$$

$$= \frac{\frac{20}{3}^\circ \cdot 3}{9^\circ \cdot 3} \qquad (\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d})$$

$$= \frac{20^\circ}{27^\circ} \qquad (\text{mult. inverse})$$

$$= \frac{20}{27} \text{ turn}$$

Therefore, we use the 20th hole of a 27-hole circle.

Note of Information: Approximate indexing in minutes.

Since for example, 22.5 is not available for a circle of holes, a 23-hole circle could be used.

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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machines

TECHNICAL ASSIGNMENT TITLE: Calculations--Indexing Degrees

INTRODUCTION:

The dividing head can be used for indexing for degrees as well as for divisions.

OBJECTIVE:

To learn how to solve problems requiring indexing for a specified number of degrees.

ASSIGNMENT:

1. Index for 82 degrees.
   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____
2. Index for 8 1/2 degrees.
   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____
3. Index for 6 3/4 degrees.
   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____
4. Index for 5 1/2 degrees.
   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

5. Index for 12°20'.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

6. Index for 6°40'.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

7. Index for 29°30'.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

8. Index for 0°40'.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

9. Index for 36°.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

10. Index for 8°20'.

   a. No. of turns:____
   b. Circle:____
   c. No. of spaces:____

ANSWERS

	No. of Turns	Circle	No. of Spaces
1.	9	18	2
2.	0	18	17
3.	0	20	15
4.	0	18	11
5.	1	27	10
6.	0	27	20
7.	3	18	5
8.	0	27	2
9.	4		
10.	0	27	25

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Shaper and Planer

TECHNICAL INFORMATION TITLE: Calculations--Cutting Speed and Number of Strokes per Minute

INTRODUCTION:

All shapers and planers must be set up properly just as any other machines, including the number of strokes per minute. There are many factors to consider: the machine in terms of size, the type of steel, the size of steel, the shape of steel, the tool bit, the cooland, the set-up, and others.

OBJECTIVE:

To provide the student an opportunity to learn how to calculate the proper number of strokes per minute and/or the cutting speed for shapers and planers.

TECHNICAL INFORMATION:

Shapers and planers have a reciprocating ram, and they cut on the forward stroke. These machines are very useful and are more economical to use in some instances than a mill because they have a single took bit. They can be set up for many angles, and tools can be made readily and quickly without great cost when compared to a mill cutter. They have limitations as does any machine; however, a skilled tradesman can find many applications such as: (a) offset shaping, (b) angular shaping, (c) grooves, (d) serrations, (e) dovetails, (f) T-Slots, (g) curves, (h) special one sided extrusing grooves, (i) internal and external squares, hexagons and various divisions including gears as well as (j) forming and bending.

We need to take a look at two different terms used for cutting speed.

One will be the forward cutting speed. This is the actual speed at which the tool cuts on the forward stroke. The second will be an average cutting speed, in which the lost time involved in the return stroke will be involved in the total time.

An example with a car may help to illustrate these terms. Suppose a car is driven at a constant speed (or velocity) of 30 miles per hour for 1/3 of an hour (or 20 minutes). Then the driver stops due to a traffic jam and doesn't move for 10 minutes. Then he starts up again. Now, the speed that he was driving before he stopped was an actual 30 miles per hour. However, the average speed will take into account the total time involved which will be 20 minutes + 10 minutes or 30 minutes (1/2 hour). Therefore, in determining the average speed we note that he went a total of 10 miles (1/3 of an hour at 30 miles per hour) and the total time was 1/2 an hour. Since he went 10 miles in 1/2 an hour the average speed would then be 20 miles per hour.

In any velocity (or speed) problem we have the following relationship:

$$v = \frac{d}{t}$$ (where v is the velocity, d is the distance, and t is the time required for this distance)

For example if you travel 120 miles in 2 hours, then:

$$v = \frac{120 \text{ miles}}{2 \text{ hours}}$$

$$= 60 \text{ miles per hour}$$

Now let us return to the shaper

Example 1. Suppose a shaper operates at 15 strokes (counting forward and return as one stroke) per minute, the length of the stroke is 8 inches, and the return stroke is 4 times as fast as the forward stroke (this may be noted by saying that the return ratio is 4:1). Find the average cutting speed and the forward cutting speed.

First of all, let us find the average cutting speed.

$$\text{avg. CS} = \frac{d \text{ (the usable forward stroke distance for a certain time)}}{t \text{ (total time for both forward and return strokes for the distance in the numerator)}}$$

The usable cutting forward distance in 1 minute will be the product of the length of the forward stroke and the number of strokes in one minute.

Thus:

$$\text{avg. CS} = \frac{L \cdot N}{1}$$ (L is the stroke length, N is the number of strokes per minute, and the 1 represents 1 minute as the time.)

$$\text{avg. CS} = L \cdot N$$ (this will be in feet per minute if the length of the stroke is in feet, and N is the number of strokes per minute)

In our problem:

$$\text{avg. CS} = \frac{2}{3} \cdot 15 \text{ feet per minute}$$

$$\text{avg. CS} = 10 \text{ feet per minute}$$

For the forward cutting speed:

$$\text{forward CS} = \frac{d \text{ (the usable forward stroke distance)}}{t \text{ (the time for forward strokes only)}}$$

During one minute the usable cutting distance will again be:

$$d = 15 \cdot \frac{2}{3}$$

$$= 10 \text{ feet (in one minute)}$$

Now we need to find the time involved for the forward strokes only in a total period of 1 minute. Since the ratio of speed for return and forward strokes is 4:1, let us divide the total time of 1 minute into 5 (4 + 1) time units. Four of the five units or 4/5 min. will be involved for the forward stroke. Since the return stroke is 4 times as fast as the forward stroke, then the time on the return stroke will be 1/4 of the time of the forward stroke or 1/4 times 4 or 1 time unit of the total five. Therefore, 1/5 min. will be spent on the return strokes.

Therefore, for the forward cutting speed:

$$\text{forward CS} = \frac{d}{t}$$

$$= \frac{10 \text{ feet}}{\frac{4}{5} \text{ min.}}$$

$$= \frac{10 \cdot \frac{5}{4}}{\frac{4}{5} \cdot \frac{5}{4}} \quad \text{(multiply the numerator and denominator by 5/4)}$$

$$= \frac{10 \cdot \frac{5}{4}}{1}$$

$$= 10 \cdot \frac{5}{4}$$

$$= \frac{10 \cdot 5}{4}$$

$$= \frac{50}{4}$$

$$= 12.5 \text{ feet per minute}$$

Example 2. A shaper operates at 30 strokes per minute, the length of the stroke is 6 in., and the return ratio is 3:1. Find the average cutting speed and the forward cutting speed.

$$\text{avg. CS} = N \cdot L$$

$$= 30 \cdot \frac{1}{2}$$

$$= 15 \text{ feet per minute}$$

$$\text{forward CS} = \frac{d \text{ (total forward stroke distance for specified time)}}{t \text{ (time for forward strokes only for distance d)}}$$

$$d = 30 \cdot \frac{1}{2}$$

$$= 15 \text{ feet (in one minute)}$$

Note that the value of 15 feet (in one minute) is what we found for the average cutting speed since we are interested in the total distance forward in one minute.

To find t we use the ratio of 3:1. We divide a minute into 3 + 1 or 4 parts. One part of the 4 or 1/4 min. is spent on the return strokes, and 3 parts of the 4 or 3/4 min. is spent on the forward strokes.

Therefore,

$$\text{forward CS} = \frac{15 \text{ feet}}{\frac{3}{4} \text{ min.}}$$

$$= \frac{15 \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} \quad \text{(multiply the numerator and denominator by 4/3)}$$

$$= \frac{15 \cdot \frac{4}{3}}{1}$$

$$= 15 \cdot \frac{4}{3}$$

$$= \frac{15 \cdot 4}{3}$$

$$= \frac{60}{3}$$

$$= 20 \text{ feet per minute}$$

In both of these first two examples, the number of strokes per minute, the length of the stroke, and the return rate were given. Suppose that instead of the number of strokes per minute, the forward cutting speed is given.

Example 3. If the forward cutting speed is 24 feet per min., the length of the stroke is 6 inches, and the return ratio is 2:1, find the average cutting speed and the number of strokes per minute.

First, let us find the average cutting speed.

$$\text{avg. CS} = \frac{d}{t}$$

We do not know the total distance traveled forward in one minute. Therefore, let us take the distance to be one forward stroke, and then find the time for one complete stroke (forward and return).

Let us find a general formula to use.

$$\text{avg. CS} = \frac{d}{t}$$

$$= \frac{L}{t_f + t_r}$$ (where t_f is the time spent on forward strokes, and t_r is the time spent on return strokes)

From the general formula, $v = \frac{d}{t}$, let us solve for t.

$$v = \frac{d}{t}$$

$$v \cdot t = \frac{d}{t} \cdot t$$ (multiply both sides by t)

$$v \cdot t = d$$

$$\frac{1}{v} \cdot v \cdot t = \frac{1}{v} \cdot d$$ (multiply both sides by 1/v)

$$t = \frac{d}{v}$$

Therefore:

$$t_f = \frac{L}{v_f}$$ (where v_f is the forward speed)

$$t_r = \frac{L}{v_r}$$ (where v_r is the return speed)

Now let us find the value for $\frac{L}{v_f} + \frac{L}{v_r}$. The least common denominator will be $v_f \cdot v_r$.

$$\frac{L}{v_f} = \frac{L \cdot v_r}{v_f \cdot v_r}$$ (multiply the numerator and denominator by v_r)

$$\frac{L}{v_r} = \frac{L \cdot v_f}{v_r \cdot v_f}$$ (multiply the numerator and denominator by v_f)

Now,

$$\frac{L}{v_f} + \frac{L}{v_r} = \frac{L \cdot v_r}{v_f \cdot v_r} + \frac{L \cdot v_f}{v_r \cdot v_f}$$

$$= \frac{L \cdot v_r + L \cdot v_f}{v_f \cdot v_r} \qquad \left(\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}\right)$$

Therefore:

$$\text{avg. CS} = \frac{L}{\frac{L \cdot v_r + L \cdot v_f}{v_f \cdot v_r}}$$

$$= \frac{L}{\frac{L \cdot v_r + L \cdot v_f}{v_f \cdot v_r}} \frac{\cdot v_f \cdot v_r}{\cdot v_f \cdot v_r} \qquad \text{(multiply both the numerator and denominator by } v_f \cdot v_r\text{)}$$

$$= \frac{L \cdot v_f \cdot v_r}{L \cdot v_r + L \cdot v_f}$$

$$= \frac{L \cdot v_f \cdot v_r}{L(v_r + v_f)} \qquad (a \cdot b + a \cdot c = a(b + c))$$

$$= \frac{v_f \cdot v_r}{v_r + v_f}$$

Now, in Example 3:

$$\text{avg. CS} = \frac{v_f \cdot v_r}{v_r + v_f}$$

$$= \frac{24 \cdot 48}{24 + 48} \qquad \text{(note that the return velocity is 48 feet per minute since the return ratio is 2:1)}$$

$$= \frac{1152}{72}$$

$$= 16 \text{ feet per minute}$$

Now to find the number of strokes per minute. We will use the relationship that avg. CS = N·L.

$$\text{avg. CS} = N \cdot L$$

$$CS(avg.)\cdot\frac{1}{L} = N\cdot L\cdot\frac{1}{L}$$

$$\frac{CS(avg.)}{L} = N$$

Thus:

$$N = \frac{16}{\frac{1}{2}}$$

$$= \frac{16\cdot 2}{\frac{1}{2}\cdot 2}$$ (multiply both the numerator and denominator by 2)

$$= \frac{32}{1}$$

= 32 strokes per minute

Example 4. If the forward cutting speed is 30 feet per minute, the length of the stroke is 9 in., and the return ratio is 1:1, find the average cutting speed and the number of strokes per minute.

$$\text{avg. } CS = \frac{v_f\cdot v_r}{v_f + v_r}$$

$$= \frac{30\cdot 30}{30 + 30}$$

$$= \frac{900}{60}$$

= 15 feet per minute

$$N = \frac{CS(avg.)}{L}$$

$$= \frac{15}{\frac{3}{4}}$$

$$= \frac{15\cdot\frac{4}{3}}{\frac{3}{4}\cdot\frac{4}{3}}$$ (multiply both the numerator and denominator by 4/3)

$$= \frac{15\cdot\frac{4}{3}}{1}$$

$$= 15 \cdot \frac{4}{3}$$

$$= \frac{15 \cdot 4}{3}$$

$$= \frac{60}{3}$$

= 20 strokes per minute

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Shaper and Planer

TECHNICAL ASSIGNMENT TITLE: Calculations--Number of Strokes Per Minute and Cutting Speed

INTRODUCTION:

The speed factor is very important in setting up any machine. In setting up the shaper, the number of strokes per minute must be considered for efficiency as well as for effective machining.

OBJECTIVE:

To learn how to calculate the number of strokes per minute and the cutting speed for a shaper or planer.

ASSIGNMENT:

1. How many strokes per minute should a shaper be set for a stroke of 6 inches if the forward cutting speed is to be 90 feet per minute and the return ratio is 1:1?

2. What is the forward cutting speed if the stroke is 10 inches and the number of strokes per minute is 42 and the return ratio is 1:1?

3. If the forward cutting speed of a shaper is 44 feet per minute and the return is 3:1, what is the average cutting speed?

4. If the length of a planer stroke is 10 feet and the return is 3:1, how many strokes per minute will a planer make with a forward cutting speed of 40 feet per minute?

5. If the forward cutting speed of a shaper is 50 feet per minute and the return is 4:1, what is the average cutting speed and the number of strokes per minute? The stroke is 15 inches.

6. If the forward cutting speed of a shaper is 40 feet per minute, the return is 4:1, and the stroke is 12 in., what is the average cutting speed and the number of strokes per minute?

7. If a planer makes a 10 foot stroke at the rate of 5 strokes per minute, what is the average cutting speed when the return ratio is 4:1?

8. If the forward cutting stroke of a shaper is 20 feet per minute and the return stroke is two times as fast, what is the average cutting speed of the shaper?

9. If the length of a shaper stroke is 12 in. and the return ratio is 4:1, how many strokes per minute will the shaper make if the forward cutting speed is 20 feet per minute?

10. If a shaper does 20 strokes per minute, has a stroke of 9 in., and has a return ratio of 4:1, find the forward cutting speed and the average cutting speed.

ANSWERS

1. 90 strokes per minute
2. 70 feet per minute
3. 33 feet per minute
4. 3 strokes per minute
5. 40 feet per minute, 32 strokes per minute
6. 32 feet per minute, 32 strokes per minute
7. 50 feet per minute
8. 13.333 feet per minute
9. 16 strokes per minute
10. avg. CS = 15 feet per minute, forward CS = 18.75 feet per minute

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Drilling and Tapping--Bench Work

JOB TITLE: Tapped Tee Head

INTRODUCTION:

In machine trades work the tradesman will encounter bench work which involves layout and machining. Drilling and tapping are common daily occurences. In spite of modern techniques, bench work is as much a part of work today as it was years ago. A knowledge of taps, their use, and how to work with them without breakage is of utmost importance.

OBJECTIVE:

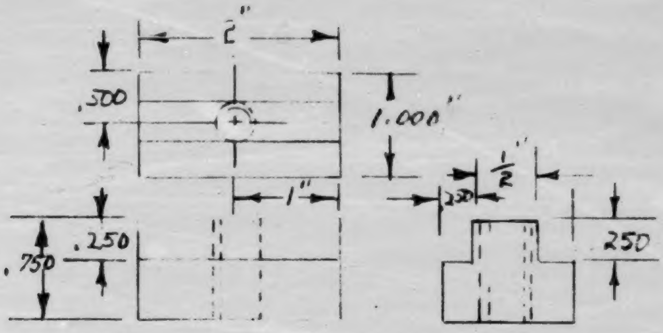
To provide the student an opportunity to develop bench work skills including layout work, filing, and tapping.

JOB SPECIFICATIONS:

Tee slot nuts are used on all machines which have T-slots in their tables. All machine shops should always carry a good supply of these in various sizes. All T-slots are not the same size.

Be careful to read the drawings before proceeding to work. Make certain to follow the plan of procedure. Use the proper tap drill for the tap to be used.

DRAWINGS: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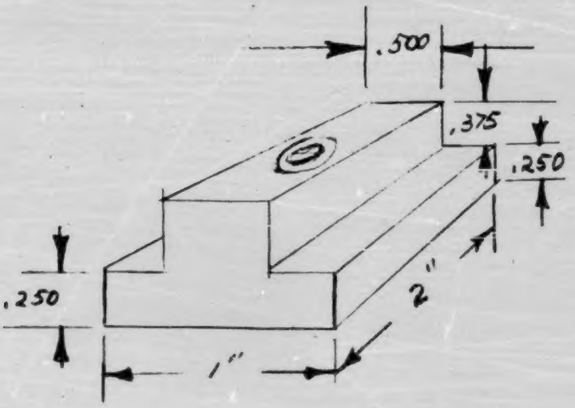
1.000"
1"
.250
.750
1/2"
.250
250


.500
.375
.250
.250
2"
1"

TOOLS:

- Micrometers
- Combination rule
- Scriber
- Gage block and scriber
- Center punch
- Tap drill
- Tap
- Tap wrench
- Shaper or mill

MATERIALS:

High speed steel

PROCEDURE:

(Operations)	(Related Information)
1. Procure materials.	1. Use high carbon steel, preferably oil-hardening steel.
2. Measure and saw rough stock.	2. Cut oversize enough to square up on shaper or mill.
3. Shape or mill stock to required dimensions.	3. Refer to drawing as you work.
4. Mill offset for Tee.	4. Use side milling cutters.
5. Layout for tapped hole.	5. Use purple layout dye before machining.
6. Center punch.	6. For more accuracy first use prick punch or automatic center punch.
7. Set up in vise on drill press.	7. Set piece on parallels. Use strips of paper for checking set-up.
8. Center drill.	
9. Drill all the way through.	9. Use correct size drill for tap.
10. Tap the hole.	10. Start with starting tap, and finish with plug or bottoming tap.
11. Heat treat.	11. Refer to Machinery Handbook or steel specifications before heat treating.
12. Inspect. Then have instructor check your work.	

QUESTIONS:

1. Explain how to cut the offsets of the Tee head.
2. What is a tap drill?
3. How do you know what size tap drill should be used if you do not have a chart?
4. What are the taps used for a blind hole?
5. What tap or taps are used for a through hole?

SELF-EVALUATION:

1. Did you read and follow the procedure carefully?
2. Did you ask questions concerning any operation that you did not know or had doubts?

3. Did you make any mistakes? List.

4. Explain how you would correct your mistakes.

5. Did you profit from your mistakes?

6. What grade would you reward yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Tapering--Lathe

JOB TITLE: Lathe Centers

INTRODUCTION:

This job is primarily designed to give training in turning tapers on a lathe. There are various systems of standard tapers such as the Morse Taper, the Brown and Sharpe Taper, the Jarno Taper, and the Taper Pin Taper. Also, there are three methods of turning tapers on a lathe: (1) by offsetting the tailstock; (2) by using the compound rest; and (3) by using the taper attachment method. Another one is also used commonly. This is the forming method. The method used depends on the following: (1) the length of the taper; (2) the angle of the taper; (3) the number of pieces to be machined; and (4) whether the piece is held in a chuck or between centers.

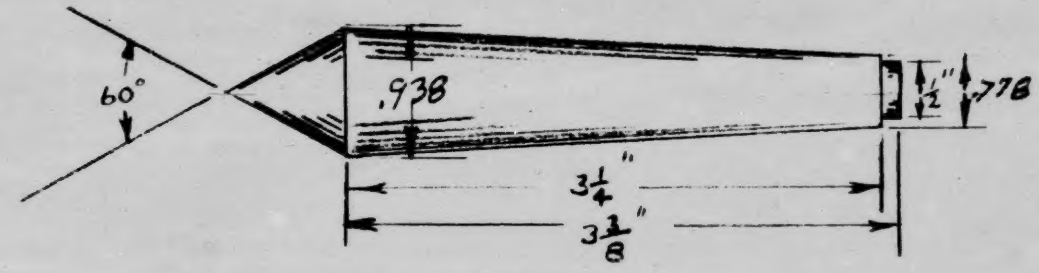
OBJECTIVE:

To provide the student an opportunity to develop skills in turning tapers on a lathe.

JOB SPECIFICATIONS:

Be sure to get all of the specifications concerning the measurements. Check the machinery handbook regarding tapers, and use the Technical Information Sheets for calculating a taper in terms of TPF, angles, or offsetting the tailstock.

DRAWING:



TOOLS:

Micrometers
Sleeve for gaging
Tool bits
Rule

MATERIALS:

High speed steel or alloy steel

PROCEDURE:

(Operations)	(Related Information)
1. Procure correct steel.	1. Steel must be high enough in carbon for heat treating. It should preferably be oil-harden steel.
2. Measure and saw rough steel.	2. Allow sufficient material to cut center hole in one end (at least 1/2" longer than needed).
3. Mount stock in chuck and face first end.	3. Just face off enough to clean up.
4. Center drill first end.	4. Use proper size center drill.
5. Reverse stock in chuck and face second end.	5. Just face off enough to clean up.
6. Remove chuck and mount face plate.	6. Use board across ways of lathe as a precaution.
7. Mount stock between centers using a dog.	7. Be sure to lubricate tailstock center and have proper tension between centers.

8.	Using roughing cuts and finishing cuts turn to .010" over largest diameter.	8.	The .010" oversize is the grinding allowance.
9.	Set up and turn long taper by use of taper attachment or offsetting the tailstock.	9.	Refer to correct operation sheet and Technical Information sheets on tapers. Check taper occasionally with the taper sleeve. Be sure small end is .010" over specified size.
10.	Using properly ground tool bit or square nose cut-off tool, cut shoulder to size on small end.	10.	Do not allow the .010" for grinding on the shoulder.
11.	Slightly chamfer or round the sharp corner on end of taper.	11.	Use high RPM on the lathe and a flat mill file.
12.	Mark the 3 1/4" length on the taper.	12.	Use hermaphrodite caliper with the curved leg against the shoulder at small end.
13.	Remove headstock lathe center and insert the center you are making.	13.	If the lathe is large you may need to use a sleeve.
14.	Swing compound rest to correct angle for the point of the center.	14.	Refer to Operation Sheet and Technical Information Sheet on compound rest tapering.
15.	Cut short 60° included taper. Cut almost to the line scribed in step number 12.	15.	The short 60° taper must be ground also.
16.	Heat treat.	16.	Refer to correct procedure for the type of steel used. Refer to Machinery Handbook.
17.	Grind long taper.	17.	Use cylindrical grinder and special adapter for point. Grind to exact size.
18.	Grind short 60° included angle.	18.	Use tool post grinder on lathe with center inserted in headstock.
19.	Inspect and have instructor check your work.		

QUESTIONS:

1. List three types of taper systems.
2. List four methods for tapering on a lathe.
3. What does TPF mean?
4. How can you check a taper?
5. What are the advantages of using each of the four methods of tapering in question 2?

6. What are the limits and/or disadvantages for each of the methods in question 2?

SELF-EVALUATION:

1. Did you follow the procedure carefully?

2. Did you have any questions?

3. Did you make any mistakes?

4. List all of the mistakes that you made.

5. List how you would correct all of the mistakes.

6. What grade would you give yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Turning--Lathe

JOB TITLE: Mandrel

INTRODUCTION:

Turning on a lathe is accomplished by revolving the work as the tool progressively moves along and into the metal. The center line of the lathe is the axis and is parallel to the ways of the lathe. Allignment of the lathe centers is essential for accurate work. Roughing cuts as well as finishing cuts are important phases of the procedure in order to obtain efficient and accurate work.

OBJECTIVE:

To provide the student an opportunity to develop skill in turning on a lathe.

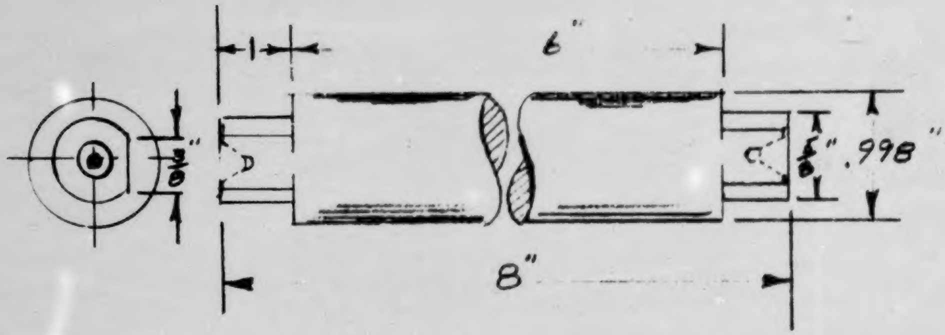
JOB SPECIFICATIONS:

There are many types of mandrels: the expansion mandrel, the gang mandrel, and the standard solid mandrel for example.

The solid mandrel will be the type you are to make. The mandrel tapers about 0.008 inch per foot, and the nominal size is near the middle. The ends are turned somewhat smaller than the body, so that any nicks or burrs caused by the clamping of the dog will not injure the accuracy of the mandrel.

It is very important that the centers are large enough to withstand the strain caused by turning. The centers are recessed (cut out around the center) for protection. The size of the mandrel is always marked on the large end.

DRAWING:



TOOLS:

Micrometers
Rule
Tool bits
Mill
Lathe

End Mill
Vise
Numbers
Hammer
Center drill

Dog
Drive plate
Drill chuck
Lathe centers

MATERIALS:

High speed steel or alloy steel

PROCEDURE:

(Operations)	(Related Information)
1. Procure material.	1. Refer to drawing.
2 Measure and saw rough stock.	2. Allow sufficient stock to face ends.
3. Secure in lathe chuck and face first end.	3. Just clean up first end. Refer to Technical Information Sheet for proper RPM.
4. Center drill first end.	4. The size of the mandrel will determine the size of the center drill.
5. Recess first end.	5. Use facing tool. Refer to drawing.
6. Reverse stock in chuck and face second end.	6. Face stock to exact mandrel length.
7. Center drill second end.	7. Use same size center drill as on the first end.
8. Recess second end.	8. Use facing tool. Refer to drawing for size of recess.

9. Remove chuck and mount stock between centers.	
10. Turn first end to smaller O.D. size.	10. Use finishing tool and finishing cut.
11. Put a slight chamfer on first end.	11. Use 10" mill file and set lathe to filing speed.
12. Turn large O.D.	12. Stock should be turned .010" oversize. This is the allowance for grinding.
13. Reverse stock between centers and turn small diameter on second end.	13. Use finishing tool and finishing cut on final cut.
14. Put a slight chamfer on second end.	14. Use mill file and set lathe to filing speed.
15. Heat treat.	15. Harden and temper according to type of use.
16. Grind to proper O.D. size with .008" taper.	16. Measure the O.D. in center of stock. Use tool post grinder in lathe or set external grinder to proper taper.
17. Mount mandrel in vise on magnetic chuck and grind flat as indicated on drawing.	17. Place mandrel on parallels. Use paper strips to check set up.
18. Inspect.	18. Be sure the center O.D. is the correct size and check each end of the taper for .008" taper.

QUESTIONS:

1. List at least three types of mandrels used between centers.
2. Give exmples of uses for each of the mandrels in question 1.
3. Why is the nominal size at the center of the mandrel?
4. Why is the O.D. turned smaller at the ends?
5. Why are the centers recessed on the mandrel?

SELF-EVALUATION:

1. Did you follow the procedure carefully:
2. Did you have any questions before you started?
3. Did you make any mistakes?
4. List all the mistakes that you made.

5. List how you would correct all of the mistakes and if you have gained from them.

6. What grade would you give yourself?

MODERN MATHEMATICS
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THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Threading--Lathe

JOB TITLE: Stud Bolt--American Standard Threads

INTRODUCTION:

In this job sheet the student will involve himself primarily with cutting threads on a lathe. The student will learn terminology, symbols, shapes, sizes, dimensions, calculations, gearing of a lathe, the proper set up for American Standard right hand external threads, and the proper method for measurement of threads.

OBJECTIVE:

To provide the student an opportunity to develop skills in thread cutting on a lathe.

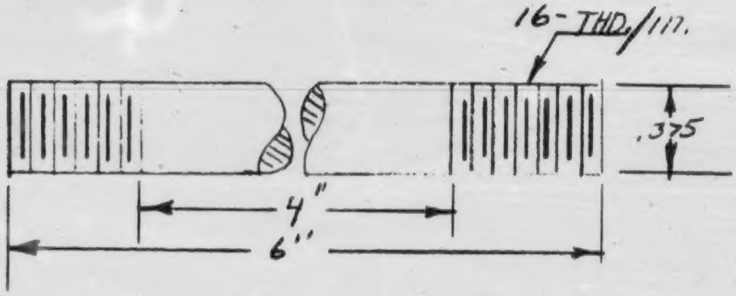
JOB SPECIFICATIONS:

There are a number of systems of threads such as: (1) American National Thread system, (2) Unified Thread system, (3) National Fine and National Course Thread systems, (4) American National Acme Thread system, and (5) American National Pipe Thread system.

Terms involved with threads are as follows: screw thread, external thread, major diameter, minor diameter, pitch diameter, pitch, axis of the screw, lead, crest, root, width of flat, depth of thread, included angle, fit, tolerance, and right hand and left hand threads.

Many of the above must be calculated before setting up the lathe for the actual threading operation. Note: The student should be sure to study the technical information sheet on American Standard Threads and Unified Threads before starting to work.

DRAWING:



TOOLS:

Micrometer
Rule
Tool bit
Center gage
Thread pitch gage
Center drill
Dog
Thread micrometer and/or
Male thread gage (Check with
machinery handbook

MATERIALS:

Round cold rolled steel

PROCEDURE:

	(Operations)		
1.	Procure materials.	1.	See drawing.
2.	Measure rough stock.	2.	Allow proper amount for chucking in 4 jaw or runnin between centers.
3.	Saw		
4.	Secure in lathe chuck.		
5.	Face first end.	5.	Just face off enough to square end.
6.	Center drill first end.	6.	Use proper size center drill.
7.	Turn around in chuck and face second end to exact length.	7.	Refer to drawing.
8.	Center drill second end.	8.	Use proper center drill. Drill to proper depth.
9.	Take stock out of chuck.		
10.	Remove chuck and mount face plate.	10.	Use chuck board across ways of lathe.
11.	Mount stock between centers using driving dog.	11.	Be sure tail of dog is in the main slot of face plate

12. Turn correct outside diameter on first end and center.	12. Refer to drawing.
13. Reverse stock in lathe.	13. Unturned end should be at tail stock.
14. Turn second end to major diameter.	14. Refer to drawing.
15. Set up lathe gear box for proper number of threads.	15. Refer to drawing. Have instructor check gear box setting.
16. Swing compound to right, 29°.	16. Refer to Technical Information Sheet on American Standard Threads. Also refer to operation sheet.
17. Grind tool bit to included angle of 60°.	17. Check tool with center gage.
18. Line up threading bit.	18. Use fish tail against tail stock sleeve.
19. Put slight chamfer on end of stock.	19. Use side of threading tool bit.
20. Using cuts of .010-.015 in. start threading on one end.	20. Check the number of threads with thread gage.
21. Continue until threads are almost down to size.	21. Use thread micrometer.
22. Finish the thread to size using .003-.004 in. cuts.	22. Use thread micrometer or male thread gage.
23. Turn stock around in lathe.	23. Be sure to wrap finished threads in brass, tin, or copper for protection of threads.
24. Repeat steps 19 through 22.	
25. Remove from lathe and submit to instructor for inspection.	

QUESTIONS:

1. Name the various systems of threads.
2. List and define all of the thread terms (Refer to Technical Information Sheet on American Standard and Unified Threads).
3. Calculate all of the necessary dimensions for a 1/2 - 13 N.C. Thread.
4. What angle do you set the compound rest for cutting American Standard Threads?

SELF-EVALUATION:

1. Did you follow the procedure carefully?
2. Did you have any questions?

3. Did you make any mistakes?

4. List all of the mistakes that you made.

5. List how you would correct all of the mistakes.

6. What grade would you give yourself?

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Gear Cutting--Using the Milling Machine

JOB TITLE: Spur Gear

INTRODUCTION:

Gears are commonly designed into practically all types of machines for transmitting power. In plants where machine shops are well equipped, the machinist can cut gears which are either out of supply, for experimental equipment, or he may need to cut a few teeth in a gear that has been broken off and welded. In any case, a good machinist should know various gears used and their functions. He should know about various definitions and the calculations involved regardless of whether he is going to write up the specifications so that a new gear may be purchased or whether he is going to mill the teeth himself.

OBJECTIVE:

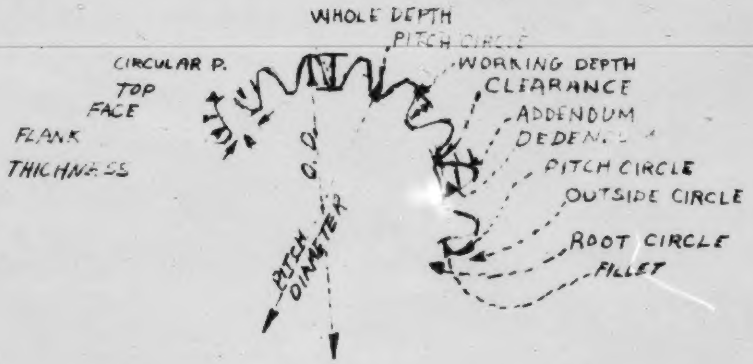
To provide the student an opportunity to learn both the mathematical and the manipulative skills regarding gear cutting.

JOB SPECIFICATIONS:

One needs to acquaint himself with the definitions of each part of a gear before attempting to cut a gear on a milling machine. There are certain minimum basic terms which must be understood such as the following: (a) the number of teeth, (b) the diametral pitch, (c) the number of the cutter, (d) the major diameter, and (e) the depth of the teeth.

Also, he must be sure to know how to calculate how many turns, what circle, and the number of spaces that are required on the dividing head for a specified number of teeth on a gear.

DRAWING:


WHOLE DEPTH
PITCH CIRCLE
CIRCULAR P.
TOP
FACE
FLANK
THICHNESS
O.D.
PITCH DIAMETER
WORKING DEPTH
CLEARANCE
ADDENDUM
DEDEN
PITCH CIRCLE
OUTSIDE CIRCLE
ROOT CIRCLE
FILLET

TOOLS:

Gear tooth vernier
Micrometers
Mandrel
Dog
Gear cutter
Dividing head
Footstock
Center rest

MATERIALS:

Round gear blank stock--steel or cast iron

PROCEDURE:

(Operations)	(Related Information)
1. Secure materials.	1. Refer to Bill of Materials.
2. Machine gear blank.	2. Stock should be turned to size while on a solid mandrel.
3. Set up gear on solid mandrel.	3. Use while mounted on the same mandrel on which it was turned to size.
4. Set up gear blank between footstock and index head centers.	4. Set headstock, footstock and driving dog with pressure on the mandrel.
5. Set up index head for proper divisions.	5. Refer to operation and technic information sheets.
6. Set up gear tooth cutter for center of blank.	6. Be sure cutter touches only th highest point on the gear blan This is the center line.
7. Set gear tooth cutter for very slight (.010"-.015") depth.	7. This small depth is used for checking the setting of index centers.

8.	Set up milling machine for correct speed.	8.	Refer to Technical Information Sheet.
9.	Set milling machine for proper table feed.	9.	Refer to Technical Information Sheet.
10.	Cut blank to the nick depth.	10.	This double checks your indexing set-up. The teeth should be counted to see that you have the correct number.
11.	Set milling tooth cutter for correct depth.	11.	Refer to Technical Information Sheet for calculation of the depth.
12.	Using the correct table feed cut all teeth in the gear blank.	12.	Double check the first tooth cut with a gear tooth vernier to be sure it is correct.
13.	Remove gear and arbor from mill.	13.	Handle the gear carefully.
14.	File off all burrs made by the milling cutter.	14.	This is done at the bench with a double-cut file.
15.	Remove gear from mandrel.	15.	Use arbor press.
16.	Heat treat if required.	16.	Check print for type of steel used and Rockwell Hardness Test Number. Refer to machinery handbook.

QUESTIONS:

1. Name various types of gears.

2. List all the definitions and rules: spur-gear elements and tooth parts.

3. List rules and formulas for dimensions of spur gears.

4. What is the formula for calculating the chordal thickness of a gear?

SELF-EVALUATION:

1. Did you read carefully all of the steps in the procedure and answer the questions above?

2. Did you ask any questions concerning the job assignment?

3. Did you make any mistakes?

4. List all of the mistakes.

5. Correct all your mistakes.

6. Did you profit by your mistakes?

7. What grade would you give yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Squaring--Shaper, Mill

JOB TITLE: Parallels

INTRODUCTION:

This job is very excellent for beginning students because it involves many basic operations which are prerequisites for advanced work. Being able to perfectly shape, mill, and grind a workpiece square or rectangular is the first step toward becoming a good toolmaker.

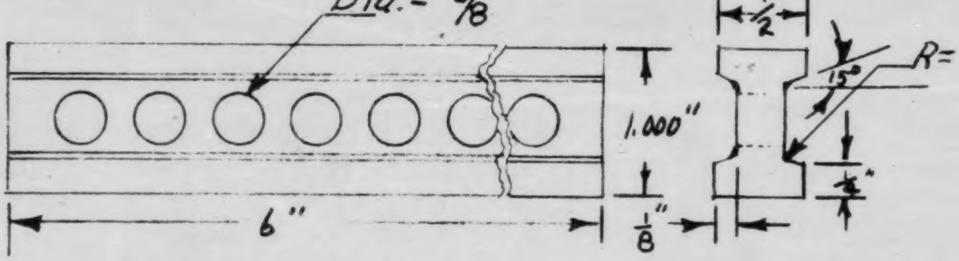
OBJECTIVE:

To provide the student an opportunity to learn how to shape, mill, and/or grind a workpiece square or rectangular.

JOB SPECIFICATIONS:

The important things in this job are as follows: (a) set up the workpiece in a vise properly, (b) the workpiece must be shaped or milled perfectly square, (c) the workpiece must be ground to tolerance as specified by the instructor. Read the print or drawing and follow the procedure very carefully.

DRAWING:



TOOLS:

Hold down clamps
Angle plate
Clamps
Verniers
Center-drill

Drill
Reamer
Shaper tool or
Mill cutter
Shaper or Mill

Micrometer
Dividers
Master square
Combinating square
Surface gage

MATERIALS:

High speed steel or alloy

PROCEDURE:

(Operations)	(Related Information)
1. Procure proper material.	1. Refer to Bill of Material.
2. Measure rough stock.	2. Cut stock at least 1/16 to 1/8 inch longer than shown on print and as near to the rectangular size as possible.
3. Saw stock.	3. Cut at least 1/16 inch longer than finished size.
4. Set correctly in vise for shaping first face.	4. Mount in vise in parallels using paper strips to check set-up.
5. Set shaper for correct speed and feed.	5. Refer to Technical Informatio Sheets to determine the corre speed and feed.
6. Using correct tool, just clean up face number 1.	6. Be sure tool cuts under the scale.
7. Set up for second face shaping.	7. Use face number 1 as referenc for set-up.
8. Using correct tool shape face number 2 until it is to size.	8. Allow at least .015" for mill and at least .005" for grindi
9. Set in vise correctly for first edge shaping.	9. Use finished surfaces and pap strips to assure correct set-
10. Clean up first edge.	10. Cut deep enough to get under surface scale.
11. Set up second edge shaping.	11. Use first edge as reference and strips of paper to check set-up.
12. Shape second edge to near print size.	12. Allow .015" for milling and .005" for grinding.
13. Set up correctly for first end shaping.	13. Use finished edges and faces as reference using paper stri for checking set-up.
14. Shape first end. Clean up only.	14. Plan to cut extra length off next set-up.

15.	Set up for second end shaping.	15.	Follow note in number 13.
16.	Shape second end to print size plus allowances.	16.	Allow .015" for milling, .005" for grinding.
17.	Remove all burrs and sharp edges with a file.	17.	Handle carefully to avoid cuts. Use 10" double-cut mill file.
18.	Remount in vise carefully for shaping undercut section.	18.	Use parallel bars and paper strips to insure correct set-up.
19.	Cut first center undercut to correct print size.	19.	Allow .005" for grinding.
20.	Reset in vise for shaping second center undercut.	20.	Refer to number 18.
21.	Shape to print size plus allowance.	21.	Refer to number 19.
22.	Apply layout blue dye.	22.	Apply evenly over the recessed section.
23.	Layout holes as per print dimensions.	23.	Use vernier caliper with stock clamped vertically in V plate.
24.	Lay stock on surface plate on one edge.	24.	Be sure surface plate is clean.
25.	Scribe linear center line of holes.	25.	Use vernier calipers taking into account oversize allowance for grinding.
26.	Carefully center punch the centers of the hole locations.	26.	Use automatic center punch or prick punch.
27.	Mount in vise on drill press.	27.	Use parallel bars and paper strips.
28.	Center drill all holes.	28.	Use small center drill.
29.	Drill holes 1/64 or 1/32 inch under finished size.	29.	The allowance is for reaming.
30.	Ream all holes to finished size.	30.	Use cutting oil with reamer.
31.	Remove all burrs and sharp edges.	31.	Use 10" double-cut mill file.
32.	Heat treat.	32.	Refer to Machinery Handbook for type of tool steel used.
33.	Grind all faces to size using the same procedure as was used in shaping the faces.	33.	Use soft wheel on surface grinder specified for hard steel.
34.	Temper the piece.	34.	Check with Rockwell Hardness Tester after tempering at proper temperature.
35.	Have instructor inspect your work.		

QUESTIONS:

1. Explain the procedure (in detail) for setting up a workpiece for each face, edge, and end before shaping or milling.

2. Explain how to calculate the locations of centers for holes.

3. Calculate the strokes per minute for the workpiece--refer to the Technica Information Sheet.

4. Calculate the number of revolutions per minute for drilling. Refer to th Technical Information Sheet.

5. Explain how to heat treat the workpiece.

6. Explain in detail how to grind each face, edge, and end so that it will meet the specifications given to you by the instructor.

SELF-EVALUATION:

1. Did you follow the procedure carefully?

2. Did you make any mistakes?

3. List all the mistakes that you made.

4. List how you would correct all of the mistakes.

5. What grade would you give yourself?

POSSIBLE SOURCES
FOR
ADDITIONAL STUDY

The following is a partial listing of books that might be utilized for additional study in the machine trades and in modern mathematics.

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Modern Mathematics
As Applied To
MACHINE TRADES

Volume II

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To the Reader

The materials in this volume were developed under a research grant awarded to Mid-Central Area Vocational School, Elwood, Indiana.

The project was supported by the following personnel in the Indiana State Department of Public Instruction: Mr. Harold Negley, State Superintendent; Mr. Robert E. Howard, Associate Superintendent for Vocational Affairs; Mr. Don Gentry, Executive Officer and State Director, State Board for Vocational-Technical Education; Mr. Monte Janik, Chief Consultant, Industrial Education; and Ms. Carol Hodgson, Coordinator, Research and Exemplary Projects.

PREFACE

Mathematics is one of the areas in education in which a great revolution has taken place. The "new" or "modern" approach to mathematics emphasizes understanding rather than just a series of manipulative techniques with little or no reference to basic properties, laws, and definitions.

A major purpose of the new or modern mathematics is to present mathematics as a consistent, logical, and step by step development. Then, based on this strong foundation the student is helped to develop a firm understanding of mathematics. Analysis of a problem should play a central role. However, it should be emphasized that drill is still an important aspect.

The modern approach to mathematics has developed a new way of teaching mathematics that should be carried on in the vocational related mathematics areas. If vocational machine trades education is to maintain its respectability and to progress in providing real vocational industrial education, it must update its related mathematics. This should involve stimulating the student "to react," "to do," "to discover," and "to explore." The student must have an opportunity to do more than merely follow like a sheep an instructor's command to "listen, watch, and then do as I do."

Through a research grant approved and funded by the Vocational Division of the Indiana State Department of Public Instruction, a developmental research project was undertaken to develop machine trades related mathematics materials using the terminology, concepts, and methods of the modern mathematics.

Volume I and Volume 2 of Modern Mathematics as Applied to the Machine Trades were developed through this research project. Included in each of these two volumes are technical information lead in sheets, machine trades technical information sheets, technical assignment sheets, sample technical operation sheets, and sample technical job sheets.

The technical information lead in sheets present in a simple and direct manner important terminology, concepts, and methods utilized in the modern mathematics. So that these units may be used for both practice and reference, practice problems with answers are provided with each technical information lead in sheet.

Each of the machine trades technical information sheets presents specific machine tool technology technical information utilizing the modern mathematics approach and terminology. As much as possible these units emphasize understanding of the concepts and formulas involved. Technical assignment sheets including assigned problems and answers have been included to provide the student with valuable practice.

Volume I is designed to be utilized by first year machine tool technology students, and Volume 2 is designed for students in the second year. Each volume was written with the student in mind. That is, ease of reading and understanding was a primary objective.

The two volumes are not designed to be a complete course of study for the machine trades area. However, sample operation sheets and sample job sheets have been included to illustrate the utilization of the technical information sheets in specific operations and jobs.

Since the terminology, concepts, and methods of modern mathematics have been emphasized throughout the two volumes, it is strongly advised that all teachers who plan to utilize the volumes participate either in in-service workshops or take at least one class emphasizing the modern mathematics. As educators we all realize that the success of any course or program depends heavily on the teacher's understanding and enthusiasm.

The writers of *Modern Mathematics as Applied to the Machine Trades* sincerely believe that these two volumes are a definite advancement and achievement in the area of machine tool related mathematics.

Guy J. Hale
Lester W. Hale
Daniel Rayshich

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MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Algebra of Sets

INTRODUCTION AND/OR OBJECTIVES:

Algebra of sets is considered to be the unifying concept of the modern mathematics. In this section, sets and subsets will be discussed. Also, the concepts of unions and intersections of sets will be explained. At the elementary level, sets are used to help develop an understanding of basic arithmetic operations.

TECHNICAL INFORMATION:

I. SETS

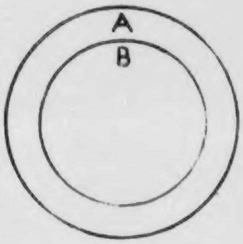
A set is simply a collection of objects. For example, set A may consist of the numbers 1, 2, 3, 4, and 5. Then, $A = \{1, 2, 3, 4, 5\}$. The numbers 1, 2, 3, 4, and 5 are called the elements of set A. The symbol $\in$ denotes "element of." Therefore, we may write $1 \in A$, $2 \in A$, $3 \in A$, $4 \in A$, and $5 \in A$. If $B = \{a, b, c, d\}$, then $a \in B$, $b \in B$, $c \in B$, and $d \in B$.

A set A is equal to a set B if they contain exactly the same elements.

Example 1. If $A = \{1, 2, 3, 4\}$, and $B = \{1, 2, 3, 4\}$, then $A = B$.

II. SUBSETS

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5\}$. Then set B is called a subset of set A. This is denoted as follows: $B \subseteq A$. Notice that every element of set B is an element of set A. B is defined to be a subset of A if every element of B is an element of A. $B \subseteq A$ is read as "B is a subset of A." In Figure 1 B is a subset of A.

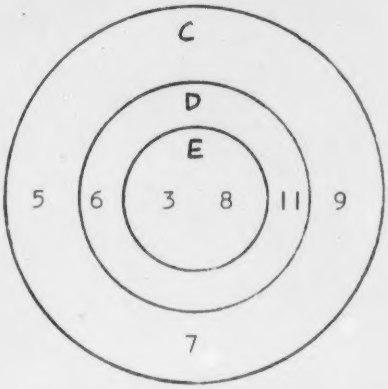


$B \subseteq A$

Figure 1

It is often helpful to use what are called Venn diagrams to provide a visual interpretation of set operations. These were used in Figure 1.

Example 2. If $C = \{3, 5, 6, 7, 8, 9, 11\}$, $D = \{3, 6, 8, 11\}$, and $E = \{3, 8\}$, then $D \subseteq C$, $E \subseteq C$, and $E \subseteq D$. See Figure 2.



$D \subseteq C$

and

$E \subseteq C$

and

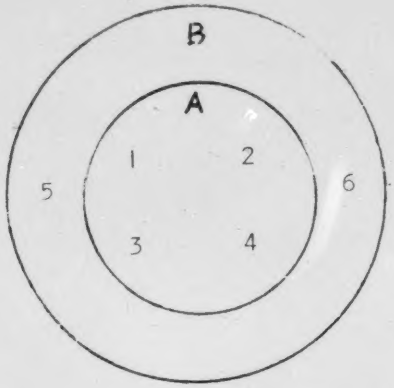
$E \subseteq D$

Figure 2

Notice that if $A = \{1, 2, 3, 4\}$, then $A \subseteq A$ by the definition of subset. That is, any set is a subset of itself.

Some textbooks use only the above notation for subsets. Other texts use another notation. As discussed above, $A \subseteq B$ will allow the possibility that $A = B$. The notation, $A \subset B$, (read as "A is a proper subset of B") is used when A is a subset of B, but it is definitely known that A is not equal to B. That is, $A \subset B$ if $A \subseteq B$ and $A \neq B$.

Example 3. $A = \{1, 2, 3, 4\}$, and $B = \{1, 2, 3, 4, 5, 6\}$ will imply that $A \subseteq B$. It is also true that $A \subset B$ (since $A \subseteq B$ and $A \neq B$). Therefore, in this example A is a subset of B, and, also, A is a proper subset of B. See Figure 3.



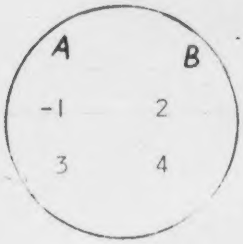
$A \subseteq B$

and

$A \subset B$

Figure 3

Example 4. If $A = \{-1, 2, 3, 4\}$ and $B = \{-1, 2, 3, 4\}$, then $A \subseteq B$ and $A = B$. Therefore, it is not true that $A \subset B$. See Figure 4.



$A \subseteq B$

and

$A = B$

Figure 4

III. UNION OF SETS

If A = {1, 2, 3, 4} and B = {2, 3, 5, 7, 9}, then A U B is defined to be the set {1, 2, 3, 4, 5, 7, 9}. That is, in this example A U B = {1, 2, 3, 4, 5, 7, 9}. A U B is read as "A union B." In the union of two sets, all elements in both sets are listed, but common elements of the two sets are not listed twice.

For any two sets A and B, A U B is defined to be the set of those elements which are either in A or in B (or in both).

Example 5. In illustration 1 and in illustration 2 in Figure 5 A U B is represented by the shaded area.

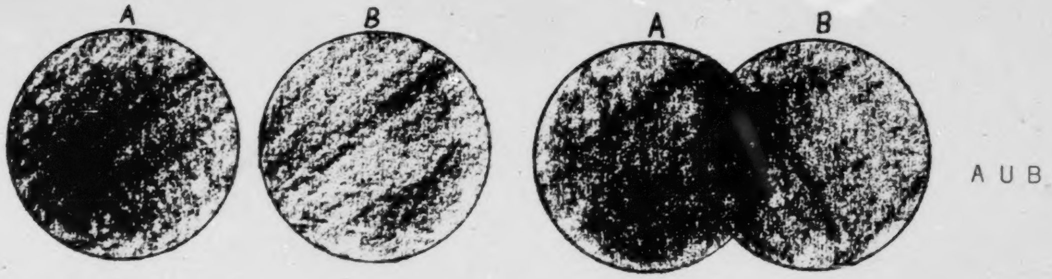


Figure 5

Example 6. If A = {-1, 2, 3, 4} and B = {-3, 0, 2}, then A U B = {-3, -1, 0, 2, 3, 4}. See Figure 6.

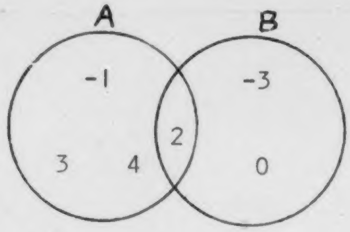
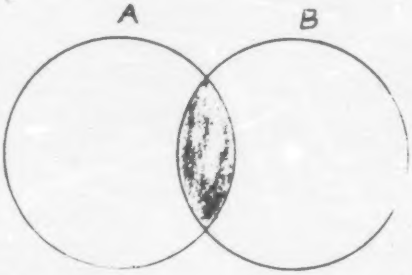


Figure 6

IV. INTERSECTION OF SETS

While the union of two sets lists all of the various elements in either set, the intersection of two sets is the set of elements which are in common to both sets. If $A = \{-1, 0, 1, 2\}$ and $B = \{-1, 1, 2, 4, 5\}$, then $A \cap B = \{-1, 1, 2\}$. $A \cap B$ is read as "A intersection B." *The intersection of two sets A and B, written as $A \cap B$, is defined to be the set of elements which are in both sets A and B.* If Figure 7, $A \cap B$ is represented by the shaded area.



$A \cap B$

Figure 7

Example 7. If $B = \{-3, -2, 3, 5, 7, 8\}$, and $C = \{-2, 5, 7, 10, 11, 12\}$, then $B \cap C = \{-2, 5, 7\}$. See Figure 8.

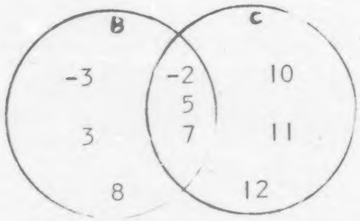


Figure 8

Example 8. If $A = \{-2, -1, 0\}$ and $B = \{2, 3, 4, 5\}$, then $A \cap B$ does not contain any elements. A set containing no elements is called the null set and is denoted by $\emptyset$. Therefore, in this example, $A \cap B = \emptyset$. See Figure 9.

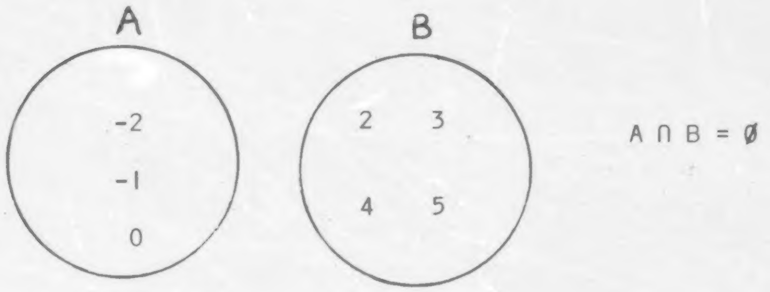


Figure 9

V. FURTHER EXAMPLES

Example 9. If $A = \{3, 4, 7, 8\}$ and $B = \{1, 3, 4, 7, 8, 9\}$, then $A \subseteq B$.

Example 10. If $A = \{-3, 4, 7, 8\}$ and $B = \{2, 3, 4, 7, 8\}$, then $A \cup B = \{-3, 2, 3, 4, 7, 8\}$.

Example 11. If $C = \{4, 5, 9, 10, 12\}$ and $D = \{5, 9, 12\}$, then $D \subseteq C$.

Example 12. If $A = \{-5, 7, 9, 12, 15\}$ and $B = \{-5, 9, 15\}$, then $B \subseteq A$.

Example 13. If $A = \{1, 2, 4, 6, 9\}$ and $B = \{2, 4, 6, 8, 10\}$, then $A \cap B = \{2, 4, 6\}$.

Example 14. If $C = \{-4, 5, 6, 9, 13, 16, 18\}$ and $D = \{6, 9, 16, 18\}$, then $C \cap D = \{6, 9, 16, 18\}$.

Example 15. If $B = \{2, 4, 6, 8\}$ and $C = \{1, 3, 5, 7, 9\}$, then $B \cap C = \emptyset$.

Example 16. If $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

SUMMARY

Symbol	Read As	Definition
$A \subseteq B$	A is a subset of B	If every element of A is an element of B
$A \subset B$	A is a proper subset of B	If $A \subseteq B$ and $A \neq B$
$A \cup B$	A union B	The set of elements either in set A or in set B (or in both)
$A \cap B$	A intersection B	The set of elements common to both sets A and B
$\emptyset$	The null set	The set containing no elements

EXERCISES

1. If $A = \{1, 3, 4, 5\}$ and $B = \{3, 4, 5\}$, is it true that $B \subseteq A$? Is it true that $B \subset A$?

2. If $A = \{-1, 0, 2, 3\}$ and $B = \{-2, -1, 0, 2\}$, is it true that $B \subseteq A$?

3. If $A = \{-1, 2, 3, 4, 5, 7\}$ and $B = \{-1, 0, 2, 4\}$, find $A \cup B$.

4. If $A = \{1, 2, 5, 8, 9\}$ and $B = \{1, 5, 9\}$, is it true that $A \subseteq B$? Find $A \cup B$.

5. If $A = \{-10, -7, -5, 0\}$ and $B = \{-7, -5, 0, 1, 2\}$, find $A \cap B$.

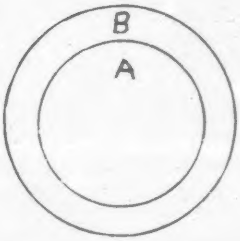
6. If $B = \{3, 5, 7, 9\}$ and $C = \{2, 4, 6, 8\}$, find $A \cup B$ and $A \cap B$.

7. If $A = \{1, 2, 3, 4\}$ and $C = \{1, 2, 3, 4\}$, is it true that $A \subseteq C$? Is it true that $A = C$? Is it true that $A \subset C$?

8. If $A = \{1, 2, 3\}$ and $B = \{2, 4, 8\}$, find $A \cup B$. Find $A \cap B$.

9. If $A = \{-2, 3, 5, 6, 7\}$ and $B = \{-3, -2, 3, 5\}$, find $A \cap B$.

10. If $A \subseteq B$, what is $A \cup B$? What is $A \cap B$?



ANSWERS

1. Yes. Yes.
2. No. (Since not all elements of B are elements of A)
3. $A \cup B = \{-1, 0, 2, 3, 4, 5, 7\}$.
4. No. $A \cup B = \{1, 2, 5, 8, 9\}$.
5. $A \cap B = \{-7, -5, 0\}$.
6. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$. $A \cap B = \emptyset$.
7. Yes. (Since every element of A is an element of C) Yes. No.
8. $A \cup B = \{1, 2, 3, 4, 8\}$. $A \cap B = \{2\}$.
9. $A \cap B = \{-2, 3, 5\}$.
10. $A \cup B = B$. $A \cap B = A$.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead-In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition, Subtraction, Multiplication and Division

INTRODUCTION AND/OR OBJECTIVES:

Of course, the operations of addition, subtraction, multiplication, and division are basic to all mathematical problems. The student should thoroughly understand the relationship between addition and subtraction and the relationship between multiplication and division.

TECHNICAL INFORMATION:

I: ADDITION AND SUBTRACTION

To gain a better understanding of the basic operations of addition and subtraction, we will look at the number line. Suppose we wish to find the value for 3 + 2.

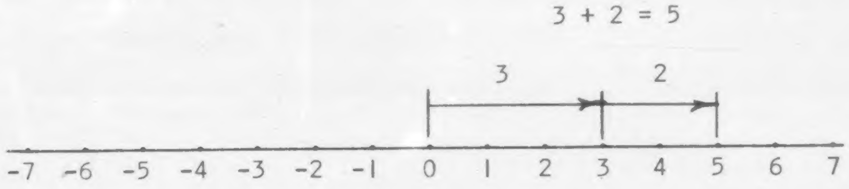


Figure 1

Study Figure 1 and you will see that 3 + 2 = 5. Note that the positive direction is to the right.

Now, suppose we wish to find the value of 5 + (-4). See Figure 2.

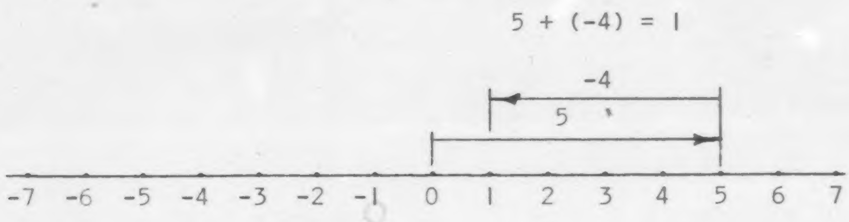


Figure 2

Since the positive direction is to the right and the negative direction is to the left, then 5 + (-4) = 1.

We now make the following definition for subtraction:

$$a - b = a + (-b)$$

Example 1. Find the value for 7 - 3.

$$7 - 3 = 7 + (-3)$$
$$= 4$$

Example 2. Find the value for 2 - 6.

$$2 - 6 = 2 + (-6)$$
$$= -4$$

Another relationship between addition and subtraction may be noted.

Suppose we wish to find the value of 100 - 98. Many people will solve this problem by noting that we must add 2 to 98 in order to get 100. In other words,

$100 - 98 = \square$ is equivalent to $98 + \square = 100$

or using an x in place of the box,

$100 - 98 = x$ is equivalent to $98 + x = 100$

Of course 2 should be placed in each box and $x = 2$.

We may write the above result in general as follows:

$a - b = \square$ is equivalent to $b + \square = a$

or

$a - b = x$ is equivalent to $b + x = a$

Example 3. Write the equivalent expression for $50 - 3 = \square$, and solve.

$50 - 3 = \square$ is equivalent to $3 + \square = 50$

47 should be placed in each box.

Example 4. Write the equivalent expression for $75 - 71 = x$, and solve.

$75 - 71 = x$ is equivalent to $71 + x = 75$

$x = 4$

II. MULTIPLICATION AND DIVISION

Suppose we wish to find the value of 8/4. The answer is 2 since $4 \cdot 2 = 8$ (4 times 2 equals 8). Here the dot indicates multiplication. In general we have the following definition for division.

$\frac{a}{b} = \square$ is equivalent to $b \cdot \square = a$

or

$\frac{a}{b} = x$ is equivalent to $b \cdot x = a$

Example 5. Write an equivalent expression for $12/3 = \boxed{}$.

$\frac{12}{3} = \boxed{}$ is equivalent to $3 \cdot \boxed{} = 12$

4 should be placed in each box

Example 6. Write an equivalent expression for $20/4 = x$.

$\frac{20}{4} = x$ is equivalent to $4 \cdot x = 20$

$x = 5$

EXERCISES

In each problem write an equivalent expression and solve.

1. $10 - 3$
2. $7 - 9$
3. $12 - 10 = \square$
4. $98 - 95 = \square$
5. $80 - 72 = x$
6. $45 - 39 = x$
7. $\frac{15}{3} = \square$
8. $\frac{12}{6} = \square$
9. $\frac{18}{6} = x$
10. $\frac{24}{3} = x$

ANSWERS

1. $10 + (-3)$
   Answer: 7

2. $7 + (-9)$
   Answer: -2

3. $10 + \square = 12$
   Answer: 2

4. $95 + \square = 98$
   Answer: 3

5. $72 + x = 80$
   Answer: $x = 8$

6. $39 + x = 45$
   Answer $x = 6$

7. $3 \cdot \square = 15$
   Answer: 5

8. $6 \cdot \square = 12$
   Answer: 2

9. $6 \cdot x = 18$
   Answer: 3

10. $3 \cdot x = 24$
    Answer: 8

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Sets of Numbers

INTRODUCTION AND/OR OBJECTIVES:

Exactly what types of numbers are available for working with practical problems? The purpose of this section is to describe the various sets of numbers: the set of counting numbers, the set of whole numbers, the set of integers, the set of rational numbers, and the set of real numbers. It should be noted that as these sets of numbers are developed, each set includes each of the sets of numbers previously discussed. That is, each set of numbers is a subset of each of the sets later discussed. In later technical information sheets, it will be assumed that the set of real numbers is the set of numbers being used.

TECHNICAL INFORMATION:

1. THE SET OF COUNTING NUMBERS

If counting the number of certain objects is all that is desired, the numbers 1, 2, 3, 4, and so on will be sufficient for the purpose. The set of numbers, {1, 2, 3, 4 . . .}, is called the set of counting numbers. This same set is also called the set of natural numbers or the set of positive integers. These numbers continue indefinitely to the right on the number line. See Figure 1.

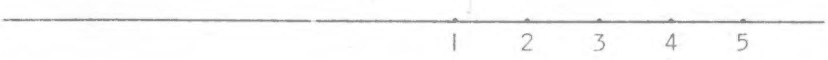


Figure 1

II. THE SET OF WHOLE NUMBERS

If the number 0 is added to the set of counting numbers, then the resulting set, {0, 1, 2, 3, 4 . . .} , is called the set of whole numbers or the set of non-negative integers. See Figure 2.

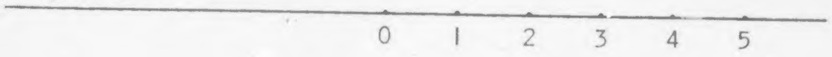


Figure 2

III. THE SET OF INTEGERS

If two whole numbers such as 2 and 3 are added, the result, 5, is again a whole number. However, subtraction will cause problems. The answer to 4 - 7 is not a whole number. Therefore, to perform subtraction, the number -3 as well as the negatives of all counting numbers must be added to the set of whole numbers. The set, {. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . .} , is called the set of integers. The integers continue indefinitely both to the right and to the left on the number line. See Figure 3.

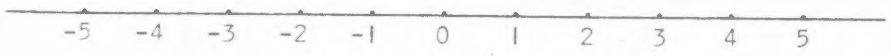


Figure 3

IV. THE SET OF RATIONAL NUMBERS

The set of integers, however, is not sufficient as a set of numbers to use in all applications. For example, in Figure 4, to measure the piece of metal, fractions of an inch are necessary.

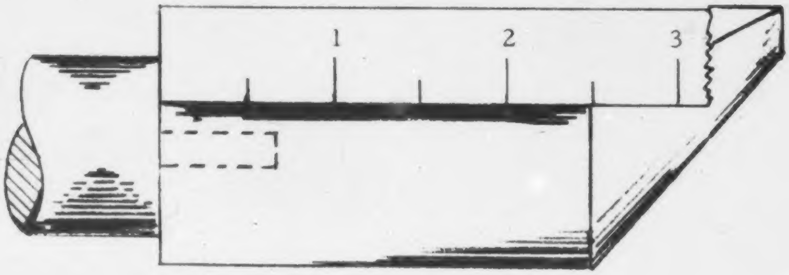


Figure 4

Therefore, to have a more usable set of numbers, positive and negative fractions need to be added to the set of integers. The set of rational numbers is defined to be the set of all numbers which can be expressed in the form $\frac{a}{b}$ where a and b are integers and b is not 0. For example, $\frac{2}{3}$ is a rational number where a = 2 and b = 3. If a = -17 and b = 4, then $\frac{a}{b}$ becomes the rational number $\frac{-17}{4}$.

Is the set of integers a subset of the set of rational numbers? That is, can we tell if the set of integers is included in the set of rational numbers? We are really asking if each integer is a rational number. That is, for example, is the integer 7 a rational number? The answer is "yes" because 7 can be expressed as $\frac{7}{1}$. Therefore, 7 can be expressed in the form $\frac{a}{b}$ where a and b are integers. The set of all rational numbers, then, includes all integers in addition to all fractions such as $\frac{1}{2}$, $\frac{3}{5}$, $\frac{51}{4}$, $\frac{-5}{7}$, $\frac{-35}{8}$, etc. It is impossible to list all of the rational numbers. The list would continue indefinitely. A few of the rational numbers are indicated on the number line in Figure 5.

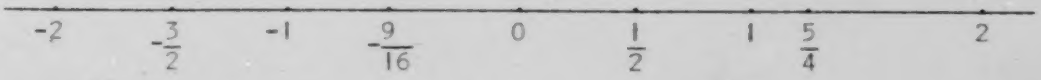


Figure 5

V. THE SET OF REAL NUMBERS

Many applications may be performed without the addition of any numbers other than the rational numbers. However, for such numbers as $\sqrt{2}$, the set of rational numbers is not sufficient. Before considering $\sqrt{2}$, an introductory example may help. In finding $\sqrt{4}$, a positive number x needs to be found such that x times x equals 4. That is, what is the positive number x such that $x \cdot x = 4$? Of course, $x = 2$. Therefore, $\sqrt{4} = 2$. Now, to go back to the problem of $\sqrt{2}$. What is the positive number x such that $x \cdot x = 2$? There is no rational number that will work. The best that can be done is to give an approximate value for $\sqrt{2}$ by using a square root table. A square root table may give the value of $\sqrt{2}$ as 1.414. This is an approximate value correct to three decimal places. Actually, 1.414 times 1.414 equals 1.999 and not 2.

The same is true for many other numbers such as $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$, π, etc. The set of all numbers of this type is called the set of irrational numbers. These comprise all numbers on the number line that are not rational numbers.

The set of real numbers is formed by combining the set of rational numbers with the set of irrational numbers. The set of real numbers comprises all numbers on the number line. It includes all of the types of numbers discussed in the above sections. It includes all integers, all positive and negative fractions, and all irrational numbers such as $\sqrt{2}$ and $\sqrt{11}$. It would certainly again be impossible to list all real numbers. A few of the real numbers are indicated on the number line in Figure 6.

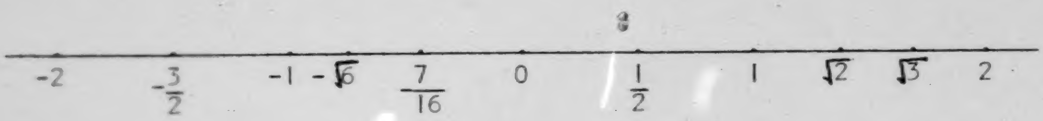


Figure 6

SUMMARY

1. The set, {1, 2, 3, 4 . . .} is called the set of counting numbers, the set of natural numbers, or the set of positive integers.

2. The set, {0, 1, 2, 3, 4 . . .} is called the set of whole numbers or the set of non-negative integers.

3. The set, {. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . .} is called the set of integers.

4. The set of elements that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ is called the set of rational numbers.

5. The set of real numbers is the set consisting of all rational numbers and all irrational numbers. This set contains all numbers on the number line.

EXERCISES

1. Why is 23 a rational number?
2. Is the number 0 a rational number?
3. To what sets does $\frac{-5}{8}$ belong?
4. Is -11 a real number? Is it a rational number? Is it an integer? Is it a counting number?
5. Is the answer to 6 - 11 a whole number? If not, what is it?
6. To what set does any number on the number line belong?

ANSWERS

1. 23 can be expressed in the form $\frac{23}{1}$.
2. Yes. 0 can be written as $\frac{0}{4}$, or $\frac{0}{1}$, etc.
3. Reals and rationals.
4. Yes. Yes. Yes. No.
5. No. An integer, rational number, and real number.
6. The set of real numbers.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead-In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Additive and Multiplicative Properties of Real Numbers

INTRODUCTION AND/OR OBJECTIVES:

This is one of the most important lead-in sections. It is important because it provides much of the basis for the structure involved in modern mathematics. This section should be well mastered. Later work with equations and formulas in the technical information sheets will depend heavily on a thorough understanding of this material.

TECHNICAL INFORMATION:

I. COMMUTATIVE AND ASSOCIATIVE PROPERTIES OF ADDITION

In Figure I, the dimension D may be found by evaluating 2 + 1 or 1 + 2. The answer in either case is certainly 3. This expresses the property that $a + b = b + a$.

The property that for any two real numbers a and b,

$$a + b = b + a$$

is called the commutative property of addition.

To find the value of dimension E in Figure I, 2 + 1 may first be determined, and then the result added to 3, or 2 could be added to the

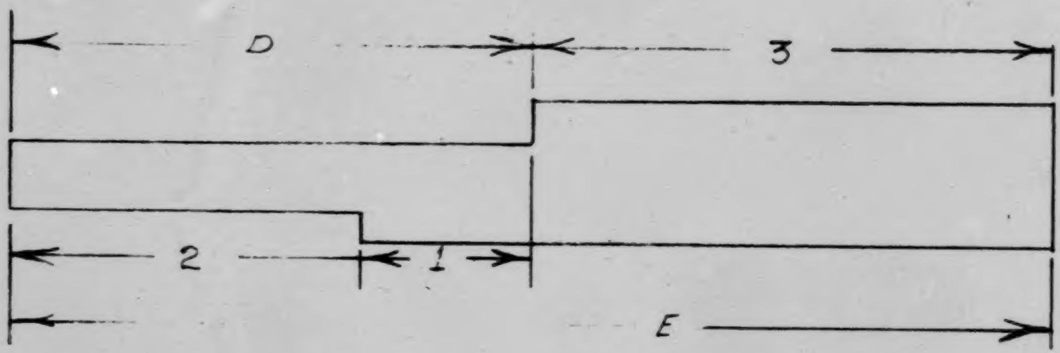


Figure 1

result of 1 + 3. In the first method, (2 + 1) + 3 = 3 + 3 = 6. Note that in evaluation, the sum 2 + 1 on the inside of the parentheses is first found to be 3. Then, this 3 is added to the second 3 to get the final answer of 6. In the second method of finding E, 2 + (1 + 3) = 2 + 4 = 6. Note that again the quantity 1 + 3 on the inside of the parentheses is first found to be 4. Then 2 is added to 4 to find the final answer of 6. This example demonstrates the property that (a + b) + c = a + (b + c).

E = (2 + 1) + 3	or	E = 2 + (1 + 3)
= 3 + 3		= 2 + 4
= 6		= 6

Therefore, (2 + 1) + 3 = 2 + (1 + 3)

The property that for any three real numbers a, b, and c, then

(a + b) + c = a + (b + c)

is called the associative property of addition.

II. COMMUTATIVE AND ASSOCIATIVE PROPERTIES OF MULTIPLICATION

In the multiplication of two numbers such as 2 and 3, it is true that $2 \cdot 3 = 3 \cdot 2$ since both sides of the equation equal 6. This illustration demonstrates the property that $ab = ba$.

$2 \cdot 3 = 6$ and $3 \cdot 2 = 6$

Therefore, $2 \cdot 3 = 3 \cdot 2$.

The property that for any two real numbers a and b, then

$$ab = ba$$

is called the commutative property of multiplication.

Note the similarity between the commutative property of addition and the commutative property of multiplication. The properties demonstrate that regardless of the order, the result is the same.

Similar to the associative property of addition, there is a comparable property for multiplication. In the evaluation of $(2 \cdot 3)4$ it is first found that $2 \cdot 3 = 6$. Then the product of $6 \cdot 4$ is found to be 24. Therefore, $(2 \cdot 3)4 = 24$. If the placement of the parentheses is changed, the problem becomes $2(3 \cdot 4)$. The product of $3 \cdot 4$ is first found to be 12. Then 2 is multiplied times 12 to get the final answer of 24. Note the procedure below:

$$(2 \cdot 3)4 = 6 \cdot 4 = 24 \quad \text{and} \quad 2(3 \cdot 4) = 2 \cdot 12 = 24$$

This example demonstrates the property that $(ab)c = a(bc)$.

The property that for any three real numbers a, b, and c, then

$$(ab)c = a(bc)$$

is called the associative property of multiplication.

III. DISTRIBUTIVE PROPERTIES

In the evaluation of 2(3 + 5), first of all, the sum of 3 + 5 is determined to be 8. Then 2 is multiplied times 8 to get the final answer of 16. This result is the same as finding $2 \cdot 3 + 2 \cdot 5$. In this expression, $2 \cdot 3 = 6$ and $2 \cdot 5 = 10$. Then $2 \cdot 3 + 2 \cdot 5 = 6 + 10 = 16$.

$$\begin{aligned} 2(3 + 5) &= 2 \cdot 8 \\ &= 16 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2 \cdot 3 + 2 \cdot 5 &= 6 + 10 \\ &= 16 \end{aligned}$$

This illustrates the property that $a(b + c) = ab + ac$.

Note that similarly $(3 + 5)2 = 3 \cdot 2 + 5 \cdot 2$.

$$\begin{aligned} (3 + 5)2 &= 8 \cdot 2 \\ &= 16 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 3 \cdot 2 + 5 \cdot 2 &= 6 + 10 \\ &= 16 \end{aligned}$$

This illustrates the property that $(b + c)a = ba + ca$.

The properties that for any three real numbers a, b, and c, then

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca$$

are called the distributive properties of multiplication over addition.

IV. ADDITIVE IDENTITY AND ADDITIVE INVERSES

It is certainly true that 5 + 0 = 5 and also that 0 + 5 = 5. This demonstrates the property that for 0 and any real number a, then $a + 0 = a$ and $0 + a = a$.

$$5 + 0 = 5 \quad \text{and} \quad 0 + 5 = 5$$

> The property that for any real number a, it is true that
>
> $$a + 0 = a \text{ and } 0 + a = a$$
>
> is called the additive property of zero. 0 is called the additive identity.

It is true that for any given number, for example 5, there exists exactly one number such that 5 plus that number is equal to 0. The number in this example is -5 since $5 + (-5) = 0$. Also, $(-5) + 5 = 0$. Given the number, -6, then there exists the number 6 such that $(-6) + 6 = 0$ and $6 + (-6) = 0$. In other words, for any given real number a, there exists a number $-a$ such that $a + (-a) = 0$ and $(-a) + a = 0$.

> The property that for any given real number a, there exists exactly one real number $-a$ such that
>
> $$a + (-a) = 0 \text{ and } (-a) + a = 0$$
>
> is called the property of additive inverses. The number $-a$ is called the additive inverse of a, and a is the additive inverse of $-a$.

Example I. Find the additive inverse of 2.

Solution: The additive inverse of 2 is -2 since,

$$2 + (-2) = 0 \text{ and } (-2) + 2 = 0$$

Example 2. Find the additive inverse of $\frac{2}{3}$.

Solution: The additive inverse of $\frac{2}{3}$ is $-\frac{2}{3}$ since,

$$\frac{2}{3} + \left(-\frac{2}{3}\right) = 0 \text{ and } \left(-\frac{2}{3}\right) + \frac{2}{3} = 0$$

Example 3. Find the additive inverse of -4.

Solution: The additive inverse of -4 is 4 since,

$$(-4) + 4 = 0 \text{ and } 4 + (-4) = 0$$

V. MULTIPLICATIVE IDENTITY AND MULTIPLICATIVE INVERSES

In multiplication, it is true that $5 \cdot 1 = 5$ and $1 \cdot 5 = 5$. This demonstrates the property that for the number 1 and any real number a, then $a \cdot 1 = 1 \cdot a = a$.

$$5 \cdot 1 = 5 \qquad \text{and} \qquad 1 \cdot 5 = 5$$

The property that for any real number a it is true that

$$a \cdot 1 = 1 \cdot a = a$$

is called the *multiplicative property of 1*. 1 is called the *multiplicative identity*.

It is true that for a given number, for example 2, there exists a number such that 2 times that number is equal to 1. The number in this example is $\frac{1}{2}$ since $2 \cdot \frac{1}{2} = 1$. Also, note that $\frac{1}{2} \cdot 2 = 1$. For the number 4 there exists what number such that 4 times that number is 1? The answer is $\frac{1}{4}$ since $4 \cdot \frac{1}{4} = 1$. In other words for any given number a except 0, there exists a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.

The property that for any real number a except 0 there exists a real number 1/a such that

$$a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$$

is called the property of multiplicative inverses. 1/a is called the multiplicative inverse of a, and a is called the multiplicative inverse of 1/a.

Example 4. Find the multiplicative inverse of 3.

Solution: The multiplicative inverse of 3 is $\frac{1}{3}$.

$$3 \cdot \frac{1}{3} = 1 \text{ and } \frac{1}{3} \cdot 3 = 1$$

Example 5. Find the multiplicative inverse of $\frac{2}{3}$.

Solution: The multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \text{ and } \frac{3}{2} \cdot \frac{2}{3} = 1$$

VI. A NOTE ON NOTATION

Referring back to the associative property of addition, since $(a + b) + c = a + (b + c)$, there is no confusion in writing $a + b + c$. Similarly, since from the associative property of multiplication, $a(bc) = (ab)c$, we may write abc.

SUMMARY

For any real numbers a, b, and c

$a + b = b + a$	the commutative property of addition
$ab = ba$	the commutative property of multiplication
$(a + b) + c = a + (b + c)$	the associative property of addition
$(ab)c = a(bc)$	the associative property of multiplication
$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	the distributive properties of multiplication over addition
$a + 0 = 0 + a = a$	the additive property of zero (0 is called the additive identity)
$a \cdot 1 = 1 \cdot a = a$	the multiplicative property of 1 (1 is called the multiplicative identity)
For any real number a, there exists a real number -a such that $a + (-a) = (-a) + a = 0$	the property of additive inverses (a and -a are additive inverses)
For any real number a except 0, there exists a real number 1/a such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$	the property of multiplicative inverses (a and 1/a are multiplicative inverses)

EXERCISES

For problems 1 through 7 state the property being used. Check the property by evaluating each side of the equation. (commutative and associative properties)

PROPERTY

1. $3 + 10 = 10 + 3$ ____________________
2. $4 \cdot 10 = 10 \cdot 4$ ____________________
3. $3 + (4 + 5) = (3 + 4) + 5$ ____________________
4. $4(3 \cdot 4) = (4 \cdot 3)4$ ____________________
5. $2 \cdot 8 = 8 \cdot 2$ ____________________
6. $2 + (4 + 1) = (2 + 4) + 1$ ____________________
7. $5(3 \cdot 4) = (5 \cdot 3)4$ ____________________

For problems 8 to 14 complete using the property indicated. Again, find the value for each side of the equation.

8. $3 \cdot 5 =$ ____________________ (commutative property of mult.)
9. $3 + (5 + 7) =$ ____________________ (associative property of add.)
10. $4 + 5 =$ ____________________ (commutative property of add.)
11. $2(6 \cdot 3) =$ ____________________ (associative property of mult.)
12. $2.4 + (2.2 + 3.1) =$ ____________________ (associative property of add.)
13. $7.1 + 2.3 =$ ____________________ (commutative property of add.)
14. $2(5 \cdot 3) =$ ____________________ (associative property of mult.)

For problems 15 to 18, complete using the property indicated. Evaluate each side of the equation. (distributive properties)

15. $3(5 + 6) =$ ____________________ (distributive property)
16. $(2 + 1)3 =$ ____________________ (distributive property)

17. 4(3.1 + 3.5) = ____________________ (distributive property)

18. (3 + 5)x = ____________________ (distributive property)

Complete (in problems 19 through 27)
(identities and inverses)

19. The additive identity is ____________________.

20. The multiplicative identity is ____________________.

21. The additive inverse of 3 is ____________________.

22. The additive inverse of 100 is ____________________.

23. The multiplicative inverse of 7 is ____________________.

24. The multiplicative inverse of 14 is ____________________.

25. The additive inverse of -6 is ____________________.

26. The multiplicative inverse of -4 is ____________________.

27. The multiplicative inverse of 3/4 is ____________________.

ANSWERS

For problems 1 through 7

	Property	Value of each side
1.	Commutative property of addition	13
2.	Commutative property of multiplication	40
3.	Associative property of addition	12
4.	Associative property of multiplication	48
5.	Commutative property of multiplication	16
6.	Associative property of addition	7
7.	Associative property of multiplication	60

For problems 8 through 14

	Completion using property	Value of each side
8.	$5 \cdot 3$	15
9.	$(3 + 5) + 7$	15
10.	$5 + 4$	9
11.	$(2 \cdot 6)3$	36
12.	$(2.4 + 2.2) + 3.1$	7.7
13.	$2.3 + 7.1$	9.4
14.	$(2 \cdot 5)3$	30

For problems 15 through 18

	Completion using property	Value of each side
15.	$3 \cdot 5 + 3 \cdot 6$	33
16.	$2 \cdot 3 + 1 \cdot 3$	9
17.	$4(3.1) + 4(3.5)$	26.4

18. $3x + 5x$

For problems 19 through 27

19. 0

20. 1

21. -3

22. -100

23. 1/7

24. 1/14

25. 6

26. -1/4

27. 4/3

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Equivalence of Fractions

INTRODUCTION AND/OR OBJECTIVES:

Since all measurements cannot be limited to working with whole numbers, fractions become very important. A certain measurement may be noted as 12/16, 24/32, or 3/4. These are equivalent fractions. How do we know if one fraction is equivalent to another? This question will be answered in this section. Also, in dealing with equivalent fractions, which is the simplest fraction to use? For example, it would certainly be burdensome to work with the fraction 228/304. It would in most cases be much nicer to work with the fraction 3/4, which is equivalent to 228/304. This section deals with finding for any given fraction the simplest fraction name or what may be called "reducing the fraction to lowest terms."

TECHNICAL INFORMATION:

Suppose that we need to measure the width of a piece of metal. In Figure 1, we use a rule in which each inch is divided into four equal parts. The width is measured as 3 of 4 equal parts. Therefore, the width is 3/4 inch.

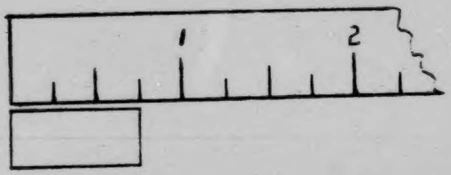


Figure 1

In Figure 2 the same piece of metal is being measured by a rule on which each inch is divided into 8 equal parts. In this case the width is measured as 6 of the 8 equal parts. Therefore, the width is 6/8 of an inch.

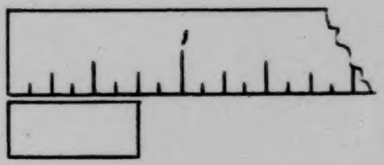


Figure 2

In Figure 3 the same piece of metal is being measured by a rule in which each inch is divided into 16 equal parts. In this case the width is measured as 12 of the 16 equal parts. Therefore, the width is 12/16 of an inch.

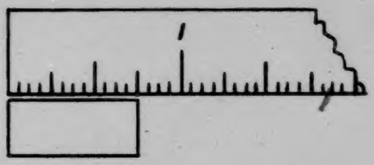


Figure 3

Similarly, if a rule divided into 32 equal parts per inch were used, the width would be measured as 24/32 of an inch.

All of these measurements were for the same piece of metal.. Therefore, all four measurements should be the same. This means that: $\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{24}{32}$. These fractions are called "equivalent fractions".

There are two interesting relationships that may be noted between these equivalent fractions. First of all, note that for the equivalent fractions 3/4 and 6/8 it is true that 3 x 8 = 4 x 6. Similarly, 6/8 and 24/32 are equivalent fractions, and it is true that 6 x 32 = 8 x 24. This suggests the following:

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if a x d = b x c.

Example 1. Are $\frac{2}{3}$ and $\frac{8}{12}$ equivalent fractions?

That is, can we check to see if it is true that:

$2 \times 12 \overset{?}{=} 3 \times 8$

$24 = 24$

Therefore, since 2 x 12 = 3 x 8, then $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent fractions.

Example 2. Are $\frac{5}{8}$ and $\frac{8}{32}$ equivalent fractions?

$5 \times 32 \overset{?}{=} 8 \times 8$

$160 \neq 64$

Therefore, since 5 x 32 ≠ 8 x 8, then $\frac{5}{8}$ and $\frac{8}{32}$ are not equivalent.

Therefore: $\frac{5}{8} \neq \frac{8}{32}$

Example 3. Are $\frac{5}{8}$ and $\frac{20}{32}$ equivalent fractions?

$5 \times 32 \overset{?}{=} 8 \times 20$

$160 = 160$

Therefore, $\frac{5}{8}$ and $\frac{20}{32}$ are equivalent fractions.

Then: $\frac{5}{8} = \frac{20}{32}$

Now, remember that previously we noted that $\frac{3}{4} = \frac{6}{8}$.

$$\frac{3}{4} = \frac{6}{8}$$

And $\frac{6}{8} = \frac{3 \times 2}{4 \times 2}$

Then $\frac{3}{4} = \frac{3 \times 2}{4 \times 2}$

Similarly, we previously recognized that $\frac{3}{4} = \frac{12}{16}$.

$$\frac{3}{4} = \frac{12}{16}$$

And $\frac{12}{16} = \frac{3 \times 4}{4 \times 4}$

Then $\frac{3}{4} = \frac{3 \times 4}{4 \times 4}$

These examples suggest the property that:

$$\frac{a \times c}{b \times c} = \frac{a}{b} \qquad (\text{if } c \neq 0)$$

That is, we may obtain an equivalent fraction from a/b by multiplying both the numerator a and the denominator b by a common factor c. This is the second relationship that we may notice about equivalent fractions.

For example,

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Similarly, we may use the idea in order to simplify a fraction by finding common factors in the numerator and the denominator.

For example,

$$\frac{14}{18} = \frac{7 \times 2}{9 \times 2} = \frac{7}{9}$$

Example 4. Find the simplest fractional name for $\frac{15}{20}$.

What do we mean to find the simplest fractional name? A fraction is said to have the simplest fractional name or be reduced to lowest terms if the numerator and the denominator have no common factors except for 1 (or -1). (Note that every number has a factor of 1 or -1 since, for example, $4 = 4 \times 1$ and $4 = -4 \times -1$.) For example 4/6 is not in lowest terms since $\frac{4}{6} = \frac{2 \times 2}{3 \times 2}$. The numerator 4 and the denominator have a common factor which is 2.

$$\frac{4}{6} = \frac{2 \times 2}{2 \times 3} = \frac{2}{3}$$

Now, 2/3 is in lowest terms.

In Example 4,

$$\frac{15}{20} = \frac{3 \times 5}{5 \times 5} = \frac{3}{5}$$ (3/5 is the simplest fractional name)

Example 5. Find the simplest fractional name for $\frac{12}{18}$.

$$\frac{12}{18} = \frac{3 \times 2 \times 2}{3 \times 3 \times 2} = \frac{2}{3}$$ (Note that the common factors are 2 and 3.)

Example 6. Find the simplest fractional name for $\frac{7}{21}$.

$$\frac{7}{21} = \frac{1 \times 7}{3 \times 7} = \frac{1}{3}$$

Example 7. Reduce $-\frac{10}{15}$ to lowest terms.

$$-\frac{10}{15} = -\frac{2 \times 5}{3 \times 5} = -\frac{2}{3}$$

Example 8. Reduce $\frac{18}{30}$ to lowest terms.

$$\frac{18}{30} = \frac{3 \times 3 \times 2}{5 \times 3 \times 2} = \frac{3}{5}$$

Or, if we can see that 6 is a common factor we could solve as follows:

$$\frac{18}{30} = \frac{3 \times 6}{5 \times 6} = \frac{3}{5}$$

EXERCISES

1. Are $\frac{2}{3}$ and $\frac{4}{6}$ equivalent fractions?

2. Is $\frac{6}{9} = \frac{8}{12}$?

3. Is $-\frac{3}{2} = -\frac{15}{10}$?

4. Is $\frac{3}{4} = \frac{8}{12}$?

In problems 5 through 12 find the simplest fractional name (or in other words, reduce to lowest terms).

5. $\frac{15}{25}$

6. $\frac{6}{10}$

7. $\frac{18}{27}$

8. $\frac{16}{24}$

9. $\frac{125}{150}$

10. $-\frac{10}{14}$

11. $\frac{33}{55}$

12. $\frac{32}{50}$

ANSWERS

1. Yes.
2. Yes.
3. Yes
4. No.
5. $\frac{3}{5}$
6. $\frac{3}{5}$
7. $\frac{2}{3}$
8. $\frac{2}{3}$
9. $\frac{5}{6}$
10. $-\frac{5}{7}$
11. $\frac{3}{5}$
12. $\frac{16}{25}$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition and Subtraction of Rational Numbers

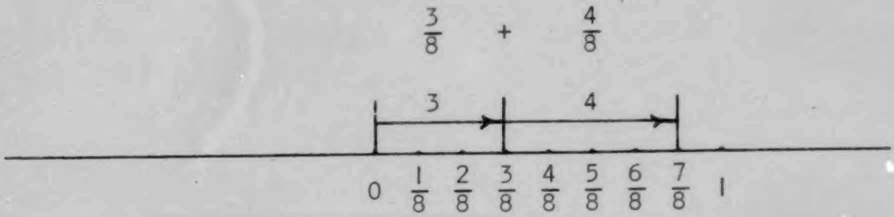
INTRODUCTION AND/OR OBJECTIVES:

In determining certain measurements it is frequently necessary to add or subtract numbers involving fractions. This section will deal with the addition and subtraction of rational numbers. The number line is used to provide a better understanding of the procedures.

TECHNICAL INFORMATION:

I. ADDITION OF RATIONAL NUMBERS

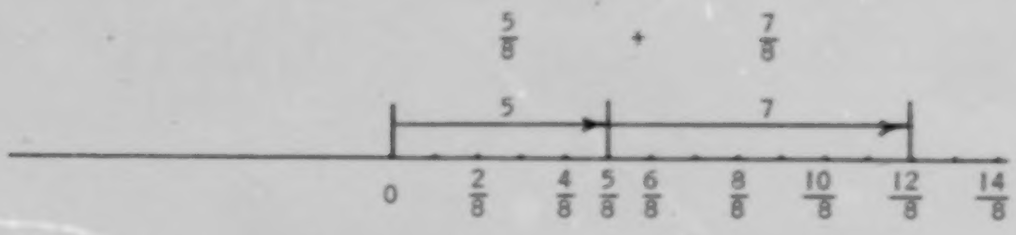
In Figure I each unit is divided into 8 equal parts just as each inch on a rule could be divided into 8 equal parts. If we wish, for example, to add 3/8 to 4/8, we can first move to the right from the origin 3 of the 8 equal parts. Then, we move to the right 4 more of the equal parts. Thus, we have totally moved 7 of the 8 equal parts. Therefore, 3/8 + 4/8 = 7/8.



$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Figure I

In Figure 2 we have added 5/8 to 7/8 to get the total of 12/8.


$\frac{5}{8}$
+
$\frac{7}{8}$
5
7
0
$\frac{2}{8}$
$\frac{4}{8}$
$\frac{5}{8}$
$\frac{6}{8}$
$\frac{8}{8}$
$\frac{10}{8}$
$\frac{12}{8}$
$\frac{14}{8}$

$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8}$$

Figure 2

These illustrations suggest a method for addition of rational numbers.

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

Example 1. Find: $\frac{5}{7} + \frac{3}{7}$

$$\frac{5}{7} + \frac{3}{7} = \frac{5 + 3}{7} = \frac{8}{7}$$

Example 2. Find: $\frac{10}{21} + \frac{7}{21}$

$$\frac{10}{21} + \frac{7}{21} = \frac{10 + 7}{21} = \frac{17}{21}$$

Example 3. Find: $\frac{3}{4} + \frac{11}{8}$

$\frac{3}{4} + \frac{11}{8} = \frac{6}{8} + \frac{11}{8} = \frac{6 + 11}{8} = \frac{17}{8}$ (Note that here we have changed $\frac{3}{4}$ to the equivalent form $\frac{6}{8}$)

This example involved the addition of fractions with different denominators. In order to add (or subtract) fractions with different denominators, we must find what is called a common denominator. It is usually best to find what is called the least common denominator. Then we find an equivalent fraction utilizing the least common denominator for each of the fractions in the original problem.

For example, suppose that we wish to find: $\frac{3}{8} + \frac{5}{18}$

To find the least common denominator (LCD) we first factor each of the denominators, 8 and 18, into prime factors. (A number is called a prime factor if it cannot be factored further except using 1 (or -1).) The numbers 2 3 5, and 7, for example, are prime.

$$8 = 2 \times 2 \times 2$$

$$18 = 2 \times 3 \times 3$$

Now, we use the product of all the different factors involved in the 2 denominators, 8 and 18. We repeat a factor if it is repeated in either the 8 or the 18. Each factor is entered the largest number of times that it appears in either 8 or 18.

Therefore:

$$\text{LCD (for 8 and 18)} = 2 \times 2 \times 2 \times 3 \times 3 = 72$$

(Note that we enter 3 twice since it appears in 18 twice. We enter 2 three times since it appears in 8 three times.)

Now, we find equivalent fractions so that the denominator of each is 72.

$$\frac{3}{8} = \frac{3 \times ?}{72} = \frac{3 \times 9}{8 \times 9} = \frac{27}{72}$$

$$\frac{5}{18} = \frac{5 \times ?}{72} = \frac{5 \times 4}{18 \times 4} = \frac{20}{72}$$

Therefore:

$$\frac{3}{8} + \frac{5}{18} = \frac{27}{72} + \frac{20}{72} = \frac{27 + 20}{72} = \frac{47}{72}$$

The student should note that the least common denominator is a multiple of each denominator. In this previous illustration: 8 x 9 = 72 and 18 x 4 = 72. In some problems it is easy to identify the least common denominator without going through the procedure of factoring each denominator. For example to find the value of $\frac{2}{3} + \frac{1}{2}$, we would use 6 as the least common denominator. 6 is the smallest number that is a multiple of 3 and also a multiple of 2.

$$\frac{2}{3} = \frac{2 \times ?}{6} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{1 \times ?}{6} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Therefore:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{4 + 3}{6} = \frac{7}{6}$$

Example 4. Find: $\frac{3}{14} + \frac{1}{6}$

$$14 = 2 \times 7$$

$$6 = 2 \times 3$$

$$\text{LCD} = 2 \times 3 \times 7 = 42$$

$$\frac{3}{14} = \frac{3 \times ?}{42} = \frac{3 \times 3}{14 \times 3} = \frac{9}{42}$$

$$\frac{1}{6} = \frac{1 \times ?}{42} = \frac{1 \times 7}{6 \times 7} = \frac{7}{42}$$

Therefore:

$$\frac{3}{14} + \frac{1}{6} = \frac{9}{42} + \frac{7}{42} = \frac{9 + 7}{42} = \frac{16}{42}$$

Example 5. Find: $\frac{1}{4} + \frac{3}{16}$

Here, it should be seen that we can use 16 as the LCD.

$$\frac{1}{4} = \frac{1 \times ?}{16} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$$

Therefore:

$$\frac{1}{4} + \frac{3}{16} = \frac{4}{16} + \frac{3}{16} = \frac{4 + 3}{16} = \frac{7}{16}$$

Example 6. Find: $\frac{1}{4} + \frac{1}{10} + \frac{3}{8}$

$4 = 2 \times 2$

$10 = 2 \times 5$

$8 = 2 \times 2 \times 2$

LCD $= 2 \times 2 \times 2 \times 5 = 40$

$$\frac{1}{4} = \frac{1 \times ?}{40} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}$$

$$\frac{1}{10} = \frac{1 \times ?}{40} = \frac{1 \times 4}{10 \times 4} = \frac{4}{40}$$

$$\frac{3}{8} = \frac{3 \times ?}{40} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

Therefore:

$$\frac{1}{4} + \frac{1}{10} + \frac{3}{8} = \frac{10}{40} + \frac{4}{40} + \frac{15}{40} = \frac{10 + 4 + 15}{40} = \frac{29}{40}$$

Example 7. $\frac{3}{8} + \frac{3}{5} + \frac{1}{6}$

$8 = 2 \times 2 \times 2$

$5 = 5$

$6 = 2 \times 3$

LCD $= 2 \times 2 \times 2 \times 3 \times 5 = 120$

$$\frac{3}{8} = \frac{3 \times ?}{120} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$$

$$\frac{3}{5} \quad \frac{3 \times ?}{120} \quad \frac{3 \times 24}{5 \times 24} \quad \frac{72}{120}$$

$$\frac{1}{6} = \frac{1 \times ?}{120} = \frac{1 \times 20}{6 \times 20} = \frac{20}{120}$$

5

$$\frac{3}{8} + \frac{3}{5} + \frac{1}{6} = \frac{45}{120} + \frac{72}{120} + \frac{20}{120} = \frac{45 + 72 + 20}{120} = \frac{137}{120}$$

We could perform these additions in column form. For example, we could have performed Example 7 as follows:

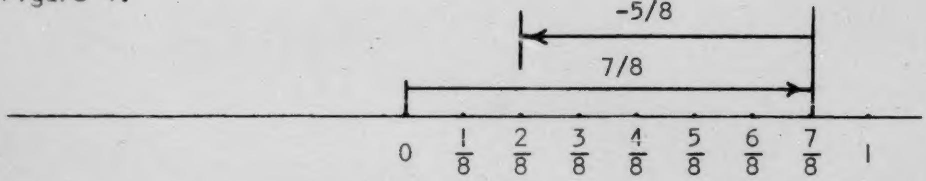
$$\begin{array}{ccc} \frac{3}{8} & \frac{3 \times 15}{8 \times 15} & \frac{45}{120} \\[1em] \frac{3}{5} & \frac{3 \times 24}{5 \times 24} & \frac{72}{120} \\[1em] \frac{1}{6} & \frac{1 \times 20}{6 \times 20} & \frac{20}{120} \\ \hline & & \frac{137}{120} \end{array}$$

(Note that we multiply the numerators and denominators in column 2 by numbers so that the denominators will all become 120, the LCD)

11. SUBTRACTION OF RATIONAL NUMBERS

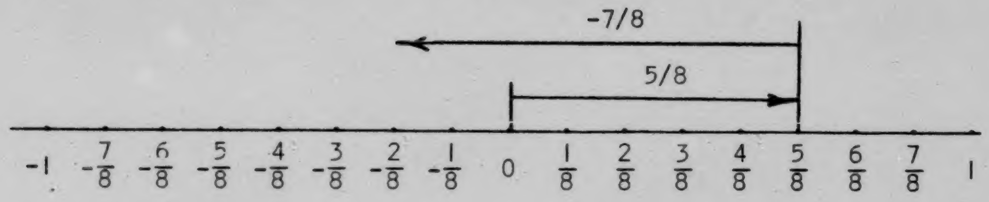
The main difference between addition and subtraction is that of direction on the number line. To find 7/8 + (-5/8) we first move to the right 7 of 8 equal parts of the unit. See Figure 3. Then, we move back to the left 5 of 8 equal parts. The result is 2/8. Therefore, 7/8 + (-5/8) = 7/8 - 5/8 = 2/8.

Similarly, to find 5/8 - 7/8, first move 5 to the right and then 7 to the left. The result is -2/8. Therefore, 5/8 + (-7/8) = 5/8 - 7/8 = -2/8. See Figure 4.



$$\frac{7}{8} - \frac{5}{8} = \frac{2}{8}$$

Figure 3



$$\frac{5}{8} - \frac{7}{8} = -\frac{2}{8}$$

Figure 4

The above examples suggest the following results:

$$\frac{a}{b} - \frac{c}{b} = \frac{a + (-c)}{b} \qquad \text{or} \qquad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

Example 8. Find: $\frac{2}{3} - \frac{4}{3}$

$$\frac{2}{3} - \frac{4}{3} = \frac{2 + (-4)}{3} = \frac{-2}{3} = -\frac{2}{3}$$

Example 9. Find: $\frac{15}{8} - \frac{5}{8}$

$$\frac{15}{8} - \frac{5}{8} = \frac{15 - 5}{8} = \frac{10}{8} = \frac{5 \times 2}{4 \times 2} = \frac{5}{4}$$

Example 10. Find: $\frac{5}{8} - \frac{1}{4}$

$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{5 - 2}{8} = \frac{3}{8}$ (Note that here the LCD = 8, and $\frac{1}{4} = \frac{2}{8}$)

Example 11. Find: $\frac{9}{10} - \frac{3}{4}$

$10 = 2 \times 5$

$4 = 2 \times 2$

$\text{LCD} = 2 \times 2 \times 5$

$$\frac{9}{10} = \frac{9 \times ?}{20} = \frac{9 \times 2}{10 \times 2} = \frac{18}{20}$$

$$\frac{3}{4} = \frac{3 \times ?}{20} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Therefore:

$$\frac{9}{10} - \frac{3}{4} = \frac{18}{20} - \frac{15}{20} = \frac{18 - 15}{20} = \frac{3}{20}$$

Or, in column form:

$$\begin{array}{rcrcr}
\frac{9}{10} & \quad & \frac{9 \times 2}{10 \times 2} & \quad & \frac{18}{20} \\[1em]
-\ \frac{3}{4} & & -\ \frac{3 \times 5}{4 \times 5} & & -\ \frac{15}{20} \\[0.5em]
\hline
 & & & & \frac{3}{20}
\end{array}$$

III. MIXED NUMERALS, PROPER FRACTIONS, AND IMPROPER FRACTIONS

A number which consists of a whole number and a fraction is called a mixed numeral. For example, $1\frac{1}{2}$, $3\frac{2}{3}$, and $5\frac{7}{8}$ are mixed numerals.

A positive fraction in which the numerator is less than the denominator is called a proper fraction. For example, $\frac{2}{3}$ is a proper fraction since the numerator, 2, is less than the denominator, 3.

An improper fraction is a fraction in which the numerator is larger than or equal to the denominator. For example $\frac{7}{4}$ is an improper fraction since the numerator, 7, is larger than the denominator, 4. Likewise, $\frac{6}{6}$ is an improper fraction since the numerator, 6, is equal to the denominator, 6.

We can replace an improper fraction by an equivalent mixed numeral as illustrated below.

Example 12. Change $\frac{11}{8}$ to a mixed numeral.

$$\frac{11}{8} = \frac{8+3}{8} = \frac{8}{8} + \frac{3}{8} = 1 + \frac{3}{8} = 1\frac{3}{8}$$

Note that $\frac{8}{8} = 1$, or in general:

$$\frac{a}{a} = 1 \qquad \text{if } a \neq 0$$

Example 13. Change $\frac{9}{4}$ to a mixed numeral.

$$\frac{9}{4} = \frac{4+4+1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 1 + 1 + \frac{1}{4} = 2\frac{1}{4}$$

Or $$\frac{9}{4} = \frac{8+1}{4} = \frac{8}{4} + \frac{1}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$$

Example 14. Convert $1\frac{7}{8}$ to an improper fraction.

$$1\frac{7}{8} = 1 + \frac{7}{8} = \frac{8}{8} + \frac{7}{8} = \frac{15}{8}$$

Example 15. Convert $5\frac{3}{4}$ to an improper fraction.

$$5\frac{3}{4} - 5 + \frac{3}{4} = \frac{5}{1} + \frac{3}{4} = \frac{5 \times 4}{1 \times 4} + \frac{3}{4} = \frac{20}{4} + \frac{3}{4} = \frac{23}{4}$$

EXERCISES

1. $\frac{3}{7} + \frac{2}{7} =$

2. $\frac{7}{16} + \frac{13}{16} =$

3. $\frac{3}{8} + \frac{5}{16} =$

4. $\frac{3}{4} + \frac{3}{16} =$

5. $\frac{1}{2} + \frac{7}{16} =$

6. $\frac{7}{8} - \frac{3}{8} =$

7. $\frac{5}{16} - \frac{10}{16} =$

8. $\frac{11}{16} - \frac{3}{8} =$

9. $\frac{7}{10} - \frac{2}{5} =$

10. $\frac{5}{6} + \frac{3}{8} =$

11. $\frac{3}{8} + \frac{1}{16} + \frac{1}{4} =$

12. $\frac{1}{10} + \frac{1}{5} + \frac{1}{4} =$

13. $\frac{1}{3} + \frac{2}{5} + \frac{3}{10} =$

14. Convert $\frac{13}{9}$ to a mixed numeral.

15. Convert $\frac{17}{7}$ to a mixed numeral.

16. Convert $1\frac{3}{16}$ to an improper fraction.

17. Convert $2\frac{5}{8}$ to an improper fraction.

18. Convert $4\frac{1}{4}$ to an improper fraction.

ANSWERS

1. $\frac{5}{7}$
2. $\frac{20}{16}$ or $\frac{5}{4}$ or $1\frac{1}{4}$
3. $\frac{11}{16}$
4. $\frac{15}{16}$
5. $\frac{15}{16}$
6. $\frac{4}{8}$ or $\frac{1}{2}$
7. $\frac{5}{16}$
8. $\frac{5}{16}$
9. $\frac{3}{10}$
10. $\frac{29}{24}$
11. $\frac{11}{16}$
12. $\frac{11}{20}$
13. $\frac{31}{30}$
14. $1\frac{4}{9}$
15. $2\frac{3}{7}$
16. $\frac{19}{16}$
17. $\frac{21}{16}$
18. $\frac{17}{4}$

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TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Multiplication of Rational Numbers

INTRODUCTION AND/OR OBJECTIVES:
Just as the addition of rational numbers is important for application problems, so also is the multiplication of rational numbers and the multiplication of whole numbers and rational numbers.

TECHNICAL INFORMATION:

To find the product of two rational numbers the following property is used:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \quad \text{or} \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Note that the · indicates multiplication just as the x sign.

Example 1. Find: $\frac{3}{4} \times \frac{5}{7}$

$$\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

Example 2. Find: $\frac{7}{5} \times \frac{5}{6}$

$$\frac{7}{5} \times \frac{5}{6} = \frac{7 \times 5}{5 \times 6} = \frac{7}{6}$$ (Note that there is a common factor of 5 in the numerator and the denominator)

Example 3. Find: $-\frac{6}{5} \times \frac{4}{5}$

$$-\frac{6}{5} \times \frac{4}{5} = -\frac{6 \times 4}{5 \times 5} = -\frac{24}{25}$$

Example 4. Find: $\frac{4}{5} \times 7$

$$\frac{4}{5} \times 7 = \frac{4}{5} \times \frac{7}{1} = \frac{4 \times 7}{5 \times 1} = \frac{28}{5}$$

Notice that in Example 4:

$$\frac{4}{5} \times 7 = \frac{4 \times 7}{5}$$

This suggests the following property:

$$\frac{a}{b} \times c = \frac{a \times c}{b} \quad \text{or} \quad \frac{a}{b} \cdot c = \frac{a \cdot c}{b}$$

Also, very similarly:

$$c \times \frac{a}{b} = \frac{c \times a}{b} \quad \text{or} \quad c \cdot \frac{a}{b} = \frac{c \cdot a}{b}$$

Example 5. Find: $\frac{2}{3} \times 8$

$$\frac{2}{3} \times 8 = \frac{2 \times 8}{3} = \frac{16}{3} = \frac{15 + 1}{3} = \frac{15}{3} + \frac{1}{3} = 5 + \frac{1}{3} = 5\frac{1}{3}$$

Example 6. Find: $5 \times \frac{3}{8}$

$$5 \times \frac{3}{8} = \frac{5 \times 3}{8} = \frac{15}{8} = \frac{8 + 7}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8} = 1\frac{7}{8}$$

EXERCISES

1. $\frac{2}{3} \times \frac{4}{7} =$
2. $\frac{1}{3} \times \frac{5}{8} =$
3. $\frac{3}{7} \times \frac{5}{6} =$
4. $\frac{11}{3} \times \frac{2}{7} =$
5. $3 \times \frac{2}{5} =$
6. $\frac{5}{6} \times 4 =$
7. $5 \times \frac{1}{8} =$
8. $\frac{3}{32} \times 8 =$

ANSWERS

1. $\frac{8}{21}$
2. $\frac{5}{24}$
3. $\frac{5}{14}$
4. $\frac{22}{21}$ or $1\frac{1}{21}$
5. $\frac{6}{5}$ or $1\frac{1}{5}$
6. $\frac{10}{3}$ or $3\frac{1}{3}$
7. $\frac{5}{8}$
8. $\frac{3}{4}$

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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Solution of Equations

INTRODUCTION AND/OR OBJECTIVES:

One of the basic operations in algebra is that of solving for an unknown quantity x (or whatever it may be) in an equation. This is an extremely important and necessary section as a lead up to the solution of unknowns in application formulas.

TECHNICAL INFORMATION:

I. ADDITIVE PROPERTY OF EQUALITY

We can easily see that since $8 = 4 \times 2$, then:

$$8 + 3 = 4 \times 2 + 3$$

$$11 = 11$$

Also, since $16 = 8 \times 2$, then:

$$16 + (-4) = 8 \times 2 + (-4)$$

$$16 - 4 = 16 - 4$$

$$12 = 12$$

In other words we may add (or subtract) the same quantity to both sides of an equation.

We will refer to the property that we may add the same quantity (either a positive or negative number) to both sides of an equation as the Additive Property of Equality (abbreviated as a. p. e.).

II. MULTIPLICATIVE PROPERTY OF EQUALITY

Since $8 = 5 + 3$, then:

$$6 \cdot 8 = 6(5 + 3)$$

$$48 = 6 \cdot 8$$

$$48 = 48$$

Also, since $8 = 5 + 3$, then:

$$\frac{1}{2} \cdot 8 = \frac{1}{2} \cdot (5 + 3)$$

$$\frac{8}{2} = \frac{5 + 3}{2}$$

$$4 = 4$$

In other words we may multiply both sides of an equation by the same number.

> We will refer to the property that we may multiply both sides of an equation by the same quantity as the Multiplicative Property of Equality (abbreviated as m. p. e.).

Notice that in the second illustration above, multiplying both sides of an equation by 1/2 is the same as dividing both sides of the equation by 2. Therefore, we can divide both sides of an equation by the same nonzero number.

III. SOLUTION OF EQUATIONS

Example 1. Find x in the following equation: $x + 3 = 8$

$x + 3 = 8$

$x + 3 - 3 = 8 - 3$ (Subtract 3 from both sides of the equation)

$x + 0 = 5$ (-3 is the additive inverse of 3)

$x = 5$ (0 is the additive identity)

Example 2. Find x in the following equation: $x - 5 = 7$

$x - 5 = 7$

$x - 5 + 5 = 7 + 5$ (Add 5 to both sides of the equation)

$x = 12$

Example 3. Find x in the following equation: $2x = 16$

$2x = 16$

$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 16$ (Multiply both sides by 1/2)

$1 \cdot x = \frac{16}{2}$ (1/2 is the multiplicative inverse of 2, $\frac{a}{b} \cdot c = \frac{a \cdot c}{b}$)

$x = 8$ (1 is the multiplicative identity)

Example 4. Find x in the following equation: $\frac{x + 3}{4} = 2$

$\frac{x + 3}{4} = 2$

$\frac{x + 3}{4} \cdot 4 = 2 \cdot 4$ (Multiply both sides by 4)

$x + 3 = 8$ (Multiplicative inverses (1/4 and 4))

$x + 3 - 3 = 8 - 3$ (Subtract 3 from both sides)

$x = 5$ (-3 is the additive inverse of 3)

Example 5. Find x in the following equation: $\frac{x - 2}{8} = \frac{3}{4}$

$\frac{x - 2}{8} = \frac{3}{4}$

$\frac{x - 2}{8} \cdot 8 = \frac{3}{4} \cdot 8$ (Multiply both sides by 8)

$x - 2 = \frac{3 \cdot 8}{4}$ (Multiplicative inverse, $\frac{a}{b} \cdot c = \frac{a \cdot c}{b}$)

$x - 2 = \frac{3 \cdot 2 \cdot 4}{4}$

$x - 2 = 3 \cdot 2$

$x - 2 = 6$

$x - 2 + 2 = 6 + 2$ (Add 2 to both sides)

$x = 8$ (2 is the additive inverse of -2)

Example 6. Find x in the following equation: $2x - 4 = x + 8$

$2x - 4 = x + 8$

$2x - 4 + 4 = x + 8 + 4$ (Add 4 to both sides)

$2x = x + 12$ (4 is the additive inverse of -4)

$2x - x = x + 12 - x$ (Subtract x from both sides)

$x = 12$ ($-x$ is the additive inverse of x)

EXERCISES

1. Solve for x: $x + 2 = 6$

2. Solve for x: $x - 5 = 7$

3. Solve for x: $x - 2 = 10$

4. Solve for x: $2x + 3 = 7$

5. Solve for x: $3x - 2 = 10$

6. Solve for x: $5x \div 7 = 32$

7. Solve for x: $\frac{x + 1}{3} = 4$

8. Solve for x: $2x - 5 = x + 3$

9. Solve for x: $\frac{x - 2}{6} = \frac{2}{3}$

10. Solve for x: $\frac{2x - 1}{4} = \frac{1}{3}$

11. Solve for x: $\frac{x + 1}{3} = \frac{2}{5}$

ANSWERS

1. $x = 4$
2. $x = 12$
3. $x = 12$
4. $x = 2$
5. $x = 4$
6. $x = 5$
7. $x = 11$
8. $x = 8$
9. $x = 6$
10. $x = \frac{7}{6}$
11. $x = \frac{1}{5}$

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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Division of Rational Numbers

INTRODUCTION AND/OR OBJECTIVES:

Just as well as it is frequently necessary to add, subtract, or multiply rational numbers in solving for dimensions or solving application problem formulas, it is frequently necessary to divide rational numbers. As will be seen, division of rational numbers can be accomplished in terms of multiplication of rational numbers.

TECHNICAL INFORMATION:

Remember that with whole numbers, $\frac{16}{8} = 2$ because $8 \times 2 = 16$.

In the division of rational numbers, $\frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2}$ because $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$.

Therefore:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{x}{y} \quad \text{means that} \quad \frac{c}{d} \times \frac{x}{y} = \frac{a}{b}$$

Or

$$\frac{a}{b} \div \frac{c}{d} = \frac{x}{y} \quad \text{means that} \quad \frac{c}{d} \times \frac{x}{y} = \frac{a}{b}$$

Example 1. Find $\frac{2}{3} \div \frac{4}{5}$.

$$\frac{2}{3} \div \frac{4}{5} = \frac{x}{y}$$

$$\frac{4}{5} \times \frac{x}{y} = \frac{2}{3}$$

$$\frac{5}{4} \times \frac{4}{5} \times \frac{x}{y} = \frac{5}{4} \times \frac{2}{3} \qquad \text{(Multiply both sides by } \tfrac{5}{4}\text{)}$$

$$1 \times \frac{x}{y} = \frac{5}{4} \times \frac{2}{3} \qquad (\tfrac{5}{4} \text{ is the multiplicative inverse of } \tfrac{4}{5})$$

$$\frac{x}{y} = \frac{2}{3} \times \frac{5}{4} \qquad \text{(1 is the multiplicative identity)}$$

Before finishing, note that the problem was to find: $\frac{x}{y} = \frac{2}{3} \div \frac{4}{5}$. In the last step above we found that: $\frac{x}{y} = \frac{2}{3} \times \frac{5}{4}$. This example suggests the fact that dividing by a rational number other than zero may be accomplished by multiplying by the multiplicative inverse of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \quad (\text{if } c \neq 0)$$

Therefore, in the above example:

$$\begin{aligned} \frac{2}{3} \div \frac{4}{5} &= \frac{2}{3} \times \frac{5}{4} \\ &= \frac{2 \times 5}{3 \times 4} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$

Example 2. Find $\frac{7}{8} \div \frac{5}{16}$.

$$\frac{7}{8} \div \frac{5}{16} = \frac{7}{8} \times \frac{16}{5}$$

$$= \frac{7 \times 16}{8 \times 5}$$

$$= \frac{7 \times 8 \times 2}{8 \times 5}$$

$$= \frac{7 \times 2}{5}$$

$$= \frac{14}{5}$$

Example 3. Find $5 \div \frac{3}{8}$.

$$5 \div \frac{3}{8} = 5 \times \frac{8}{3}$$

$$= \frac{5}{1} \times \frac{8}{3}$$

$$= \frac{5 \times 8}{1 \times 3}$$

$$= \frac{40}{3}$$

Example 4. Find $\frac{3}{4} \div 8$.

$$\frac{3}{4} \div 8 = \frac{3}{4} \div \frac{8}{1}$$

$$= \frac{3}{4} \times \frac{1}{8}$$

$$= \frac{3 \times 1}{4 \times 8}$$

$$= \frac{3}{32}$$

EXERCISES

1. Find $\frac{4}{5} \div \frac{2}{3}$

2. Find $\frac{6}{7} \div \frac{3}{4}$

3. Find $\frac{3}{4} \div \frac{5}{8}$

4. Find $\frac{5}{2} \div \frac{3}{4}$

5. Find $\frac{2}{3} \div 4$

6. Find $\frac{3}{16} \div 6$

7. Find $4 \div \frac{3}{8}$

8. Find $8 \div \frac{1}{2}$

ANSWERS

1. $\frac{6}{5}$ or $1\frac{1}{5}$
2. $\frac{8}{7}$ or $1\frac{1}{7}$
3. $\frac{6}{5}$ or $1\frac{1}{5}$
4. $\frac{10}{3}$ or $3\frac{1}{3}$
5. $\frac{1}{6}$
6. $\frac{1}{32}$
7. $\frac{32}{3}$ or $10\frac{2}{3}$
8. 16

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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Addition and Subtraction of Decimal Numbers

INTRODUCTION AND/OR OBJECTIVES:

In working with measurements it is mandatory that the student be able to work with decimal numbers. The student should understand the meaning of the placement of numbers in a decimal. In determining a various dimension it is frequently necessary to add or subtract various decimal numbers.

TECHNICAL INFORMATION:

In a decimal number, the first digit to the right of the decimal point indicates tenths, the next digit indicated hundredths, the next indicates thousandths, the next indicates ten thousandths, the next indicates hundred thousandths, and so forth. See Figure I below.

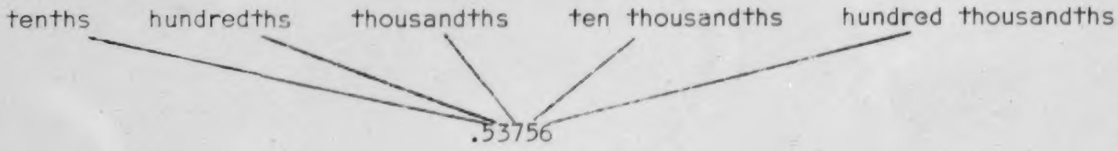

Figure I

Therefore:

$$.53756 = \frac{5}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{5}{10,000} + \frac{6}{100,000}$$

Consider the number .37

$$.37 = \frac{3}{10} + \frac{7}{100}$$

Now:

$$\frac{3}{10} = \frac{3}{10} \times \frac{10}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$$

Therefore:

$$.37 = \frac{30}{100} + \frac{7}{100}$$

$$= \frac{30 + 7}{100}$$

$$= \frac{37}{100}$$

Thus, we read .37 as "37 hundredths" (thirty seven hundredths).

Similarly:

$$.432 = \frac{4}{10} + \frac{3}{100} + \frac{2}{1000}$$

Now, we need to use the common denominator 1000.

$$\frac{4}{10} = \frac{4 \times 100}{10 \times 100} = \frac{400}{1000}$$

$$\frac{3}{100} = \frac{3 \times 10}{100 \times 10} = \frac{30}{1000}$$

Then:

$$.432 = \frac{400}{1000} + \frac{30}{1000} + \frac{2}{1000}$$

$$= \frac{400 + 30 + 2}{1000}$$

$$= \frac{432}{1000}$$

Therefore, .432 is read as "432 thousandths" (four hundred thirty two thousandths).

Example 1. Read .275

.275 is read as "275 thousandths"

Example 2. Read .63

.63 is read as "63 hundredths"

Example 3. Read .0625

.0625 is read as "625 ten thousandths"

Example 4. Find the value of .325 + .432

$$\begin{array}{r} .325 \\ +\ \underline{.432} \\ .757 \end{array}$$

Step 1. Add the 5 thousandths to the 2 thousandths to get 7 thousandths.

Step 2. Add the 2 hundredths to the 3 hundredths to get 5 hundredths.

Step 3. Add the 3 tenths to the 4 tenths to get 7 tenths.

The answer is .757 (read as "757 thousandths")

Example 5. Find the value of .47 + .36

$$\begin{array}{r} .4\ 7 \\ \underline{.3\ 6} \end{array}$$

Note that when we add the numbers in the hundredths column we get 13.

$$\begin{aligned} \frac{13}{100} &= \frac{10+3}{100} \\ &= \frac{10}{100} + \frac{3}{100} \\ &= \frac{1 \times 10}{10 \times 10} + \frac{3}{100} \\ &= \frac{1}{10} + \frac{3}{100} \end{aligned}$$

Therefore, we have a result of 3 hundredths and 1 tenth. We, then, enter 3 in the hundredths column in the answer and add 1 to the tenths column.

```
 /
.4 7

.3 6
----
.8 3
```

Now, add the numbers in the tenths column to get a total of 8 tenths.

Example 6. Find the value of .234 + .341 + .256

```
 / /
.2 3 4

.3 4 1

.2 5 6
------
.8 3 1
```

Step 1. Add the numbers in the thousandths column. The total is 11. Enter 1 in the thousandths column of the answer and add 1 to the hundredths column.

Step 2. Add the numbers in the hundredths column. The total is 13. Enter 3 in the hundredths column of the answer and add 1 to the tenths column.

Step 3. Add the numbers in the tenths column. Enter the total 8 in the tenths column of the answer.

Example 7. Find the value of .124 + .311 + .245 + .422

```
  / /
 .1 2 4

 .3 1 1

 .2 4 5

 .4 2 2
-------
1.1 0 2
```

Example 8. Find the value of .457 - .232

```
  .4 5 7

- .2 3 2
  ------
  .2 2 5
```

Step 1. Subtract in the thousandths column.
Step 2. Subtract in the hundredths column.
Step 3. Subtract in the tenths column.

Example 9. Find the value of .65 - .17

$$\begin{array}{r} .6\ 5 \\ -\ \underline{.1\ 7} \end{array}$$

Step 1. We cannot subtract 7 hundredths from 5 hundredths. We, therefore, change .65 to .5 + .15 . We have, thus, taken 1 from the tenths column (from 6). This 1 tenth is equal to 10 hundredths. The 10 hundredths plus the 5 hundredths is 15 hundredths. We now subtract the 7 hundredths from the 15 hundredths. We enter the result 8 in the hundredths column of the answer.

$$\begin{array}{r} .5\ {}^{1}5 \\ -\ \underline{.1\ 7} \\ .4\ 8 \end{array}$$

Step 2. Subtract 1 from 5 in the tenths column. Enter the result 4 in the tenths column of the answer.

Example 10. Find the value of .432 - .216

$$\begin{array}{r} .4\ 3\ 2 \\ -\ \underline{.2\ 1\ 6} \end{array} \qquad \begin{array}{r} .4\ 2\ {}^{1}2 \\ -\ \underline{.2\ 1\ 6} \\ .2\ 1\ 6 \end{array}$$

Example 11. Find the value of .522 - .237

$$\begin{array}{r} .5\ 2\ 2 \\ -\ \underline{.2\ 3\ 7} \end{array} \qquad \begin{array}{r} .5\ 1\ {}^{1}2 \\ -\ \underline{.2\ 3\ 7} \\ 5 \end{array} \qquad \begin{array}{r} .4\ {}^{1}1\ 2 \\ -\ \underline{.2\ 3\ 7} \\ .2\ 8\ 5 \end{array}$$

Example 12. Find the total length L in Figure 2.

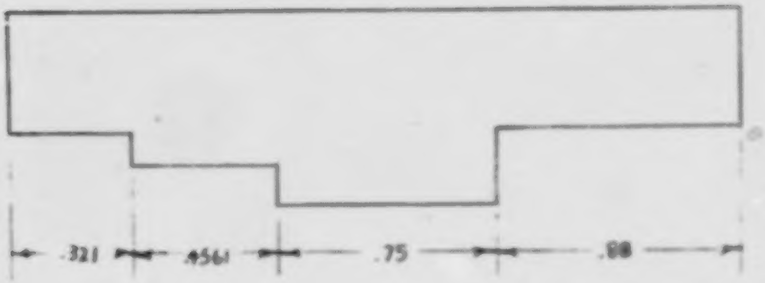

Figure 2

$L = .321 + .4561 + .75 + .88$

$$\begin{array}{r} {}^{2} \\ .3\,2\,1 \\ .4\,5\,6\,1 \\ .7\,5 \\ \underline{.8\,8} \\ 2.4\,0\,7\,1 \end{array}$$

$l = 2.4071$

or 2.41 (correct to two decimal places)

EXERCISES

In problems 1 to 4 read the decimal value.

1. .23
2. .152
3. .275
4. .3752

In problems 5 to 16 find the value.

5. .23 + .35
6. .34 + .47
7. .234 + .312
8. .337 + .435
9. .312 + .214 + .456
10. .124 + .326 + .125 + .427
11. .32 - .11
12. .72 - .37
13. .342 - .125
14. .455 - .278
15. .567 - .289
16. .682 - .391

ANSWERS

1. 23 hundredths (twenty three hundredths)
2. 152 thousandths (one hundred fifty two thousandths)
3. 275 thousandths (two hundred seventy five thousandths)
4. 3,752 ten thousandths (three thousand seven hundred fifty two ten thousandths)
5. .58
6. .81
7. .546
8. .772
9. .982
10. 1.002
11. .21
12. .35
13. .217
14. .177
15. .278
16. .291

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Multiplication and Division of Decimal Numbers

INTRODUCTION AND/OR OBJECTIVES:

Just as it is important to be able to add and subtract decimal numbers, it is necessary to be able to multiply and divide decimal numbers. In dealing with approximate numbers obtained from measurements, we should consider the number of significant digits.

TECHNICAL INFORMATION:

Suppose we wish to find .3 × .7 .

$$.3 = \frac{3}{10}$$

$$.7 = \frac{7}{10}$$

Therefore:

$$.3 \times .7 = \frac{3}{10} \times \frac{7}{10}$$

$$= \frac{3 \times 7}{100}$$

$$= \frac{21}{100}$$

$$= .21$$

Note that there is one decimal place in .3, one decimal place in .7, and the answer .21 has two decimal places.

Now, let us find the value of .4 × .43 .

$$.4 = \frac{4}{10}$$

$$.43 = \frac{43}{100}$$

Then:

$$\begin{aligned} .4 \times .43 &= \frac{4}{10} \times \frac{43}{100} \\ &= \frac{4 \times 43}{10 \times 100} \\ &= \frac{172}{1000} \\ &= .172 \end{aligned}$$

Note that there is one decimal place in .4, two decimal places in .43, and three decimal places in the answer .172 .

These two examples suggest that if two numbers are multiplied, the number of decimal places in the answer is the sum of the numbers of decimal places in the two numbers multiplied.

Example 1. Find the value of .35 × .42

$$\begin{array}{r} .35 \\ \underline{.42} \\ 70 \\ \underline{140} \\ .1470 \end{array}$$

Since .35 has 2 decimal places and .42 has 2 decimal places, then the answer should have 2 + 2 or 4 decimal places.

Example 2. Find the value of .241 × .37

$$\begin{array}{r} .241 \\ \underline{.37} \\ 1687 \\ \underline{723} \\ .08917 \end{array}$$

Since .241 has 3 decimal places and .37 has 2 decimal places, then the answer should have 3 + 2 or 5 decimal places.

Example 3. Find the value of .372 x 24

$$\begin{array}{r} .372 \\ \\ \underline{24} \\ \underline{1488} \\ \underline{744} \\ 8.928 \end{array}$$

Since .372 has 3 decimal places and 24 has none, then the answer should have 3 + 0 or 3 decimal places.

Example 4. Find the value of .24 x 3.7

$$\begin{array}{r} .24 \\ \\ \underline{3.7} \\ \underline{168} \\ \underline{72} \\ .888 \end{array}$$

Since .24 has 2 decimal places and 3.7 has one decimal place, then the answer should have 2 + 1 or 3 decimal places.

Now, let us turn to the division of decimal numbers. Let us find the value of $\frac{.543}{.3}$. In order to determine the position of the decimal point in the answer, we shall begin by multiplying the numerator and denominator by 10 in order to change the denominator to a whole number. This should help us to determine the correct position for the decimal point.

$$\frac{.543}{.3} = \frac{.543 \times 10}{.3 \times 10}$$

$$= \frac{5.43}{3}$$

$$\begin{array}{r} 181 \\ 3\,\overline{)543} \\ \underline{3} \\ 24 \\ \underline{24} \\ 03 \\ \underline{3} \end{array}$$

We have 181 as the digits in the answer, but where does the decimal point go? Since we are dividing a number over 5 (5.43) by the number 3, the answer should be between 1 and 2. Therefore, in 181, the decimal point must go after the 1. Therefore:

$$\frac{.543}{.3} = 1.81$$

Note that in this example we multiplied by 10 so that the denominator .3 will become the whole number 3. However, we also had to multiply .543 by 10. Now, how does this affect the decimal point placement if we use long division?

$$.3\;\big)\overline{.543}$$

When we multiplied by 10 we changed .3 to 3 and .543 to 5.43 . In the long division form we could accomplish this by moving the decimal point one place for both numbers .3 and .543 . This will then determine the correct position for the decimal point in the answer.

$$\begin{array}{r@{}l}
 & 1.81 \\
.3_{\wedge}\;\big) & \overline{.5_{\wedge}43} \\
 & \underline{3} \\
 & 2\,4 \\
 & \underline{2\,4} \\
 & 03 \\
 & \underline{3}
\end{array}$$

This illustration suggests that in the long division of decimals, we think of moving the decimal point in the divisor (.3 in our example) so that the divisor is a whole number. We then move the decimal point in the dividend (.543 in the example) the same number of places. This will position the decimal point correctly for the quotient (1.81 in the example).

Example 5. Find the value of $\frac{.9375}{2.5}$

$$\begin{array}{r@{}l}
 & .375 \\
2.5_{\wedge}\;\big) & \overline{.9_{\wedge}375} \\
 & \underline{7\,5} \\
 & 1\,87 \\
 & \underline{1\,75} \\
 & 125 \\
 & \underline{125}
\end{array}$$

Example 6. Find the value of $\frac{.8625}{.345}$

```
              2.5
.345  √.862 5
        690
        172 5
        172 5
```

Example 7. Find the value of $\frac{20}{.34}$.

```
              58.823529  etc.
.34  √20.00 0000000
       17 0
        3 00
        2 72
          28 0
          27 2
              80
              68
              120
              102
               180
               170
                100
                 68
                 320
                 306
                  140
```

Therefore:

$\frac{20}{.34}$ = 58.8235 (correct to four decimal places)

EXERCISES

Find the value in each of the following problems.

1. $.4 \times .8 =$

2. $.23 \times .41 =$

3. $17 \times 3.4 =$

4. $321 \times .61 =$

5. $4.2 \times 7.3 =$

6. $\frac{.2}{.1} =$

7. $\frac{.84}{2.1} =$

8. $\frac{4.62}{2.3} =$

9. $\frac{87.3}{45.2} =$

10. $\frac{3.214}{22} =$

ANSWERS

1. .32
2. .0943
3. 57.8
4. 195.81
5. 30.66
6. 2
7. .4
8. 2.0087 (to the nearest ten thousandth)
9. 1.9314 (to the nearest ten thousandth)
10. .1461 (to the nearest ten thousandth)

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Equivalent Fractional and Decimal Names

INTRODUCTION AND/OR OBJECTIVES:
In most situations a chart is available for converting a decimal to a fraction or a fraction to a decimal. However, the student should be capable of changing a number in fractional form to its equivalent decimal form or from a decimal form to a fractional form in case a conversion chart is not available.

TECHNICAL INFORMATION:

Example 1. Convert $\frac{3}{8}$ to its decimal equivalent.

To convert from the factional form to the equivalent decimal form, we use long division.

$$\begin{array}{r} .375 \\ 8\,\overline{)3.0} \\ \underline{2\ 4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Therefore:

$$\frac{3}{8} = .375$$

Example 2. Convert $\frac{5}{16}$ to its decimal equivalent.

$$\begin{array}{r} .3125 \\ 16\,\overline{)5.0} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \end{array}$$

Therefore:

$$\frac{5}{16} = .3125$$

Example 3. Convert $\frac{1}{3}$ to its decimal equivalent.

$$\begin{array}{r} .3333 \\ 3\,\overline{)1.0} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \end{array}$$

Note that we continue to get 3's. Therefore, we present our answer correct to the number of decimal places desired.

Therefore:

$\frac{1}{3} = .333$ (correct to the nearest thousandth)

or $\frac{1}{3} = .3333$ (correct to the nearest ten thousandth)

Example 4. Convert .625 to its fractional equivalent.

$$.625 = \frac{625}{1000}$$

$$= \frac{25 \times 25}{25 \times 40}$$

$$= \frac{25}{40}$$

$$= \frac{5 \times 5}{8 \times 5}$$

$$= \frac{5}{8}$$

Example 5: Convert .125 to its fractional equivalent.

$$.125 = \frac{125}{1000}$$

$$= \frac{25 \times 5}{25 \times 40}$$

$$= \frac{5}{40}$$

$$= \frac{1 \times 5}{8 \times 5}$$

$$= \frac{1}{8}$$

Example 6: Convert .25 to its fractional equivalent.

$$.25 = \frac{25}{100}$$

$$= \frac{1 \times 25}{4 \times 25}$$

$$= \frac{1}{4}$$

EXERCISES

In problems 1 to 5 convert to the decimal equivalent.

1. $\frac{3}{5}$
2. $\frac{7}{16}$
3. $\frac{5}{8}$
4. $\frac{2}{3}$
5. $\frac{3}{32}$

In problems 6 to 10 convert to the fractional equivalent.

6. .500
7. .875
8. .125
9. .750
10. .0625

ANSWERS

1. .600

2. .4375

3. .625

4. .6667 (correct to the nearest ten thousandth)
or .667 (correct to the nearest thousandth)

5. .09375
or .094 (correct to the nearest thousandth)

6. $\frac{1}{2}$

7. $\frac{7}{8}$

8. $\frac{1}{8}$

9. $\frac{3}{4}$

10. $\frac{1}{16}$

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
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OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Calculations Involving Approximate Numbers

INTRODUCTION:

If there are four people in a particular room, then the number of people is exactly four. However, when we measure something with a rule, micrometer, or a similar instrument, the value we get for the measurement is an approximation of the exact value. For example, we may measure something with an ordinary pair of micrometers and find the measurement is .376. We have indicated the measurement to the nearest thousandth. However, the actual exact measurement might be .37582, or it might be .37612. The measurement of .376 is an approximation which is accurate to the nearest thousandth. Note that there are three different digits in .376. We denote this by saying .376 has three significant digits.

A constant problem is the number of decimal places and the number of digits to include in answers which are the result of addition, subtraction, multiplication, and division involving approximate numbers.

OBJECTIVES:

1. To provide the student with an understanding of how to determine the number of significant digits in an approximate number.

2. To provide the student with an understanding of how to determine the number of decimal places and the number of significant digits which should be involved in the answer to a problem which includes addition, subtraction, multiplication, and division with approximate numbers.

TECHNICAL INFORMATION:

I. SIGNIFICANT DIGITS

First of all, we will call a digit a significant digit if it is known to be correct within the limits of the type of measurement used. For example, if we measure a piece of round stock and find that the diameter is .446 to the nearest thousandth of an inch, then there are three significant digits (4, 4, and 6) in the number .446.

Example 1. How many significant digits are there in each of the following numbers? 3.54, 4.3251, 2.001, 3.21, 3.210, .523, .21, and 321.251

	Given Approximate Number	Answer: Number of Significant Digits
a.	3.54	3
b.	4.3251	5
c.	2.001	4
d.	3.21	3
e.	3.210	4 (since the 0 placed on the end tells us that the measurement was to the nearest thousandth)
f.	.523	3
g.	.21	2
h.	321.251	6

Now, let us consider the approximate number .002 with regard to the number of significant digits. A temptation is to say that there are three significant digits. However, there is only 1 significant digit, namely 2. Notice that the number .002 is read as 2 thousandths. The two zeros in front of the 2 are merely decimal place holders and are not considered as significant digits.

Example 2. How many significant digits are in the following numbers? .032, .005, .132, 4.032, 52.001, 654.32, 654.320, .0123

	Given Approximate Number	Answer: Number of Significant Digits
a.	.032	2
b.	.005	1
c.	.132	3
d.	4.032	4 (since the 0 is preceded and followed by nonzero digits, the 0 is more than a place holder)

e.	52.001	5
f.	654.32	5
g.	654.320	6
h.	.0123	3

If the number 230 is an approximate number correct to the nearest 10, then there are only two significant digits in 230, namely 2 and 3. If 230 is measured to the nearest 1, then there are three significant digits, namely 2, 3, and 0. Therefore, we must know the accuracy of the measurement before we can exactly determine the number of significant digits in such numbers as 230, 3500, 234,00, etc. Normally, unless we know the exact method of measurement, we will indicate that 234,000, if it is an approximate number, has three significant digits, 372,500 has 4 significant digits, and 5,200,000 has 2 significant digits.

II. ADDITION AND SUBTRACTION OF APPROXIMATE NUMBERS

Suppose we wish to add 3.234 and 2.44 and we know that both of the numbers are approximate.

$$\begin{array}{r} 3.234 \\ +\ \underline{2.44} \\ 5.674 \end{array}$$

When we add, we get 4 digits in the answer. However, the number 2.44 is accurate only to the nearest hundredth. Therefore, we cannot expect the answer to be correct to the nearest thousandth. Thus, we should round the answer to the nearest hundredth, so that the answer will be 5.67 instead of 5.674. There are various ways of rounding numbers. Probably the most frequently used is to round up if the following digit is 5 to 9 and round down if the digit following is 0 to 4. Since the digit following 7 in this problem is 4 we round down to 7.

Suppose we add 5.21 and 3.747.

$$\begin{array}{r} 3.747 \\ +\ \underline{5.21} \\ 8.957 \end{array}$$

The answer should be rounded to the nearest hundredth since 5.21 is correct to only the nearest hundredth. How do we round 8.957 so that it is expressed as a number correct to the nearest hundredth (two decimal places)? That is, do we write the number as 8.95 or as 8.96? In this case we round up to 8.96 since the 7 in 8.957 is between 5 and 9. Therefore, we round up from 5 to 6 in the hundredths position in the answer.

A helpful concept to remember is that the number of decimal places in the answer of an addition (or subtraction) problem involving approximate numbers should be the same as the number of decimal places in the number in the original problem with the fewest number of decimal places.

Example 1. Add the approximate numbers 4.238 and 5.21.

$$\begin{array}{r} 4.238 \\ +\ \underline{5.21} \\ 9.448 \end{array}$$

We will round up to 9.45. (Note that the answer is now correct to the nearest hundredth as was the least accurate number 5.21 in the original problem.)

Answer: 9.45

Example 2. Add the approximate numbers 32.21 and 4.4.

$$\begin{array}{r} 32.21 \\ +\ \underline{4.4} \\ 36.61 \end{array}$$

We round down to 36.6.

Answer: 36.6

Subtraction will follow the same process as for addition.

Example 3. If 3.732 and 2.41 are approximate, find the value of 3.732 − 2.41.

$$\begin{array}{r} 3.732 \\ -\ \underline{2.41} \\ 1.322 \end{array}$$

We will round to 1.32.

Answer: 1.32

Example 4. If 5.7477 and 2.352 are approximate, find the value of 5.7477 - 2.352.

$$\begin{array}{r} 5.7477 \\ -\ \underline{2.352} \\ 3.3957 \end{array}$$

We will round to 3.396.

Answer: 3.396

III. MULTIPLICATION AND DIVISION OF APPROXIMATE NUMBERS

In multiplication and division we can expect the answer to have no larger number of significant digits than the number in the original problem with the least number of significant digits. Therefore, we examine the original problem to determine the least number of significant digits in any number in the original problem. Our answer will then be rounded to that number of significant digits.

Example 1. If 32.1 and 2.4 are approximate numbers, find the value of 32.1 x 2.4.

$$\begin{array}{r} 32.1 \\ \times\ \underline{2.4} \\ 1284 \\ \underline{642} \\ 77.04 \end{array}$$

Since 32.1 has 3 significant digits and 2.4 has 2 significant digits, our answer should have the smaller number, 2, of significant digits. Therefore, the answer should be rounded to 77.

Answer: 77

Example 2. If 3.45 and 4.321 are both approximate numbers, find the value of 3.45 x 4.321.

$$\begin{array}{r} 3.45 \\ \times\ \underline{4.321} \\ 345 \\ 690 \\ 1035 \\ \underline{1380} \\ 14.90745 \end{array}$$

Since 3.45 has only 3 significant digits, the answer should have only 3 significant digits. Therefore, the answer should be rounded to 14.9.

Answer: 14.9

Example 3. If 3.58 and 2.2 are approximate numbers, find the value of $\frac{3.58}{2.2}$.

$$\begin{array}{r} 1.51 \text{ etc.} \\ 2.2\overline{)3.34} \\ \underline{2\ 2} \\ 1\ 14 \\ \underline{1\ 10} \\ 40 \\ \underline{22} \\ 180 \end{array}$$

Since 2.2 has only 2 significant digits, the answer should have only 2 significant digits. Therefore, we round 1.51 to 1.5.

Answer: 1.5

Example 4. If 4.321 and 3.45 are approximate numbers, find the value of $\frac{4.321}{3.45}$.

$$\begin{array}{r} 1.252 \text{ etc.} \\ 3.45\overline{)4.321} \\ \underline{3\ 45} \\ 871 \\ \underline{690} \\ 1810 \\ \underline{1725} \\ 850 \\ \underline{690} \end{array}$$

Since 3.45 has only three significant digits, the answer should have only 3 significant digits. Therefore, the answer should be rounded to 1.25.

Answer: 1.25

IV. CALCULATIONS WITH BOTH EXACT AND APPROXIMATE NUMBERS

If one or more numbers are exact and one or more are approximate, then in determining the number of decimal places and the number of significant digits in the answer, we need only to consider the decimal places and significant digits in the approximate numbers. This is true since the exact number is not restricted as to decimal place or significant digit accuracy.

Example 1. Suppose 7 is exact and 2.32 is approximate, find:

a. $7 + 2.32$

b. $7 - 2.32$

c. 7×2.32

d. $\frac{7}{2.32}$

Since 7 is exact we may add zeros if it is helpful in the problem.

For part a:

$$\begin{array}{r} 7.00 \\ +\ \underline{2.32} \\ 9.32 \end{array}$$

Since 2.32 is accurate to the nearest hundredth (two decimal places), then the answer should be correct to the nearest hundredth.

Answer: 9.32

For part b:

$$\begin{array}{r} 7.00 \\ -\ \underline{2.32} \\ 4.68 \end{array}$$

Answer: 4.68

For part c:

$$\begin{array}{r} 2.32 \\ \times\ \underline{\quad 7} \\ 16.24 \end{array}$$

Since 2.32 has 3 significant digits, the answer should have three significant digits. Therefore, the answer should be rounded to 16.2.

Answer: 16.2

For part d:

$$\begin{array}{r} 3.017 \text{ etc.} \\ 2.32\overline{)7.0000} \\ \underline{6\ 96} \\ 400 \\ \underline{232} \\ 1680 \\ \underline{1624} \end{array}$$

Since 2.32 has 3 significant digits, the answer should have three significant digits. Therefore, the answer should be rounded to 3.02.

Answer: 3.02

Since the methods of handling the number of decimal places and significant digits depends on a knowledge of how the numbers were obtained (with regard to the accuracy), the constant checking of significant digits will not be stressed in this book. However, in dealing with measurements performed by the students, the knowledge of the accuracy of numbers will allow utilization of the methods in this section. Also, the teacher might wish to indicate the accuracy of particular numbers in various assignments presented in this book.

PROBLEMS

1. Indicate the number of significant digits in each of the following numbers:

   a. 35.1
   b. .021
   c. .201
   d. 36.223
   e. 2.2
   f. 2.20
   g. 3.001

2. Carry out the indicated operations involving the given approximate numbers and round your answers so that the correct number of digits appears in the answer.

   a. 35.221 + 7.25
   b. 32.238 + 5.23
   c. 14.221 + 2.32 + 3.21
   d. 31.271 - 3.14
   e. 2.778 - 1.314
   f. 9.3248 - 7.217
   g. 3.421 x 2.3
   h. 4.32 x 7.41
   i. 1.221 x 2.25
   j. 5.22/3.1
   k. 7.223/3.21
   l. 5.22/.331

3. If we know that 5 is an exact number and 2.31 is an approximate number, find each of the following:

   a. 5 + 2.31
   b. 5 - 2.31
   c. 5 x 2.31
   d. 5/2.31

ANSWERS

1.

a. 3
b. 2
c. 3
d. 5
e. 2
f. 3
g. 4

2.

a. 42.47
b. 37.47
c. 19.75
d. 28.13
e. 1.464
f. 2.108
g. 7.9
h. 32.0
i. 2.75
j. 1.7
k. 2.25
l. 15.8

3.

a. 7.31
b. 2.69
c. 11.6
d. 2.16

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements

TECHNICAL INFORMATION TITLE: Conversion of English Units of Measurements to Metric Units and Vice Versa

INTRODUCTION:

Metric units are being used more each day as a result of international trade and because of scientific developments as well as practical applications in many technical areas.

OBJECTIVE:

To provide the student an opportunity to learn how to convert English units of measurements to metric units and from metric to English units.

TECHNICAL INFORMATION:

The following will allow the change from metric to English units or vice versa:

1 inch = 2.54 centimeters

1 inch = 25.4 millimeters

1 meter = 39.37 inches

1 centimeter = .3937 inches

1 millimeter = .03937 inches

1 meter = 100 centimeters

1 meter = 1000 millimeters

1 centimeter = 10 millimeters

1 centimeter = .01 meters

1 millimeter = .001 meters

1 millimeter = .1 centimeters

1 kilometer = 1000 meters

Probably two of the most useful of the above conversions are that 1 inch is equal to 2.54 centimeters, and 1 centimeter is equal to .3937 inches. The first is equivalent to writing 2.54 cm./in. (read as 2.54 centimeters per inch). This means that there are 2.54 centimeters in 1 inch. We may rewrite 2.54 cm./in. as $\frac{2.54 \text{ cm.}}{1 \text{ in.}}$. Therefore, if a piece is 2 inches long, we need only to multiply the 2.54 by 2 (since there are 2.54 centimeters per inch). Thus, 2 inches = 2(2.54) cm. or 5.08 cm.

APPLICATION OF THE RULE:

Example 1. Convert each dimension in Figure 1 from English units of measurement (inches) to metric units. First of all, convert all measurements to centimeters.

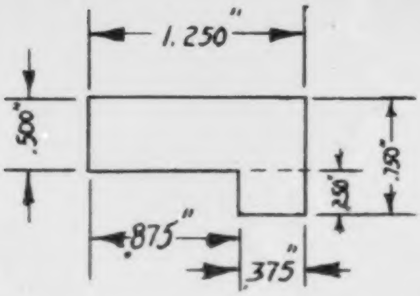

Figure 1

$$.500 \text{ inches} = .500\ \cancel{\text{in.}} \times \frac{2.54 \text{ cm.}}{1\ \cancel{\text{in.}}}$$

(Note that we are actually multiplying .500 by 1 since 2.54 cm. = 1 inch. Therefore, $\frac{2.54 \text{ cm.}}{1 \text{ in.}} = 1$.)

$$= .500 \times 2.54 \text{ cm.}$$

$$= 1.27 \text{ cm.}$$

$$.875 \text{ inches} = .875\ \cancel{\text{in.}} \times \frac{2.54 \text{ cm.}}{1\ \cancel{\text{in.}}}$$

$$= .875 \times 2.54 \text{ cm.}$$

$$= 2.22 \text{ cm.}$$

$$1.250 \text{ inches} = 1.250 \text{ in.} \times \frac{2.54 \text{ cm.}}{1 \text{ in.}}$$

$$= 1.250 \times 2.54 \text{ cm.}$$

$$= 3.18 \text{ cm.}$$

$$.250 \text{ inches} = .250 \text{ in.} \times \frac{2.54 \text{ cm.}}{1 \text{ in.}}$$

$$= .250 \times 2.54 \text{ cm.}$$

$$= .635 \text{ cm.}$$

$$.750 \text{ inches} = .750 \text{ in.} \times \frac{2.54 \text{ cm.}}{1 \text{ in.}}$$

$$= .750 \times 2.54 \text{ cm.}$$

$$= 1.91 \text{ cm.}$$

$$.375 \text{ inches} = .375 \text{ in.} \times \frac{2.54 \text{ cm.}}{1 \text{ in.}}$$

$$= .375 \times 2.54 \text{ cm.}$$

$$= .953 \text{ cm.}$$

If we wish to convert any of these measurements to millimeters, we may use the fact that 1 centimeter = 10 millimeters. So we may multiply the number of centimeters by $\frac{10 \text{ mm.}}{1 \text{ cm.}}$ (since 1 cm. equals 10 mm.).

For example:

$$1.27 \text{ centimeters} = 1.27 \cancel{\text{cm}}. \times \frac{10 \text{ mm.}}{1 \cancel{\text{cm}}.}$$

$$= 1.27 \times 10 \text{ mm.}$$

$$= 12.7 \text{ mm.}$$

Example 2. Convert all metric units of measurements in Figure 2 to English units (inches).

To convert from millimeters to inches it is easiest to use the conversion that 1 millimeter = .03937 inches. Therefore, we may multiply the dimension in millimeters by $\frac{.03937 \text{ inches}}{1 \text{ millimeter}}$ (since 1 millimeter = .03937 inches).

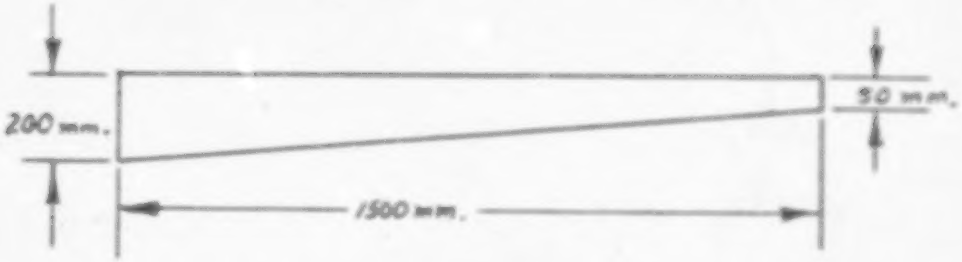

Figure 2

$$200 \text{ millimeters} = 200 \cancel{\text{mm}}. \times \frac{.03937 \text{ in.}}{1 \cancel{\text{mm}}.}$$

$$= 200 \times .03937 \text{ in.}$$

$$= 7.874 \text{ in.}$$

$$1500 \text{ millimeters} = 1500 \text{ mm.} \times \frac{.03937 \text{ in.}}{1 \text{ mm.}}$$

$$= 1500 \times .03937 \text{ in.}$$

$$= 59.055 \text{ in.}$$

$$50 \text{ millimeters} = 50 \text{ mm.} \times \frac{.03937 \text{ in.}}{1 \text{ mm.}}$$

$$= 1.969 \text{ in.}$$

Example 3. Convert the metric units of measurements in Figure 3 into English units (inches).

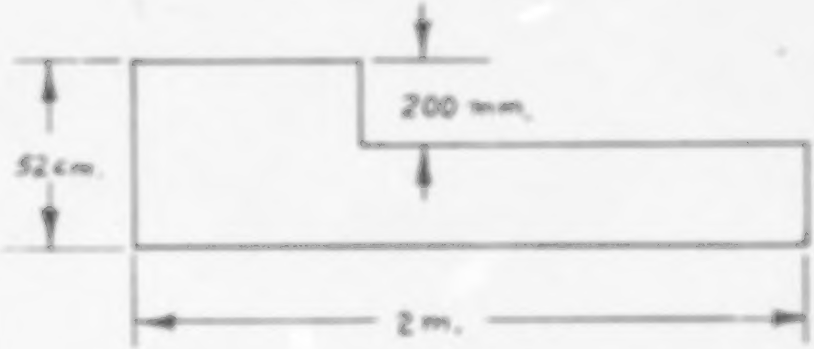

Figure 3

$$200 \text{ millimeters} = 200 \cancel{\text{mm.}} \times \frac{.03937 \text{ in.}}{1 \cancel{\text{mm.}}}$$

$$= 200 \times .03937 \text{ in.}$$

$$= 7.874 \text{ in.}$$

Since 1 meter = 39.37 inches, then to convert from meters to inches we multiply the number in meters by $\frac{39.37 \text{ in.}}{1 \text{ meter}}$.

$$2 \text{ meters} = 2 \cancel{\text{meters}} \times \frac{39.37 \text{ in.}}{1 \cancel{\text{meter}}}$$

$$= 2 \times 39.37 \text{ in.}$$

$$= 78.74 \text{ in.}$$

Since 1 centimeter equals .3937 inches, then to convert from centimeters to inches, we multiply the number in centimeters by $\frac{.3937 \text{ in.}}{1 \text{ cm.}}$.

$$52 \text{ centimeters} = 52 \cancel{\text{cm.}} \times \frac{.3937 \text{ in.}}{1 \cancel{\text{cm.}}}$$

$$= 52 \times .3937 \text{ in.}$$

$$= 20.47 \text{ in.}$$

Example 4. Convert 4.609 kilometers to a measurement in inches.

Since 1 kilometer = 1000 meters, we can multiply the number of kilometers by $\frac{1000 \text{ meters}}{1 \text{ kilometer}}$ to find the number of meters. Then we can convert from meters to inches as performed in Example 3.

$$4.609 \text{ kilometers} = 4.609 \cancel{\text{kilometers}} \times \frac{1000 \text{ meters}}{1 \cancel{\text{kilometer}}}$$

$$= 4.609 \times 1000 \text{ meters}$$

$$= 4609 \text{ meters}$$

$$4609 \text{ meters} = 4609 \cancel{\text{meters}} \times \frac{39.37 \text{ in.}}{1 \cancel{\text{meter}}}$$

$$= 4609 \times 39.37 \text{ in.}$$

$$= 181456.33 \text{ in.}$$

$$= 181{,}500 \text{ in.} \quad \text{(correct to four significant figures)}$$

Example 5. Convert 20 meters to a measurement in centimeters.

To change from meters to centimeters, we use the conversion that 1 meter = 100 centimeters. Therefore, multiply the number of meters by $\frac{100 \text{ cm.}}{1 \text{ meter}}$.

$$20 \text{ meters} = 20 \cancel{\text{meters}} \times \frac{100 \text{ cm.}}{1 \cancel{\text{meter}}}$$

$$= 20 \times 100 \text{ cm.}$$

$$= 2000 \text{ cm.}$$

Example 6. Find the pitch in Figure 4 in millimeters.

To change inches to millimeters use the conversion that 1 inch = 25.4

millimeters. Therefore, multiply the number in inches by $\frac{25.4 \text{ mm.}}{1 \text{ in.}}$.

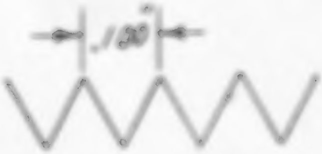

Figure 4

$.100 \text{ inches} = .100 \cancel{\text{in.}} \times \frac{25.4 \text{ mm.}}{1 \cancel{\text{in.}}}$

$= .100 \times 25.4 \text{ mm.}$

$= 2.54 \text{ mm.}$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Conversion of English Units of Measurement to Metric Units and from Metric to English Units

INTRODUCTION:

Today, more of our plants are using metric units than ever before. This is largely due to our international trade. It is, therefore, necessary to learn how to convert from one system to the other.

OBJECTIVE:

To learn how to convert from English units to metric units and from metric units to English units.

ASSIGNMENT:

Fill in the blank in each of the following problems:

1. .420 in. = ______cm.
2. 450 ft. = ______meters
3. .420 in. = ______mm.
4. 2.482 in. = ______mm.
5. 2.480 in. = ______cm.
6. .002 in. = ______cm.
7. .001 in. = ______mm.
8. .005 in. = ______cm.
9. 4 yards = ______cm.
10. 100 yards = ______cm.
11. 5280 ft. = ______meters
12. 5280 ft. = ______kilometers

ANSWERS

1. 1.07 cm. (three significant figures)
2. 137 meters (three significant figures)
3. 10.7 mm. (three significant figures)
4. 63.0 mm. (three significant figures)
5. 6.30 cm. (three significant figures)
6. .00508 cm. (three significant figures)
7. .0254 mm. (three significant figures)
8. .0127 cm. (three significant figures)
9. 366. cm. (three significant figures)
10. 9140. (three significant figures)
11. 1610. meters (three significant figures)
12. 1.61 kilometers (three significant figures)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Integral Exponents

INTRODUCTION AND/OR OBJECTIVES:

In the use of scientific notation, the knowledge of how to work with integral exponents is essential. Work with exponents is very fundamental and necessary in algebra. This unit will deal with exactly what is meant by an exponent, how to multiply terms involving exponents, how to divide terms involving exponents, and the use of negative exponents.

TECHNICAL INFORMATION:

I. POSITIVE INTEGRAL EXPONENTS

Consider the following numbers: 10^4, 7^3, and 6^2. The numbers 4, 3, and 2 are called exponents. They are integral exponents since they are integers. 10^4 is read "10 to the 4th power," 7^3 is read "7 to the 3rd power," and 6^2 is read "6 to the 2nd power or 6 squared."

An exponent is a power.

10^1 means 10

10^2 means 10×10

10^3 means $10 \times 10 \times 10$

10^4 means $10 \times 10 \times 10 \times 10$

10^5 means $10 \times 10 \times 10 \times 10 \times 10$

10^6 means $10 \times 10 \times 10 \times 10 \times 10 \times 10$

10^7 means $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

Similarly:

7^1 means 7

7^2 means 7×7

7^3 means $7 \times 7 \times 7$

7^4 means $7 \times 7 \times 7 \times 7$

7^5 means $7 \times 7 \times 7 \times 7 \times 7$

Therefore:

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$ or $1,000$

$10^4 = 10000$ or $10,000$

$10^5 = 100,000$

$10^6 = 1,000,000$

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

$7^5 = 16,807$

II. PRODUCTS OF TERMS INVOLVING EXPONENTS

What is the value of $10^3 \times 10^4$?

$$10^3 \times 10^4 = (10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10)$$
$$= 10^7$$

(Note that $3 + 4 = 7$)

Similarly:

$$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$$
$$= 3^6$$

(Note that $4 + 2 = 6$)

The above suggests the property that if m and n are positive integers (natural numbers) then:

$$10^m \times 10^n = 10^{m + n}$$

$$3^m \times 3^n = 3^{m + n}$$

or in general:

$$a^m \times a^n = a^{m + n}$$ (for m and n being any natural numbers)

Example 1. Find $6^5 \times 6^3$

$$6^5 \times 6^3 = 6^{5 + 3} = 6^8$$

Example 2. Find $2^{10} \times 2^{21}$

$$2^{10} \times 2^{21} = 2^{10 + 21} = 2^{31}$$

Example 3. Find $10^7 \times 10^{13}$

$$10^7 \times 10^{13} = 10^{7 + 13} = 10^{20}$$

III. DIVISION OF TERMS INVOLVING EXPONENTS

How can we find the value of $\frac{10^3}{10^2}$?

$$\frac{10^3}{10^2} = \frac{10 \times 10 \times 10}{10 \times 10}$$

$$= \frac{(10 \times 10) \times 10}{(10 \times 10)}$$

$$= 10$$

Similarly:

$$\frac{10^7}{10^4} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10}$$

$$= \frac{(10 \times 10 \times 10 \times 10) \times 10 \times 10 \times 10}{(10 \times 10 \times 10 \times 10)}$$

$$= 10 \times 10 \times 10$$

$$= 10^3 \text{ or } 1000$$

(Note that 7 - 4 = 3)

This suggests the rule that:

$$\frac{10^m}{10^n} = 10^{m - n}$$

or in general:

$$\frac{a^m}{a^n} = a^{m - n}$$ (If m and n are natural numbers, m is larger than n, and a is not zero)

Example 4. Find $\frac{10^8}{10^5}$

$$\frac{10^8}{10^5} = 10^{8 - 5}$$

$$= 10^3 \text{ or } 1000$$

Example 5. Find $\frac{3^4}{3^2}$

$$\frac{3^4}{3^2} = 3^{4 - 2}$$

$$= 3^2 \text{ or } 9$$

[redacted]

Example 6. Find $\frac{10^6}{10}$

$$\frac{10^6}{10} = \frac{10^6}{10^1}$$

$$= 10^{6-1}$$

$$= 10^5 \quad \text{or } 100{,}000$$

IV. THE EXPONENT ZERO

What is the meaning or value of 10^0? Let us look at the rules for division that we discussed above. If extended these should suggest a value for 10^0.

First of all is the property that:

$$\frac{10^m}{10^n} = 10^{m-n}$$

If we would allow the possibility that $m = n$, then, for example:

$$\frac{10^2}{10^2} = 10^{2-2}$$

$$= 10^0$$

But, $\frac{10^2}{10^2} = \frac{10 \times 10}{10 \times 10}$

$$= 1$$

This, then, would suggest that $10^0 = 1$.

Similarly: $\frac{4^3}{4^3} = 4^{3-3}$

$$= 4^0$$

But, again, $\frac{4^3}{4^3} = 1$

This again would suggest that $4^0 = 1$.

Because of these suggested values, we define:

$a^0 = 1$ (if a is not zero)

Example 7. $1075^0 = 1$

Example 8. $742^0 = 1$

V. NEGATIVE INTEGRAL EXPONENTS

Assume that in the rule: $10^m \times 10^n = 10^{m+n}$, it is possible to allow m or n to take on negative values. Then if the rule still works,

$$10^2 \times 10^{-2} = 10^{2+(-2)}$$
$$= 10^0$$
$$= 1$$

Similarly: $10^3 \times 10^{-3} = 10^{3+(-3)}$
$$= 10^0$$
$$= 1$$

Similarly: $5^4 \times 5^{-4} = 5^{4+(-4)}$
$$= 5^0$$
$$= 1$$

If we now remember the idea of the multiplicative inverse of a number, we can then determine the values of 10^{-2}, 10^{-3}, and 5^{-4}. The multiplicative inverse of the number 2 is the number p such that $2 \times p = 1$. Of course, $p = 1/2$. Similarly the number p such that $4 \times p = 1$ is $1/4$.

Since $10^2 \times 10^{-2} = 1$, then 10^{-2} must be the multiplicative inverse of 10^2, or $10^{-2} = \frac{1}{10^2}$.

Similarly: From above, $10^3 \times 10^{-3} = 1$

and also, $10^3 \times \frac{1}{10^3} = 1$

Therefore: $10^{-3} = \frac{1}{10^3}$

These illustrations suggest the property that:

$10^{-m} = \frac{1}{10^m}$ (if m is any integer except 0)

and in general:

$a^{-m} = \frac{1}{a^m}$ (if m is any integer except 0, and a is not zero)

Example 9. Find 10^{-1}

$$10^{-1} = \frac{1}{10^1}$$

$$= \frac{1}{10}$$

$$= .1$$

Example 10. Find 3^{-2}

$$3^{-2} = \frac{1}{3^2}$$

$$= \frac{1}{9}$$

$$= .1111$$

VI. ADDITIONAL PROBLEMS WITH EXPONENTS

In the sections above we talked about the possibility of extending the multiplication and division rules involving exponents. These rules can be extended such that the following are true.

$$a^m \times a^n = a^{m+n}$$

and (for any integers m and n, as long as a is not 0)

$$\frac{a^m}{a^n} = a^{m-n}$$

Example 11. Find $6^3 \times 6^{-2}$

$$6^3 \times 6^{-2} = 6^{3+(-2)}$$
$$= 6^1$$
$$= 6$$

Example 12. Find $2^4 \times 2^{-5}$

$$2^4 \times 2^{-5} = 2^{4+(-5)}$$
$$= 2^{-1}$$
$$= \frac{1}{2^1}$$
$$= \frac{1}{2}$$

Example 13. Find $\frac{6^2}{6^4}$

$$\frac{6^2}{6^4} = 6^{2-4}$$
$$= 6^{-2}$$
$$= \frac{1}{6^2}$$
$$= \frac{1}{36}$$

Example 14. Find $\frac{2^3}{2^6}$

$$\frac{2^3}{2^6} = 2^{3-6}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

VII. POWER OF A POWER

Suppose we wish to find the value of $(2^2)^3$.

$$(2^2)^3 = 2^2 \times 2^2 \times 2^2$$

$$= (2 \times 2) \times (2 \times 2) \times (2 \times 2)$$

$$= 2^6 \text{ or } 64$$

(Note that 2 × 3 = 6)

Similarly:

$$(3^4)^2 = (3^4) \times (3^4)$$

$$= (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$$

$$= 3^8 \text{ or } 6561$$

(Note that 4 × 2 = 8)

These illustrations suggest the property that:

$(a^m)^n = a^{m \times n}$ (where m and n are any natural numbers)

Example 15. Find $(4^3)^2$

$$(4^3)^2 = 4^{3 \times 2}$$

4^6 or 4096

Example 16. Find $(2^3)^3$

$(2^3)^3 = 2^{3 \times 3}$

$= 2^9$ or 512

EXERCISES

1. 6^3
2. 4^4
3. 5^2
4. 10^8
5. $10^2 \times 10^7$
6. $10^3 \times 10^3$
7. $5^3 \times 5^6$
8. $\frac{10^6}{10^3}$
9. $\frac{10^5}{10}$
10. $\frac{3^5}{3^3}$
11. 10^{-3}
12. 10^{-4}
13. 5^{-2}
14. 6^{-1}
15. $5^2 \times 5^{-1}$
16. $10^5 \times 10^{-2}$
17. $(2^2)^4$
18. $(10^5)^2$

ANSWERS

1. 216
2. 256
3. 25
4. 100,000,000
5. $10^9 = 1{,}000{,}000{,}000$
6. $10^6 = 1{,}000{,}000$
7. $5^9 = 1{,}953{,}125$
8. $10^3 = 1{,}000$
9. $10^4 = 10{,}000$
10. $3^2 = 9$
11. $\frac{1}{10^3} = \frac{1}{1000} = .001$
12. $\frac{1}{10^4} = \frac{1}{10{,}000} = .0001$
13. $\frac{1}{5^2} = \frac{1}{25} = .04$
14. $\frac{1}{6} = .1667$
15. $5^1 = 5$
16. $10^3 = 1{,}000$
17. 2^8 or 256
18. 10^{10}

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements

TECHNICAL INFORMATION TITLE: Scientific Notation

INTRODUCTION:

In a modern era of technology, the use of scientific notation is essential because of the high degree of accuracy needed. The exponential method is extremely useful in terms of slide rule calculations.

OBJECTIVE:

To provide the student an opportunity to learn scientific notation and then be able to apply it in areas of arithmetic calculations.

TECHNICAL INFORMATION:

The main idea of scientific notation is utilizing the powers of 10 to minimize the complexity of multiplication and division of extremely large or extremely small numbers. First of all, consider the number .1. This is a decimal number representing the fraction 1/10.

$$\frac{1}{10} = \frac{1}{10^1}$$

$$= 10^{-1}$$

Therefore, $.1 = 10^{-1}$

Similarly:

$$.01 = \frac{1}{100}$$

$$= \frac{1}{10^2}$$

$$= 10^{-2}$$

Therefore, $.01 = 10^{-2}$

Similarly:

$$
\begin{aligned}
.001 &= \frac{1}{1000} \\
&= \frac{1}{10^3} \\
&= 10^{-3}
\end{aligned}
$$

Therefore, $.001 = 10^{-3}$

This leads us to the values:

$$
\begin{aligned}
.1 &= 10^{-1} \\
.01 &= 10^{-2} \\
.001 &= 10^{-3} \\
.0001 &= 10^{-4} \\
.00001 &= 10^{-5}
\end{aligned}
$$

etc.

Notice that the negative exponent can be determined by noting the number of places needed to move the decimal point so that it is immediately to the right of 1. For example in $\underbrace{.0001}$ the decimal point would have to be moved 4 positions. We know that $.0001 = 10^{-4}$. Similarly in $\underbrace{.000001}$, the decimal point has to be moved 6 places, and, therefore, $.000001 = 10^{-6}$. Similarly, $\underbrace{.00000001}$ should equal 10^{-8}, since the decimal point would have to be moved 8 places.

Now:

$$
\begin{aligned}
.00002 &= 2 \times \underbrace{.00001} \\
&= 2 \times 10^{-5}
\end{aligned}
$$

Also:

$$.000004 = 4 \times .000001$$

$= 4 \times 10^{-6}$

Note that $.000004 = 4 \times 10^{-6}$ since the decimal point would have to be moved 6 places.

We can work with .0022 similarly by writing this as 22×10^{-4}. However, in scientific notation instead of the number 22 as the multiplier, we prefer to have the number be between 1 and 10 (actually less than 10). Therefore, instead of writing .0022 as 22×10^{-4}, we write .0022 as 2.2×10^{-3}. That is, we move the decimal only 3 places so that the multiplier is 2.2 (a number between 1 and 10).

APPLICATION OF THE RULE:

Example 1. Change .0000000024 using scientific notation.

$.0000000024 = 2.4 \times 10^{-9}$

Example 2. Change .00000456 using scientific notation.

$.00000456 = 4.56 \times 10^{-6}$

Now, remember that:

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$ or 1,000

$10^4 = 10000$ or 10,000

$10^5 = 100000$ or 100,000

etc.

Again, note that in 10000 the decimal point would have to be moved 4 places to the left to be in the position immediately to the right of 1. Also note that the value of 10000 is 10^4. Similarly for 100000000, since the decimal point must be moved 8 places to the left, the value of 100000000 is 10^8.

Example 3. Find the value of 1000000 (using scientific notation).

$$1\underbrace{000000} = 10^6$$

Example 4. Find the value of 200000 (using scientific notation).

$$\begin{aligned} 200000 &= 2 \times 100000 \\ &= 2 \times 10^5 \end{aligned}$$

Example 5. Find the value of 40000000 (using scientific notation).

$$\begin{aligned} 40000000 &= 4 \times 10000000 \\ &= 4 \times 10^7 \end{aligned}$$

Example 6. Find the value of 234000 (using scientific notation).

$$2\underbrace{34000} = 2.34 \times 10^5$$

Example 7. Find the value of 7650000000 (using scientific notation).

$$7\underbrace{650000000} = 7.65 \times 10^9$$

I. ADDITION USING SCIENTIFIC NOTATION

Example 8. Find the sum of .000042 and .0000048 (using scientific notation).

$$\begin{array}{r} .000042 \\ \underline{.0000048} \\ .\underbrace{00004}68 \end{array} \quad = \quad 4.68 \times 10^{-5}$$

II. MULTIPLICATION USING SCIENTIFIC NOTATION

Example 9. Find the value of the product of .00065 and .00025 by first changing the numbers into scientific notation.

$$\begin{aligned} (.00065) \times (.00025) &= (6.5 \times 10^{-4}) \times (2.5 \times 10^{-4}) \\ &= (6.5 \times 2.5) \times (10^{-4} \times 10^{-4}) \end{aligned}$$

(By the

commutative and associative
properties of multiplication)

$$= 16.25 \times 10^{-8}$$

$$= 1.625 \times 10^{1} \times 10^{-8}$$

$$= 1.625 \times 10^{1+(-8)} \quad (a^m \times a^n = a^{m+n})$$

$$= 1.625 \times 10^{-7}$$

or .0000001625

This should be a little easier in column form:

$$\begin{array}{r} 6.5 \times 10^{-4} \\ \underline{2.5 \times 10^{-4}} \\ 16.25 \times 10^{-8} \end{array} = 1.625 \times 10^{1} \times 10^{-8}$$

$$= 1.625 \times 10^{-7}$$

Example 10. Find the value of .0034 × .0006.

$$\begin{array}{rr} & .0034 \\ \times & \underline{.0006} \end{array} \qquad \begin{array}{r} 3.4 \times 10^{-3} \\ \underline{6 \;\; \times 10^{-4}} \\ 20.4 \times 10^{-7} \end{array} = 2.04 \times 10^{1} \times 10^{-7}$$

$$= 2.04 \times 10^{-6}$$

or .00000204

Example 11. Find the value of 4400000 × 350000.

$$\begin{array}{rr} & 4400000 \\ \times & \underline{350000} \end{array} \qquad \begin{array}{l} \;\;4.4 \times 10^{6} \\ \;\;\underline{3.5 \times 10^{5}} \\ \;\;220 \\ \underline{132 \qquad\quad} \\ 15.40 \times 10^{11} \end{array} = 1.540 \times 10^{1} \times 10^{11}$$

$$= 1.540 \times 10^{1+11}$$

$$= 1.540 \times 10^{12}$$

or 1,540,000,000,000

Example 12. Find the value of .00024 × 430,000.

$$\begin{array}{r} .00024 \\ \times\ \underline{430{,}000} \end{array} \qquad \begin{array}{l} 2.4 \times 10^{-4} \\ \underline{4.3 \times 10^{5}} \\ 72 \\ \underline{96} \\ 10.32 \times 10^{1} = 1.032 \times 10^{1} \times 10^{1} \\ \qquad\qquad\quad = 1.032 \times 10^{2} \\ \qquad\qquad\qquad \text{or } 103.2 \end{array}$$

III. DIVISION USING SCIENTIFIC NOTATION

Example 13. Find the value of $\frac{.00042}{.000021}$.

$$\frac{.00042}{.000021} = \frac{4.2 \times 10^{-4}}{2.1 \times 10^{-5}}$$

$$= \frac{4.2}{2.1} \times \frac{10^{-4}}{10^{-5}} \qquad \text{(Using the rule that } \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}\text{)}$$

$$= 2 \times 10^{(-4)\ -\ (-5)} \qquad \text{(Using the rule that } \frac{a^m}{a^n} = a^{m\ -\ n}\text{)}$$

$$= 2 \times 10^{-4\ +\ 5}$$

$$= 2 \times 10^{1}$$

or 20

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: Scientific Notation

INTRODUCTION:

The use of scientific is very helpful when dealing with extremely large or extremely small numbers.

OBJECTIVE:

To learn how to use scientific notation.

ASSIGNMENT:

Fill in the blank in each of the following problems:

1. .0000000000025 = ________ (in scientific notation)
2. 250,000,000,000,000 = ________ (in scientific notation)
3. .0000000003345 = ________ (in scientific notation)
4. 3546000 = ________ (in scientific notation)
5. .0000001 = ________ (in scientific notation)
6. 99,000,000,000,000 = ________ (in scientific notation)
7. (.000000045)(250,000,000) = ________ (use scientific notation to multiply)
8. (270,000,000(40,000,000,000) = ________ (use scientific notation to multiply)
9. (.0000025)(.000035) = ________ (use scientific notation to multiply)
10. (345,000,000)(.000003) = ________ (use scientific notation to multiply)
11. $\frac{4{,}000{,}000}{.00000035}$ = ________ (use scientific notation to divide)
12. $\frac{.000000045}{85{,}000{,}000}$ = ________ (use scientific notation to divide)

ANSWERS

1. 2.5×10^{-12}
2. 2.5×10^{14}
3. 3.345×10^{-10}
4. 3.546×10^{6}
5. 1×10^{-7}
6. 9.9×10^{13}
7. 1.125×10^{1} or 11.25
8. 1.08×10^{19} or
9. 8.75×10^{-11}
10. 1.035×10^{3} or 1035
11. 1.143×10^{13} (rounded to three decimal places) or 11,430,000,000,000
12. 5.294×10^{-16} (rounded to three decimal places)
    or .0000000000000005294

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements

TECHNICAL INFORMATION TITLE: The Use of Scientific Notation in the Conversion of Units of Measurement

INTRODUCTION:

In a modern technological era practical problems involve the conversion of metric measurements to English and vice versa. Problems also involve the use of scientific notation. This is true, especially, with extremely large or extremely small numbers. The two types of problems combined provide a very ideal method (especially when using the slide rule).

OBJECTIVE:

To provide the student an opportunity to learn how to convert English units to metric units or vice versa in combination with the use of scientific notation.

TECHNICAL INFORMATION:

Care with English and metric units of measurements is essential. When extremely large or extremely small numbers are involved, then the use of scientific notation can be very helpful. The combination of the two types of problems can best be illustrated through the use of examples such as in the following section.

APPLICATION OF THE RULE:

Example I. Convert .00078 inches to a measurement in millimeters.

$$.00078 \text{ in.} = 7.8 \times 10^{-4} \text{ in.}$$

Then:

$$7.8 \times 10^{-4} \text{ in.} = 7.8 \times 10^{-4} \cancel{\text{in.}} \times \frac{25.4 \text{ mm.}}{1 \cancel{\text{in.}}}$$

$$= 7.8 \times 10^{-4} \times 25.4 \text{ mm.}$$

$$= 198.12 \times 10^{-4} \text{ mm.}$$

(or 200×10^{-4} mm. correct to 2 significant figures)

Example 2. Convert 240 cm. to inches.

$$240 \text{ centimeters} = 2.40 \times 10^{2} \text{ cm.}$$

$$= 2.40 \times 10^{2}\ \cancel{\text{cm}}. \times \frac{.3937 \text{ in.}}{1\ \cancel{\text{cm}}.}$$

$$= 2.40 \times 10^{2}\ \cancel{\text{cm}}. \times \frac{3.937 \times 10^{-1} \text{ in.}}{1\ \cancel{\text{cm}}.}$$

$$= 2.40 \times 10^{2} \times 3.937 \times 10^{-1} \text{ inches}$$

$$= 2.40 \times 3.937 \times 10^{1} \text{ inches}$$

$$= 9.449 \times 10^{1} \text{ inches}$$

$$= 94.49 \text{ inches}$$

(or 94.5 inches correct to three significant figures)

Example 3. Convert .00034 inches to centimeters.

$$.00034 \text{ inches} = 3.4 \times 10^{-4}\ \cancel{\text{in}}. \times \frac{2.54 \text{ cm.}}{1\ \cancel{\text{in}}.}$$

$$= 3.4 \times 2.54 \times 10^{-4} \text{ cm.}$$

$$= 8.6 \times 10^{-4} \text{ cm.}$$

$$= .00086 \text{ cm.}$$ (correct to two significant figures)

Example 4. Convert 20000 cm. to millimeters.

$$20000 \text{ centimeters} = 2 \times 10^{4}\ \cancel{\text{cm}}. \times \frac{10 \text{ mm.}}{1\ \cancel{\text{cm}}.}$$

$$= 2 \times 10^{4} \times 10 \text{ mm.}$$

$$= 2 \times 10^{5} \text{ mm.}$$ (or 200000 mm.)

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Precision Measurements

TECHNICAL ASSIGNMENT TITLE: The Use of Scientific Notation in the Conversion of Units of Measurement

INTRODUCTION:

Once the metric system becomes reality in terms of everyday use, and accuracy must be more accountable, then both systems will be used simultaneously.

OBJECTIVE:

To learn how to convert units of measurement using scientific notation.

ASSIGNMENT:

Fill in the blank in each of the following problems:

1. .000000025 in. = ______mm.
2. 2.035642 in. = ______cm.
3. 58400000 mm. = ______in.
4. 200,000 cm. = ______mm.
5. 8,540,000 ft. = ______cm.
6. 54,000 yards = ______in.
7. .000000025 in. = ______yards
8. 240,000 miles = ______meters
9. 500,000 kilometers = ______miles
10. .00000025 in. = ______cm.

[redacted]

ANSWERS

1. 6.35×10^{-7} mm. or .000000635 mm. (three significant figures)
2. 5.17 cm. (three significant figures)
3. 2.299×10^{6} in. or 2,299,000 (four significant figures)
4. 2×10^{6} mm. or 2,000,000 mm.
5. 2.60×10^{8} cm. or 260,000,000 (three significant figures)
6. 1.944×10^{6} in. or 1,944,000 in. (three significant figures)
7. 6.9×10^{-10} yards or .00000000069 yards
8. 3.86×10^{8} meters or 386,000,000 meters (three significant figures)
9. 3.108×10^{5} miles or 310,800 miles (four significant figures)
10. 6.35×10^{-7} cm. or .000000635 cm. (three significant figures)

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Change Gears--Compound Gearing

INTRODUCTION:
Since it is not always possible to obtain gears that will make a simple train to cut desired threads, then we must rely on compound trains.

OBJECTIVE:
To provide the student an opportunity to learn how to calculate for a set of gears when necessary to rely on a compound train.

TECHNICAL INFORMATION:

In order to set up gear trains, a machine is equipped with gear train adaptors which permit gears to be attached and adjustments to be made.

The formula when using a compound train of gears is:

$$\frac{\text{Threads per inch}}{\text{Lathe gear constant}} = \frac{\text{Product of teeth in driven gears}}{\text{Product of teeth in driving gears}}$$

APPLICATION OF THE RULE:

Example. Determine the change gears to be used to cut 24 threads per inch when the lathe constant is 6. The gears available have from 30 teeth to 100 teeth in multiples of 5.

Gears available: 30, 35, 40, 45, 50, 55, 60, 65, 70
75, 80, 85, 90, 95, 100

Let us try to use the formula for simple gearing:

$$\frac{\text{Threads per inch}}{\text{Lathe gear constant}} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

$$\frac{24}{6} = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

$$4 = \frac{\text{Lead screw gear}}{\text{Spindle stud gear}}$$

If we use the gear with the fewest number of teeth, 30, to be the spindle stud gear, then:

$$4 = \frac{\text{Lead screw gear}}{30}$$

$$30 \cdot 4 = 30 \cdot \frac{\text{Lead screw gear}}{30} \qquad \text{(Mult. both sides by 30)}$$

$$120 = \text{Lead screw gear} \qquad \text{(Mult. inverse)}$$

However, this gear is not available in the set. Therefore, simple gearing cannot be used.

The alternative, then, is to use compound trains of gears. In this case we use the formula:

$$\frac{\text{Threads per inch}}{\text{Lathe gear constant}} = \frac{\text{Product of teeth in driven gears}}{\text{Product of teeth in driving gears}}$$

$$\frac{24}{6} = \frac{\text{Product of teeth in driven gears}}{\text{Product of teeth in driving gears}}$$

$$\frac{4}{1} = \frac{\text{Product of teeth in driven gears}}{\text{Product of teeth in driving gears}}$$

Therefore, the ratio of the product of the teeth in the driven gears to the product of the teeth in the driving gears should be 4 to 1.

One method of trying to find such gears is to select two of the gears with a small number of teeth for the driving gears, and then determine the gears needed for the driven gears.

In the given example, select the gears having 30 and 40 teeth (30 and 35 could have been selected) to be the driving gears.

Therefore,

$$4 = \frac{\text{Product of teeth in driven gears}}{30 \cdot 40}$$

$$4 = \frac{\text{Product of teeth in driven gears}}{1200}$$

$$1200 \cdot 4 = 1200 \cdot \frac{\text{Product of teeth in driven gears}}{1200} \qquad \text{(Mult. both sides by 1200)}$$

$$4800 = \text{Product of teeth in driven gears} \qquad \text{(Mult. inverse)}$$

Then, any two gears may be selected so that the product of the two is 4800.

For example,

Select 60 and 80

since $60 \cdot 80 = 4800$.

Therefore, the two driven gears to be used are 60 and 80 when driving gears of 30 and 40 are used. See Figure 1 below.

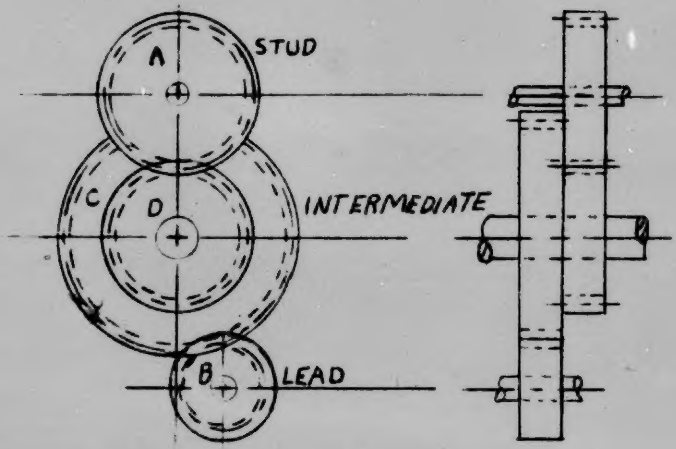

Figure 1

Now, what is the relation between the number of teeth and the revolutions per minute of the gears in a compound train? If we find the product of the number of teeth of rhe driving gears and the number of revolutions per minute of the first driving gear, the answer should equal the product of the number of teeth of the driven gears and the number of revolutions per minute of the last driven gear.

Thus, in Figure 2,

$$N_1 \times N_2 \times \text{R.P.M.(of } N_1) = n_1 \times n_2 \times \text{r.p.m.(of } n_2)$$

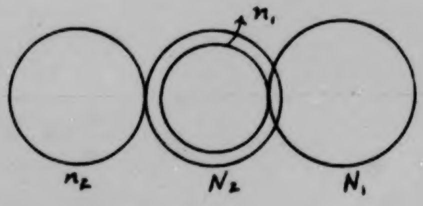

Figure 2

Example. In Figure 2, if N_1 is 50, N_2 is 40, n_1 is 30, n_2 is 40, and the gear with N_1 (or 50) teeth revolves at 150 revolutions per minute, at what speed does the gear with n_2 teeth revolve?

$$50 \times 40 \times 150 = 30 \times 40 \times \text{r.p.m.(of } n_2)$$

$$\frac{1}{30 \times 40} \times 50 \times 40 \times 150 = \frac{1}{30 \times 40} \times 30 \times 40 \times \text{r.p.m.(of } n_2)$$

(we multiplied both sides by 1/(30 × 40).)

$$\frac{50 \times 40 \times 150}{30 \times 40} = \text{r.p.m.(of } n_2)$$

$$250 = \text{r.p.m.(of } n_2)$$

[redacted]

MODERN MATHEMATICS
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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Change Gears--Compound Gearing

INTRODUCTION:
Since it is not always possible to obtain gears that will make a simple train to cut desired threads, then we may rely on compound trains.

OBJECTIVE:
To calculate a proper set of gears for a specific thread.

ASSIGNMENT:

1. Find change gears that can be used to cut 28 threads per inch if the lathe constant is 8.

2. Find change gears that can be used to cut 32 threads per inch if the lathe constant is 6.

3. Find change gears that can be used to cut 40 threads per inch if the lathe constant is 8.

4. Find the change gears that can be used to cut 16 threads per inch when the lathe constant is 6. One driven gear has 40 teeth, and one driving gear has 30 teeth.

5. One driving gear has 16 teeth, and one driven gear has 48 teeth. What change gears could be used to cut 18 threads per inch when the lathe constant is 8?

6. What change gears could be used to cut 10 threads per inch if the lathe constant is 5? The ratio of one driving gear to one driven gear is 1 to 1.

7. Gear up a lathe with a constant of 6 to cut 12 1/2 threads per inch. One driven gear has 50 teeth, and one driving gear has 40 teeth.

8. A machinist is to cut 4 1/2 threads per inch on a lathe with a screw constant of 6. If one driving gear has 30 teeth, and one driven gear has 40 teeth, what other gears could be used?

9. If Figure 1 if gear A revolves at 40 revolutions per minute, how fast will gear B revolve?

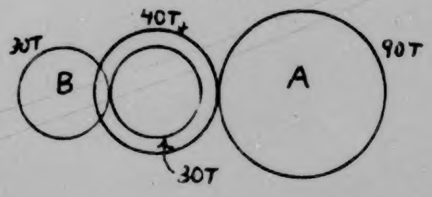


Figure 1

10. In Figure 1 if B is required to make 2 revolutions for each revolution of gear A, gear A would have to be replaced by a gear with how many teeth?

11. If gear A in Figure 2 revolves at 100 revolutions per minute, how fast will gear D revolve?

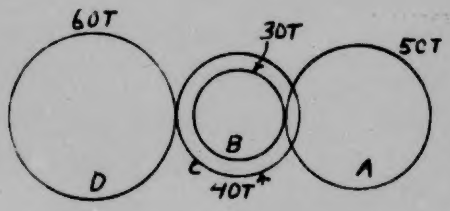


Figure 2

12. In Figure 2 how many teeth should a gear replacing gear B have so that when gear A revolves at 120 revolutions per minute, gear D will revolve at 160 revolutions per minute?

ANSWERS

In problems 1 through 8 the following are possible gears that could be used. There are certainly other combinations of gears that will work.

1. Driven gears: 70, 75
   Driving gears: 30, 50
2. Driven gears: 80, 80
   Driving gears: 30, 40
3. Driven gears: 60, 100
   Driving gears: 30, 40
4. Driven gear: 100
   Driving gear: 50
5. Driven gear: 36
   Driving gear: 48
6. Driven gear: 80
   Driving gear: 40
7. Driven gear: 50
   Driving gear: 30
8. Driven gear: 36
   Driving gear: 64
9. 90 revolutions per minute
10. 80
11. 111.11 revolutions per minute
12. 25

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION TITLE: Square Threads

INTRODUCTION:

Generally speaking, most threads, especially the National Standard Thread (or Unified Thread Form), are used to fasten two pieces of metal together. The primary purpose of the square thread is to transmit motion under heavy stress or load situations. This form of thread is normally used on large metal bench vises, house jacks, etc. Generally speaking, the thread should be able to withstand a large amount of thrust and stress at all sections of the thread. For this reason, in general, the depth and width are the same. However, practically speaking, there must be clearance allowances for the threads to operate properly.

OBJECTIVES:

1. To provide the student an opportunity to learn how to calculate the various dimensions of the square thread.
2. To enable the student to learn how to calculate the proper dimensions for the square threading tool and, also, how to grind it properly.

TECHNICAL INFORMATION:

It is a known fact that a piece of steel which has exactly the same outside diameter as the inside diameter of the hole in which it is to fit, will not turn freely unless there is some allowance made for clearance. This clearance should be considered in determining the various dimensions of the square thread. In general the clearance should be between .005 in. and .010 in.

The following formulas may be used in computing the various thread dimensions and the tool width. See Figure 1 and Figure 2.

Pitch (P) = $\frac{1}{N}$ where N is the number of threads per inch)

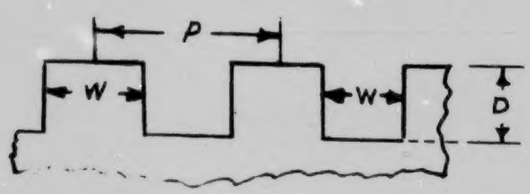
Depth (D) = $\frac{P}{2}$ or $\frac{1}{2N}$

Tool Width (W) = $\frac{P}{2}$ or $\frac{1}{2N}$ (+.005 in. for clearance)

Outside diameter (O.D.) = Major diameter (-.005 in. for clearance)

Minor diameter = Major diameter - P (-.005 in. for clearance)

Minor diameter (hole size) for cutting internal threads = Minor diameter of external threads (+.010)



The Square Thread

Figure 1

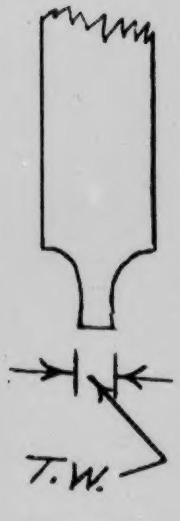


Figure 2

APPLICATION OF THE RULE:

Example 1. Find the pitch for a square 1 in., 8 threads per inch screw.

$$P = \frac{1}{N}$$

$$= \frac{1}{8}$$

$$= .125 \text{ in.}$$

Example 2. Find the depth for the thread in Example 1.

$$D = \frac{P}{2}$$

$$= \frac{.125}{2}$$

$$= .063 \text{ in. (to the nearest thousandth)}$$

Example 3. Find the width of the tool needed to cut the thread in Example 1.

$$W = \frac{P}{2} + .005 \text{ in.}$$

$$= .063 + .005$$

$$= .068 \text{ in.}$$

Example 4. What are the outside and minor diameters for the thread in Example 1?

$$\text{Outside diameter} = \text{Major diameter} - .005 \text{ in.}$$

$$= 1.000 - .005$$

$$= .995 \text{ in.}$$

$$\text{Minor diameter} = \text{Major diameter} - P - .005 \text{ in.}$$

$$= 1.000 - .125 - .005$$

$$= 1.000 - .130$$

$$= .870 \text{ in.}$$

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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: Square Threads

INTRODUCTION:

In order to cut square threads the student must be able to calculate the various dimensions involved and, also, to determine the size of the threading tool.

OBJECTIVE:

To provide the student practice in determining various dimensions associated with square threads.

ASSIGNMENT:

1. What is the pitch for a square 1 in., 4 threads per inch screw?
2. What is the depth (D) for the screw in problem 1?
3. What is the minor diameter for the thread in problem 1?
4. You will be required to grind a tool for the above thread. What size would the tool width be for cutting the above thread?
5. You are required to cut the internal threads for the above thread. To what size should the hole be bored before cutting the internal threads for the above screw? How would you measure the depth of the internal threads being cut?
6. The lead screw for a metal working vise is being made in the shop. The square threads on the screw are 3/4 - 6. What would be the major diameter of the shaft before any threads are turned? What should be the final outside diameter?
7. To what size should the tool be ground for cutting the above thread?
8. To what size should the minor diameter be turned for the vise screw in problem 6? What tools should be used for measuring this minor diameter?

ANSWERS

1. .250 in.

2. .125 in.

3. .745 in.

4. .130 in.

5. .755 in.
   Internal calipers, outside micrometers, or fit screw to hole.

6. .750 in.
   .745 in.

7. .089 in.

8. .578 in.
   O.D. calipers and vernier calipers

MODERN MATHEMATICS
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OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Setting Up and Cutting Square Threads

COURSE UNIT TITLE: Lathe--Square External Threads

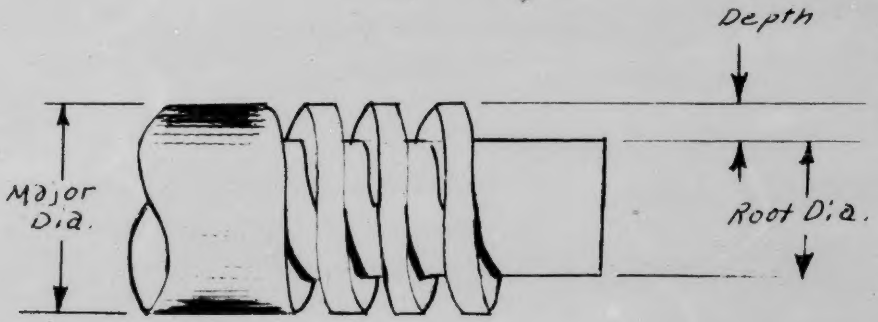
INTRODUCTION:

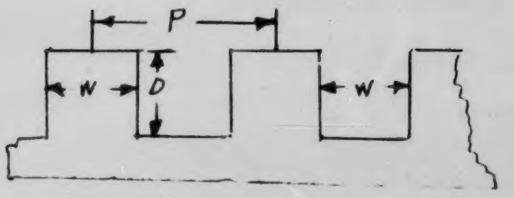
Square threads are generally used for transmitting motion under heavy stress situations. Square threads are used for the lead screw and the feed screw on a lathe. They are also used for vise screws and jackscrews. Square threads are stronger than Acme threads. In general, the width and depth are the same. However, there must be clearance allowances so that the threads may operate.

OBJECTIVE:

To provide the student practice in cutting square threads on a lathe.

DRAWING:





SQUARE THREAD

TOOLS AND MATERIALS REQUIRED:

Lathe	Micrometer
Dog	Round steel stock
Tool bit for lathe	Outside calipers

PROCEDURE:

(Operations)	(Related Information)
1. Procure material.	1. Refer to print or drawing.
2. Secure in three jaw chuck.	2. Project 1/2 in. out of chuck.
3. Face first end.	3. Use facing tool and face only enough to clean up the end.
4. Center drill first end.	4. Use center drill of proper size.
5. Face second end.	5. Same as no. 3.
6. Center drill second end.	6. Same as no. 4.
7. Secure between centers.	7. Use dog and drive plate.
8. Turn O.D.	8. Use fine finish tool for finishing cut.
9. Turn root diameter at recess.	9. Refer to Technical Information Sheet.
10. Set compound rest.	10. Set parallel to axis of workpiece.
11. Set tool bit (roughing).	11. Set to left of rest, on center, and square.
12. Set quick change gear box for proper number of threads.	12. Set for proper number of threads per inch.
13. Set up machine feed levers for thread cutting.	13. Check whether to use odd or even on particular lathe on thread dial.
14. Take trial run.	14. Use pencil. Check pitch.
15. Take first cut.	15. Cut .002 to .003 in. with crossfeed.
16. Repeat no. 15.	16. Leave .015 in. for finishing.
17. Replace rough tool with finishing tool.	17. Repeat cuts.
18. Check root diameter.	18. Use outside calipers and telescope gage or equivalent.

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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL INFORMATION SHEET: American National Acme Threads

INTRODUCTION:
The National Acme Thread is classified as a power transmission type of thread. It is a modification of both the square thread and the National Standard 60° form of thread. It is both easier to cut and stronger than the square thread.

OBJECTIVE:
To provide the student with practice in computing the various dimensions associated with American National Acme Threads.

TECHNICAL INFORMATION:

The following procedure is followed for the Acme type of thread. After setting the compound rest at 14 1/2 degrees to the right (for cutting a right hand external Acme thread), the tool is set at 90° to the centerline of the part to be threaded.

The outside diameter is checked with regular micrometers, and the minor diameter may be checked by using outside calipers in conjunction with micrometers and a telescope gage. If the mating part is available, it should be used in preference to the methods mentioned above.

Refer to Figure I with regard to the following formulas regarding dimensions associated with Acme threads. The pitch (P) is equal to 1 divided by the number of threads per inch (N).

$P = \frac{1}{N}$ (where P is the pitch, and N is the number of threads per inch)

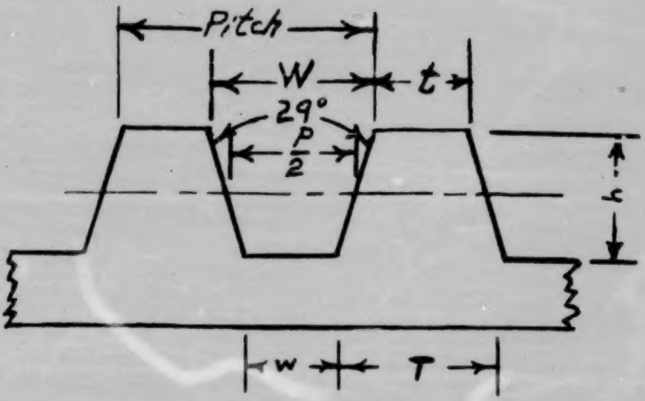
Pitch
W
t
29°
$\frac{P}{2}$
h
w
T

Figure 1

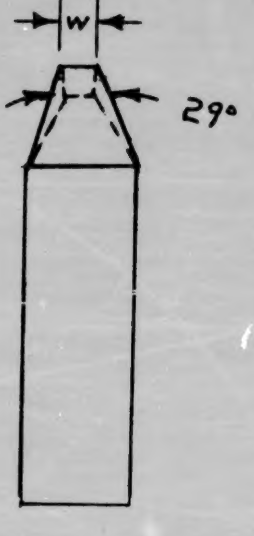

Figure 2

The depth (h) of an Acme thread is equal to one half of the pitch plus .01". The .01" is the amount for clearance when the threads are mated. The formula for the depth (h) is as follows:

$$h = \frac{P}{2} + .01 \qquad \text{(where h is the depth and P is the pitch)}$$

The minor diameter is equal to the major diameter minus twice the depth.

$$\begin{aligned} \text{Minor diameter} &= \text{Major diameter} - 2h \\ &= \text{Major diameter} - 2(\frac{P}{2} + .01) \\ &= \text{Major diameter} - 2\cdot\frac{P}{2} - 2(.01) \qquad \text{(Distributive property)} \\ &= \text{Major diameter} - P - .02 \qquad \text{(Mult. inverse)} \end{aligned}$$

Thus, the final formula for the minor diameter is as follows:

$$\text{Minor diameter} = \text{Major diameter} - P - .02 \qquad \text{(where P is the pitch)}$$

The distance across the crest of the thread (t) is always .3707 times the pitch. The distance across the root (w) is .0052" less than the width of the crest.

$t = .3707P$ (where t is the distance across the crest of the thread, and P is the pitch)

$w = t - .0052''$ (where w is the distance across the root, and t is the distance across the crest of the thread)

or $w = .3707P - .0052''$ (where w is the distance across the root, and P is the pitch)

The distance between the crests (W) is always .6293 times the pitch. The distance between the roots, or the width of the tooth (T), is .0052" greater than the distance between the crests.

$W = .6293P$ (where W is the distance between the crests, and P is the pitch)

$T = W + .0052''$ (where T is the width of the tooth, and W is the distance between the crests)

or $T = .6293P + .0052''$ (where T is the width of the tooth, and P is the pitch)

The working clearance allowed between an Acme thread and a nut is .01". Since a working clearance is necessary on both sides of the diameter of the screw and the nut, the following relationship should exist:

$K = \text{Major diameter} + .02''$ (where K is the major diameter of the internal thread)

$k = \text{Minor diameter} + .02''$ (where k is the diameter of the bored hole)

We have already noted that the minor diameter is equal to the major diameter minus P minus .02". Therefore,

$k = \text{Major diameter} - P - .02'' + .02''$

Thus, $k = \text{Major diameter} - P$ (where k is the diameter of the bored hole, and P is the pitch)

The tool for cutting National Acme Threads is ground to the included angle and tool tip size as indicated by an Acme thread gage. This gage should also be used for setting the tool bit properly. See Figure 2.

Example 1. Find the depth (h) for a 2" Acme thread having 4 threads per inch.

First we will find the pitch.

$$P = \frac{1}{N}$$

$$= \frac{1}{4}$$

$$= .250''$$

Now,

$$h = \frac{P}{2} + .010$$

$$= \frac{.250}{2} + .010$$

$$= .125 + .010$$

$$= .135 \text{ in.}$$

Example 2. For the Acme thread in Example 1 find the distance across the crest of the thread (t).

$$t = .3707P$$

$$= (.3707)(.250)$$

$$= .093 \text{ in.} \quad \text{(to the nearest thousandth)}$$

Example 3. For the Acme thread in Example 1 find the distance across the root (w).

$$w = t - .0052$$

$$= .093 - .0052$$

$$= .088 \text{ in.} \quad \text{(to the nearest thousandth)}$$

Example 4. Find the size of the drilled hole for the Acme thread in Example 1.

$$k = \text{Major diameter} - P$$

$$= 2.000 - .250$$

$$= 1.750 \text{ in.} \quad \text{(or } 1\ 3/4 \text{ in.)}$$

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Lathe

TECHNICAL ASSIGNMENT TITLE: American National Acme Threads

INTRODUCTION:
The student should be able to calculate the various dimensions involved with American National Acme Threads.

OBJECTIVE:
To provide the student practice in evaluating dimensions involved with American National Acme Threads.

ASSIGNMENT:

1. What is the pitch for a 1" - 5 N. A. (National Acme Thread)?
2. What is the depth (h) for the above thread?
3. What is the width (t) of the crest of the above thread?
4. What is the width (w) at the root of the above thread?
5. What is the minor diameter of a 3" - 2 N. A. Thread?
6. Find (a) the distance between the crests (W) and (b) the width of the root of a 1 3/4" - 4 National Acme Thread screw.
7. Find the following for a 5/8" - 8 Acme thread:
    a. The depth (h) of the thread
    b. The distance across the top flats (t)
    c. The width of the point on the tool bit or width (w) across the bottom of the flats.
8. Find the diameter for the bored hole for each of the following Acme threads.
    a. 2 1/4" - 3 N. A. Thread
    b. 1 3/4" - 4 N. A. Thread
    c. 3" - 2 N. A. Thread
    d. 3/4" - 6 N. A. Thread

ANSWERS

1. .200 in.

2. .110 in.

3. .074 in.

4. .069 in.

5. 2.48 in.

6. a. .157 in.
   b. .088 in.

7. a. .073 in.
   b. .046 in.
   c. .041 in.

8. a. 1.917 in.
   b. 1.500 in.
   c. 2.50 in.
   d. .583 in.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Setting Up and Cutting an Acme Thread

COURSE UNIT TITLE: Lathe--Acme Threads

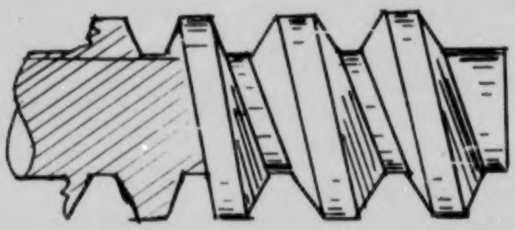
INTRODUCTION:

The Acme thread has sides forming an included angle of 29 degrees, and the normal or working depth is equal to 1/2 of the pitch. The clearance, for both crest and root, is .010 in. for all sizes of Acme threads. The major diameter of taps is .020 in. oversize to give an .010 in. major diameter clearance in the nuts. Also, the Acme thread tools are properly shaped to gage, and the screws are cut .010 in. deeper to give a minor diameter clearance in the threads.

OBJECTIVE:

To provide the student practice in cutting an Acme thread on a lathe.

DRAWING:



29° WORM THREAD

TOOLS AND MATERIALS REQUIRED:

Vernier bevel protractor
Gear tooth vernier
Micrometer
Tool bit ground for Acme thread
Acme thread gage
Round stock

PROCEDURE:

(Operations)	(Related Information)
1. Procure material.	1. Write Bill of Materials using print or drawing.
2. Measure rough stock.	2. Allow extra length for facing the two ends.
3. Saw.	3. Use power saw.
4. Secure in three jaw lathe chuck.	4. Extend 1/2 inch.
5. Face first end.	5. Use facing tool. Face only enough to clean up end.
6. Center drill.	6. Center drill using proper size center drill based on diameter of stock.
7. Face second end.	7. Same as no. 5.
8. Center drill.	8. Same as no. 6.
9. Secure between centers.	9. Use dog.
10. Turn large O.D.	10. See Technical Information Sheet.
11. Turn root diameter at recess.	11. See Technical Information Sheet.
12. Set compound rest.	12. Set at 14 1/2 degrees to the right.
13. Set tool bit (roughing).	13. Set to left of tool rest, on center, and perpendicular to workpiece. Use Acme gage on tailstock sleeve.
14. Set for threads per inch.	14. Set quick change gear box.
15. Take trial run.	15. Use pencil to check pitch. Use thread dial and half nut lever.
16. Take first cut.	16. Cut .002 to .003 in. using compound rest.
17. Repeat cuts.	17. Use half nut lever and observe thread dial.
18. Finish and inspect.	18. Use gear tooth verniers.

MODERN MATHEMATICS
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THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Rectangular Coordinate System

INTRODUCTION AND/OR OBJECTIVES:

The rectangular coordinate system is basic to both an understanding of the problems involving trigonometry and also an understanding of the concepts in numerical control. This section will be restricted to two dimensions, that is, points lying in the same plane.

TECHNICAL INFORMATION:

Consider two perpendicular lines (two lines which intersect so that the angles formed are right angles) as in Figure I. Call the vertical line

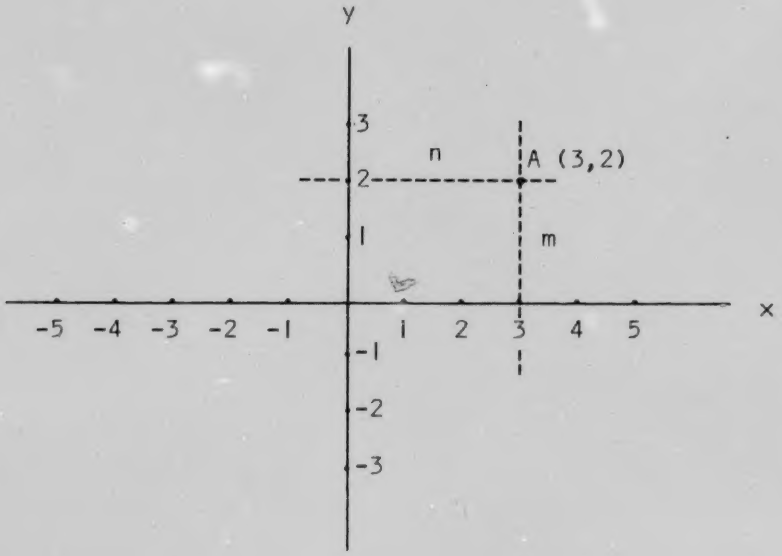


Figure I

the y-axis and the horizontal line the x-axis. Call the point of intersection, 0, of the two lines the origin. Now, a scale is selected for both the x-axis and the y-axis. Normally, positive numbers are assigned to points on the right end of the x-axis and negative numbers to points on the left end of the x-axis. Similarly, positive numbers are associated with points on the upper end of the y-axis and negative numbers with points on the lower end of the y-axis. 0 is assigned to each axis at the origin.

To find the coordinates for point A (See Figure 1), first of all, consider the line m through A parallel to the y-axis. This line intersects the x-axis at 3. The first coordinate for A is defined to be 3. Then, consider the line n through A parallel to the x-axis. This line intersects the y-axis at 2. The second coordinate of point A is then defined to be 2. The coordinates for A are defined to be (3, 2). The first coordinate, 3,

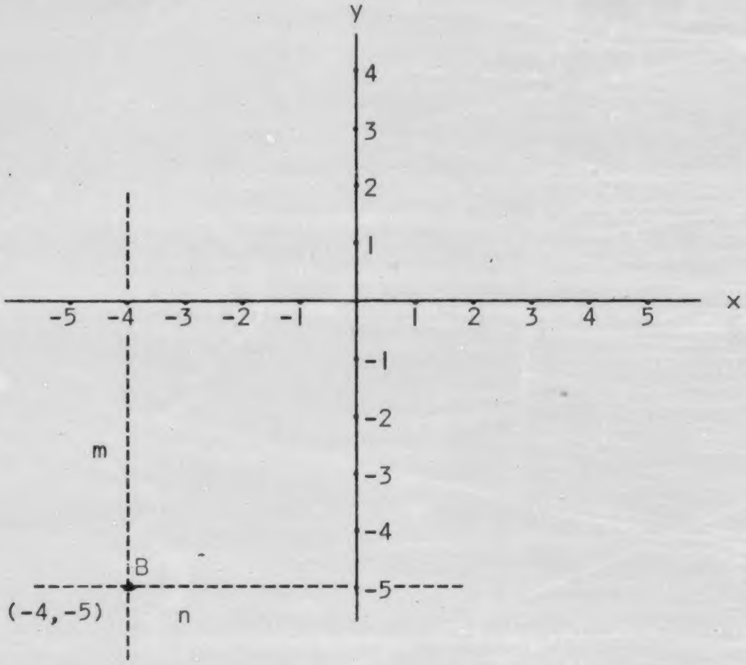


Figure 2

is called the abcissa. The second coordinate, 2, is called the ordinate.

In Figure 2, the line m through B parallel to the y-axis intersects the x-axis at -4. The line n through B parallel to the x-axis intersects the y-axis at -5. The coordinates for B are, therefore, (-4, -5).

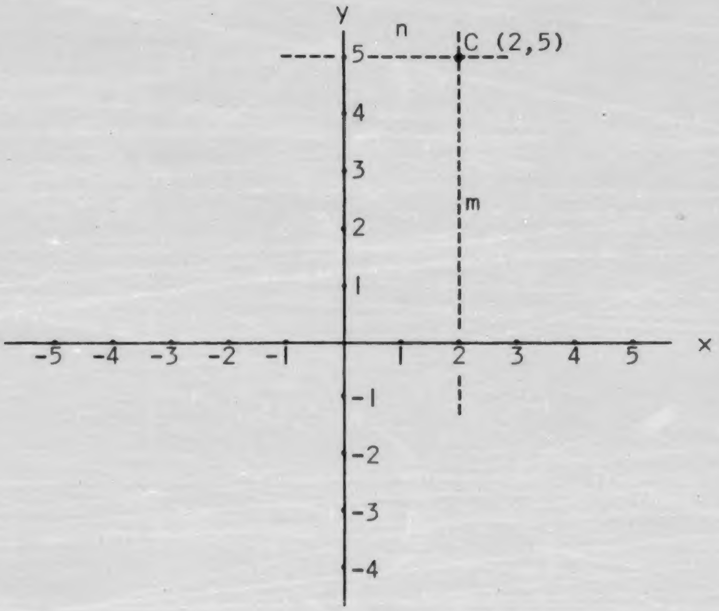


Figure 3

In Figure 3, to find the point with coordinates (2, 5) first of all draw the line m through 2 on the x-axis parallel to the y-axis. Next, draw the line n through 5 on the y-axis parallel to the x-axis. Lines m and n intersect at some point C. Point C has coordinates (2, 5).

Now, what about the coordinates for points on the x-axis and points on the y-axis? The point A (See Figure 4) at 3 on the x-axis is assigned the coordinates (3, 0). Likewise, point B at -2 on the x-axis is assigned the coordinates (-2, 0). The point C at 2 on the y-axis is assigned the coordinates (0, 2). Likewise, the point D at -4 on the y-axis is assigned the coordinates (0, -4). Similarly, other points on the x-axis and y-axis can be assigned coordinates. What about the coordinates for 0, the origin? Since the origin is at 0 on both axes, then its coordinates will be (0, 0).

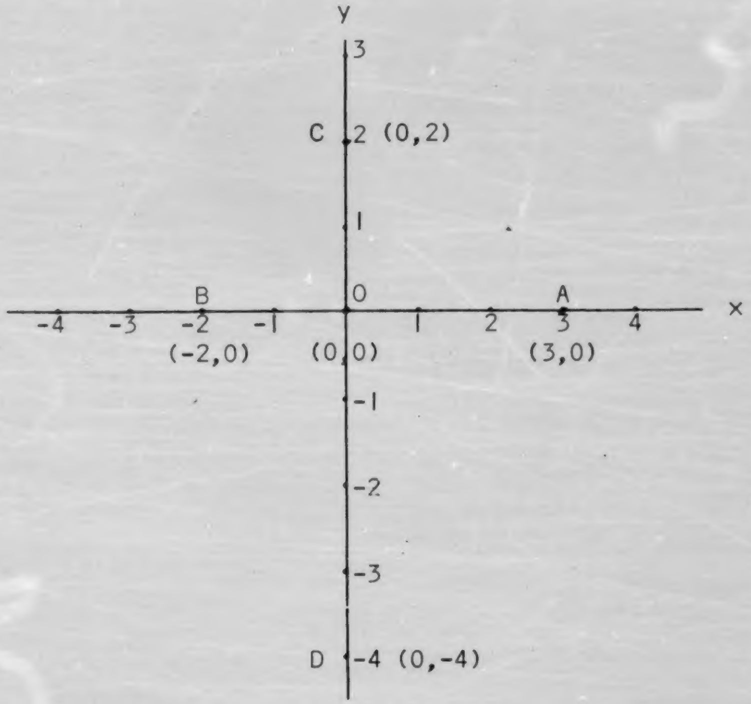
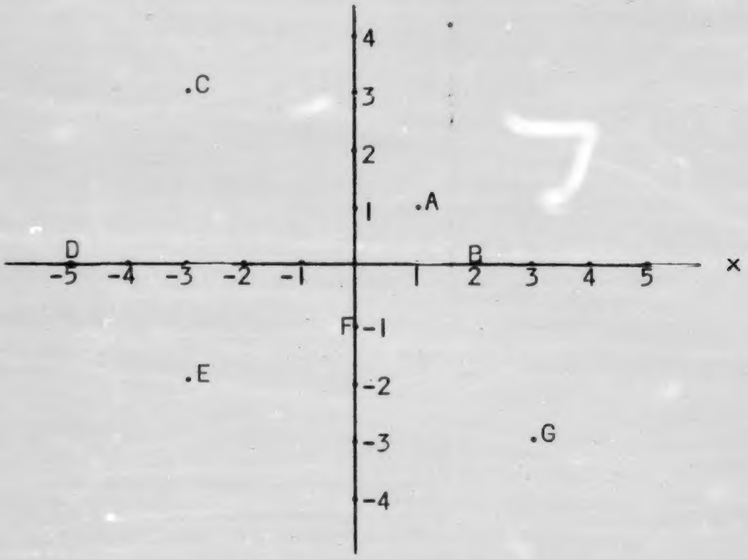


Figure 4

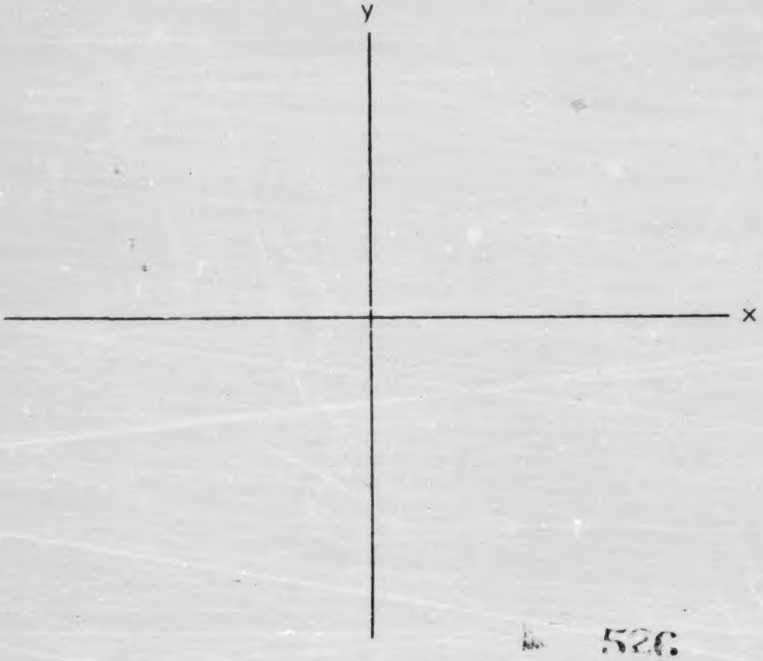
EXERCISES

1. Find the coordinates for points A, B, C, D, E, F, and G in the figure below.


4
3
2
1
.C
.A
D
B
-5
-4
-3
-2
-1
1
2
3
4
5
x
F
-1
-2
-3
-4
.E
.G

2. In the figure below, plot the points with the given coordinates.

A: (1, 2)
B: (-3, 5)
C: (0, -3)
D: (2, -1)
E: (-5, 0)
F: (3, -4)



ANSWERS

1. A: (1, 1)
   B: (0, 2)
   C: (-3, 3)
   D: (-5, 0)
   E: (-3, -2)
   F: (0, -1)
   G: (3, -3)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: The Trigonometric Functions

INTRODUCTION AND/OR OBJECTIVES:

Trigonometry must be considered one of the most important areas of applied mathematics in the machine trades. Trigonometry is a very valuable tool in being able to work with the measures of angles and the measures of sides of a triangle. By using trigonometry, the lengths of sides will determine the measures of angles in the triangle, and vice versa. This section will deal entirely with right triangles.

I. RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM

In Figure I consider the point B having coordinates (3, 4). The length of the segment directed from O to A (denoted by OA) is equal to 3. The length of the segment directed from A to B (denoted by AB) is equal

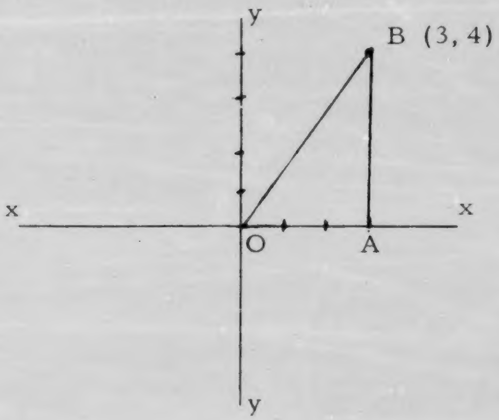


Figure I

to 4. How can we find the length of the segment from O to B (OB)?

The three points O, A, and B form a triangle which we may denote by $\triangle OAB$. This triangle is called a right triangle since one angle ($\angle OAB$) is a right angle. The segments $\overline{OA}$ and $\overline{AB}$ are called the legs of the triangle and $\overline{OB}$ is called the hypotenuse. (Note that $\overline{AB}$ denotes the segment whereas AB denotes the directed length of the segment.) $\overline{OA}$, $\overline{AB}$, and $\overline{OB}$ are all called sides of the triangle.

The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. That is: (In Figure 1)

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$(OB)^2 = (3)^2 + (4)^2$$

$$(OB)^2 = 3 \cdot 3 + 4 \cdot 4$$

$$(OB)^2 = 9 + 16$$

$$(OB)^2 = 25$$

Therefore:

$$OB = \sqrt{25}$$

$$OB = 5$$

Example 1. In triangle ABC in Figure 2 below, find the length of the hypotenuse (AC).

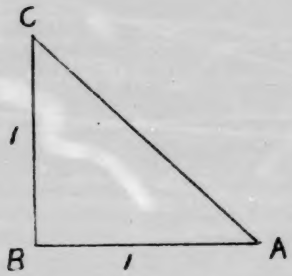


Figure 2

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (1)^2 + (1)^2$

$(AC)^2 = 1 + 1$

$(AC)^2 = 2$

$AC = \sqrt{2}$ or approximately 1.414

II. THE TRIGONOMETRIC FUNCTIONS

Consider triangle ABC in Figure 3.

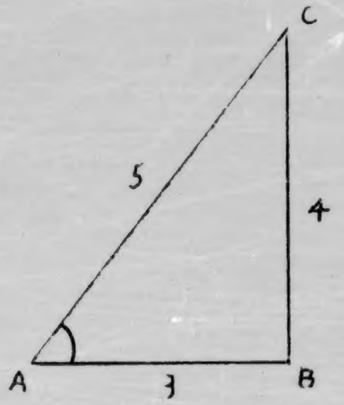


Figure 3

Note that AB = 3, BC = 4, and AC = 5. For ease in writing let us refer to ∠CAB as just ∠A. We can now set up ratios of the lengths of the legs and the length of the hypotenuse of △ABC. These various ratios are called trigonometric functions. These trigonometric functions allow us to find the lengths of various sides of the triangle if we know an angle or allow us to find the angle if we know lengths of the various sides.

The first trigonometric function that we will define is the sine of ∠A. The sine of ∠A or more simply sin ∠A is defined as the length of the side opposite $\angle A$ divided by the length of the hypotenuse.

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

In Figure 3, $\overline{BC}$ is the side opposite $\angle A$ and $\overline{AC}$ is the hypotenuse. Therefore:

$$\begin{aligned}\sin \angle A &= \frac{BC}{AC}\\ &= \frac{4}{5}\\ &= .8000\end{aligned}$$

A second trigonometric function of $\angle A$ is the cosine of $\angle A$ or more simply cos $\angle A$. The cos $\angle A$ is defined as the length of the side adjacent to $\angle A$ divided by the length of the hypotenuse.

$$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

In Figure 3, $\overline{AB}$ is the side adjacent to $\angle A$ and $\overline{AC}$ is the hypotenuse. Therefore:

$$\begin{aligned}\cos \angle A &= \frac{AB}{AC}\\ &= \frac{3}{5}\\ &= .6000\end{aligned}$$

A third trigonometric function of $\angle A$ is the tangent of $\angle A$. The tangent of $\angle A$ or more simply tan $\angle A$ is defined as the length of the side opposite $\angle A$ divided by the side adjacent to $\angle A$.

$$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

[redacted]

In Figure 3, BC is the side opposite to ∠A and AB is the side adjacent to ∠A.

$$\tan \angle A = \frac{BC}{AB}$$

$$= \frac{4}{3}$$

$$= 1.3333$$

Note that the letters of the vertices can change in different problems. Therefore, check to see which side is the opposite side, which is the adjacent side, and which is the hypotenuse for the particular angle used in the problem.

Example 1. In Figure 4, find sin ∠B, cos ∠B, tan ∠B.

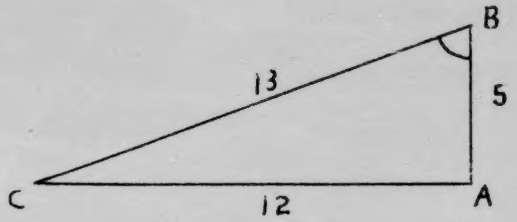


Figure 4

$$\sin \angle B = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{12}{13}$$

$$\cos \angle B = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$= \frac{AB}{BC}$$

$$= \frac{5}{13}$$

____

$$\tan \angle B = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{AC}{AB}$$

$$= \frac{12}{5}$$

____

There are three other trigonometric functions which are probably not used as frequently as the sine, cosine and tangent functions.

One of these is the cotangent of an angle. The cotangent of ∠A or more simply cot ∠A is defined as the length of the adjacent side divided by the length of the opposite side.

$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

In Figure 3, $\overline{AB}$ is the side adjacent ∠A and $\overline{BC}$ is the side opposite ∠A.

$$\cot \angle A = \frac{AB}{BC}$$

$$= \frac{3}{4}$$

$$= .750$$

The cosecant of ∠A abbreviated to csc ∠A is defined as the length of the hypotenuse divided by the length of the side opposite to ∠A.

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

In Figure 3, $\overline{AC}$ is the hypotenuse and $\overline{BC}$ is the side opposite $\angle A$.

$$\begin{aligned} \csc \angle A &= \frac{AC}{BC} \\ &= \frac{5}{4} \\ &= 1.250 \end{aligned}$$

The secant of $\angle A$ abbreviated to sec $\angle A$ is defined as the length of the hypotenuse divided by the length of the adjacent side.

$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

In Figure 3, $\overline{AC}$ is the hypotenuse and $\overline{AB}$ is the side adjacent to $\angle A$.

$$\begin{aligned} \sec \angle A &= \frac{AC}{AB} \\ &= \frac{5}{3} \\ &= 1.6667 \end{aligned}$$

Example 2. Find the cot $\angle B$, csc $\angle B$, and sec $\angle B$ in Figure 4.

$$\begin{aligned} \cot \angle B &= \frac{\text{length of adjacent side}}{\text{length of opposite side}} \\ &= \frac{AB}{AC} \\ &= \frac{5}{12} \end{aligned}$$

____

$$\begin{aligned} \csc \angle B &= \frac{\text{length of hypotenuse}}{\text{length of opposite side}} \\ &= \frac{BC}{AC} \\ &= \frac{13}{12} \end{aligned}$$

____

$$\sec \angle B = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

$$= \frac{BC}{AB}$$

$$= \frac{13}{5}$$

Example 3. Find the values of sin ∠A, cos ∠A, tan ∠A, cot ∠A, csc ∠A, and sec ∠A in Figure 5.

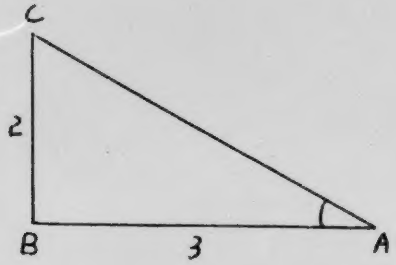


Figure 5

First of all, we need to find the value of AC.

From the Pythagorean Theorem:

$$\begin{aligned}(AC)^2 &= (AB)^2 + (BC)^2\\ &= (3)^2 + (2)^2\\ &= 9 + 4\\ &= 13\\ AC &= \sqrt{13}\\ &= 3.6056\end{aligned}$$

Then:

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$= \frac{BC}{AC}$$

$= \frac{2}{\sqrt{13}}$

$= \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$ (We multiply the numerator and the denominator by $\sqrt{13}$ so that we can change the denominator from a square root to a whole number. This will change the problem so that instead of dividing by a square root (which will be 3.6056) we will multiply by the square root.

$= \frac{2 \cdot \sqrt{13}}{13}$ $(\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d})$

$= \frac{2(3.6056)}{13}$

$= \frac{7.2112}{13}$

$= .5547$

$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$

$= \frac{AB}{AC}$

$= \frac{3}{\sqrt{13}}$

$= \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$= \frac{3 \cdot \sqrt{13}}{13}$

$= \frac{3(3.6056)}{13}$

$= \frac{10.8168}{13}$

$= .8321$

$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$

$= \frac{BC}{AB}$

$$= \frac{2}{3}$$

$$= .6667$$

$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$= \frac{AB}{BC}$$

$$= \frac{3}{2}$$

$$= 1.5000$$

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

$$= \frac{AC}{BC}$$

$$= \frac{\sqrt{13}}{2}$$

$$= \frac{3.6056}{2}$$

$$= 1.8028$$

$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

$$= \frac{AC}{AB}$$

$$= \frac{\sqrt{13}}{3}$$

$$= \frac{3.6056}{3}$$

$$= 1.2019$$

SUMMARY

$$\sin \angle A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\cos \angle A = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\tan \angle A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

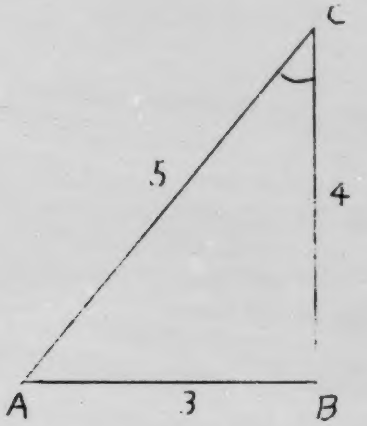
$$\cot \angle A = \frac{\text{length of adjacent side}}{\text{length of opposite side}}$$

$$\csc \angle A = \frac{\text{length of hypotenuse}}{\text{length of opposite side}}$$

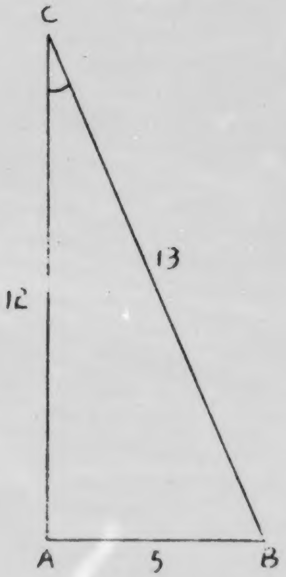
$$\sec \angle A = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}}$$

EXERCISES

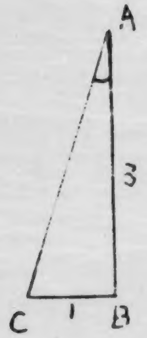
1. Find the values of the six trigonometric functions for ∠C in the triangle below.



2. Find the values of the six trigonometric functions for ∠C in the triangle below.



3. Find the values of the six trigonometric functions for ∠A in the triangle below.



ANSWERS

1. $\sin \angle C = \frac{3}{5} = .6000$

$\cos \angle C = \frac{4}{5} = .8000$

$\tan \angle C = \frac{3}{4} = .7500$

$\cot \angle C = \frac{4}{3} = 1.3333$

$\csc \angle C = \frac{5}{3} = 1.6667$

$\sec \angle C = \frac{5}{4} = 1.2500$

2. $\sin \angle C = \frac{5}{13} = .3846$

$\cos \angle C = \frac{12}{13} = .9231$

$\tan \angle C = \frac{5}{12} = .4167$

$\cot \angle C = \frac{12}{5} = 2.4000$

$\csc \angle C = \frac{13}{5} = 2.6000$

$\sec \angle C = \frac{13}{12} = 1.0833$

3. $\sin \angle A = \frac{1}{\sqrt{10}} = .3162$

$\cos \angle A = \frac{3}{\sqrt{10}} = .9487$

$\tan \angle A = \frac{1}{3} = .3333$

$\cot \angle A = \frac{3}{1} = 3.0000$

$\csc \angle A = \frac{\sqrt{10}}{1} = 3.1623$

$\sec \angle A = \frac{\sqrt{10}}{3} = 1.0541$

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET
(Lead in)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Modern Related Mathematics

TECHNICAL INFORMATION TITLE: Using Trigonometric Tables

INTRODUCTION AND/OR OBJECTIVES:

In the solution for measures of angles in a triangle or the lengths of sides of a triangle, the accurate use of trigonometric tables is a must. Very frequent practice with the use of tables including interpolation to determine values not in the tables will provide the user with confidence in his ability.

TECHNICAL INFORMATION:

For a given angle the value for each of the six trigonometric functions for that angle may be read directly from trigonometric tables or can be found by using a method called interpolation. It is also possible to find the angle if the value for any one of the six trigonometric functions is given. (For the following discussion and examples the student should refer to prepared trigonometric tables of the instructor's choice in which values are listed to the nearest minute for angles from 0° to 90°.)

In these tables the student should check the following readings:

$\sin 15^\circ 22' = .26499$ $\tan 64^\circ 38' = 2.1092$

$\cos 27^\circ 32' = .88674$ $\sec 85^\circ 45' = 13.494$

Now let us look at two examples in which the angle is to be found when a function value is known.

Example 1. If $\cos A = .92421$, find A. By checking the cosine values in the tables, it is found that the angle A is $22^\circ 27'$.

Example 2. If in a right triangle, $\tan B = 1.2131$, then from the tables, $B = 50^\circ30'$.

If angle measurements involve seconds, the table may still be used, but a process called interpolation must be utilized.

Example 3. Find the value of $\sin 37^\circ23'20''$.

First, find the values for $\sin 37^\circ23'$ and $\sin 37^\circ24'$. See below.

Angles Sine Values

$$60''\left[\begin{array}{l} 20''\left[\begin{array}{l}37^\circ23'\\ 37^\circ23'20''\end{array}\right.\\ \quad\;\; 37^\circ24'\end{array}\right. \qquad \sin 37^\circ23'20'' = \left.\begin{array}{r}\left.\begin{array}{r}.60714\\ \underline{\qquad\quad}\end{array}\right]x\\ .60737\end{array}\right]\quad .00023$$

The difference between $37^\circ23'$ and $37^\circ24'$ is $1'$ or $60''$. The difference between $37^\circ23'$ and $37^\circ23'20''$ is $20''$. These differences are written by the brackets as above. The difference between .60714 and .60737 is .00023.

Now, x (the difference between .60714 and the number we are after) divided by .00023 should be in the same ratio as 20 divided by 60. That is:

$$\frac{x}{.00023} = \frac{20}{60}$$

$$\frac{x}{.00023} = \frac{1}{3}$$

$$3x = (.00023)1 \qquad \left(\frac{a}{b} = \frac{c}{d} \text{ implies that } a \cdot d = b \cdot c\right)$$

$$3x = .00023$$

$$\frac{1}{3}3x = \frac{1}{3}(.00023) \qquad \text{(Multiply both sides by } 1/3\text{)}$$

$$x = \frac{.00023}{3} \qquad \left(\text{Multiplicative inverse, } \frac{a}{b}\cdot c = \frac{a \cdot c}{b}\right)$$

$x = .00008$ (to the nearest hundred thousandth)

Therefore:

$$\sin 37^\circ23'20'' = .60714 + .00008$$

$$= .60722$$

Example 4. Find the value of $\cos 27^\circ32'15''$.

Angles — Cosine Values

$$60''\left[\begin{array}{l}15''\left[\begin{array}{l}27^\circ32'\\27^\circ32'15''\end{array}\right.\\ \qquad 27^\circ33'\end{array}\right.\qquad \cos 27^\circ32'15'' = \left.\begin{array}{r}\left.\begin{array}{r}.88674\\ \underline{\qquad\quad}\end{array}\right]x\\ .88661\end{array}\right].00013$$

Therefore:

$$\frac{x}{.00013} = \frac{15}{60}$$

$$\frac{x}{.00013} = \frac{1}{4}$$

$$4x = (.00013)1$$

$$\frac{1}{4}4x = \frac{1}{4}\cdot(.00013)$$

$$x = \frac{.00013}{4}$$

$x = .00003$ (to the nearest hundred thousandth)

Therefore:

$$\cos 27^\circ32'15'' = .88674 - .00003$$

$= .88671$ (Notice that we subtract .00003 since the bottom cosine value is less than the upper cosine value.)

Example 5. Find A if $\tan A = .66030$.

$$\begin{array}{c} \underline{\text{Angle}} \\ 60''\left[\begin{array}{l} x\left[\begin{array}{l} 33^\circ 26' \\ \underline{\qquad\quad} \end{array}\right. = A \\ 33^\circ 27' \end{array}\right. \end{array} \qquad\qquad \begin{array}{c} \underline{\text{Tangent Values}} \\ \left.\begin{array}{l}\left.\begin{array}{l} .66021 \\ .66030 \end{array}\right] .00009 \\ .66063 \end{array}\right] .00042 \end{array}$$

Therefore:

$$\frac{x}{60} = \frac{.00009}{.00042}$$

$$\frac{x}{60}\cdot 60 = \frac{.00009}{.00042}\cdot 60 \qquad \text{(Multiply both sides by 60)}$$

$$x = \frac{(.00009)60}{.00042} \qquad \text{(Multiplicative inverse, } \frac{a}{b}\cdot c = \frac{a\cdot c}{b}\text{)}$$

$$x = \frac{.00540}{.00042}$$

$$x = \frac{540}{42}$$

$$x = \frac{90\cdot 6}{7\cdot 6}$$

$$x = \frac{90}{7}$$

$$x = 13 \qquad \text{(to the nearest whole number)}$$

Therefore:

$$A = 33^\circ 26' + 13''$$

$$= 33^\circ 26' 13''$$

EXERCISES

1. Find tan $12^\circ 15'$
2. Find cos $6^\circ 49'$
3. Find sin $78^\circ 2'$
4. Find sec $38^\circ 16'$
5. Find cot $2^\circ 49'$
6. Find csc $87^\circ 12'$
7. Find sin $31^\circ 12' 15''$
8. Find tan $68^\circ 17' 34''$
9. Find cos $42^\circ 48' 30''$
10. Find tan $17^\circ 32' 45''$

ANSWERS

1. .21712
2. .99293
3. .97827
4. 1.2737
5. 20.325
6. 1.0012
7. .51809
8. 2.5120
9. .73363
10. .31618

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET
(Lead in)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements and Layouts

TECHNICAL INFORMATION TITLE: The Trigonometric Functions for Angles in the First and Second Quadrants

INTRODUCTION AND/OR OBJECTIVES:

In addition to working with acute angles in right triangles, it is many times necessary to work with oblique triangles involving angles which are larger than 90° in measurement. This involves working with angles which are many times not directly listed in the tables. Thus, it is necessary to work with reference angles to determine the values of the trigonometric functions of given angles which are larger than 90°. It is important to fully understand the definitions of the trigonometric functions in terms of the coordinates of a point on the terminal side of an angle.

TECHNICAL INFORMATION:

I. BASIC DEFINITIONS FOR THE SIX TRIGONOMETRIC FUNCTIONS

In an earlier Technical Information Sheet the six trigonometric functions for angles in a right triangle were discussed. The trigonometric functions were defined as follows: (See Figure I)

$$\sin \angle BOA = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{AB}{OA}$$

$$\cos \angle BOA = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{OB}{OA}$$

$$\tan \angle BOA = \frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{AB}{OB}$$

$$\cot \angle BOA = \frac{\text{length of adjacent side}}{\text{length of opposite side}} = \frac{OB}{AB}$$

$$\csc \angle BOA = \frac{\text{length of hypotenuse}}{\text{length of opposite side}} = \frac{OA}{BA}$$

$$\sec \angle BOA = \frac{\text{length of hypotenuse}}{\text{length of adjacent side}} = \frac{OA}{OB}$$

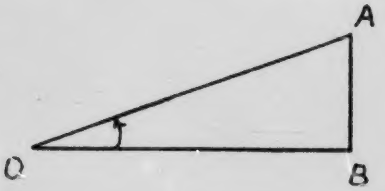


Figure 1

We may also define the trigonometric functions of an angle in another way. But, first, we need to know what it means for an angle to be in "standard position." An angle is in "standard position" if one side of the angle is on the positive x-axis (this side is called the initial side), and if the vertex of the angle is at the origin. The other side of the angle, regardless of its position, is called the terminal side of the angle. Therefore, in Figure 2, $\angle BOA$ is in standard position. $\overline{OB}$ is the initial side of the angle, and $\overline{OA}$ is the terminal side.

We may now define the six trigonometric functions of $\angle BOA$ in terms of the coordinates (x, y) for any point on the terminal side of the angle. Note, that we may choose any point (x, y) on the terminal side to find the trigonometric function values. Assume that we select the point A as this point on the terminal side. Then, the segment $\overline{OA}$ is called the radius vector for point A. The length OA, which is the length of the radius vector, is designated by r. Regardless of the coordinates for the point A, r is always taken as a positive value.

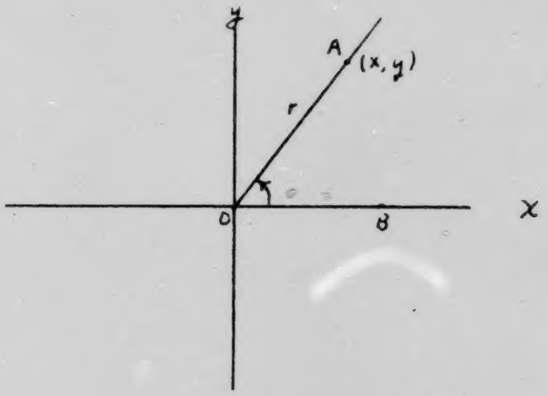


Figure 2

Then, the trigonometric functions are defined as follows:

$\sin \angle BOA = \frac{y}{r}$

$\cos \angle BOA = \frac{x}{r}$

$\tan \angle BOA = \frac{y}{x}$

$\cot \angle BOA = \frac{x}{y}$

$\csc \angle BOA = \frac{r}{y}$

$\sec \angle BOA = \frac{r}{x}$

Note that all of these definitions depend on the coordinates for the point A and r (the length from O to A).

Example 1. Find the six trigonometric function values for angle θ in Figure 3. Point B on the terminal side of the angle has coordinates (3, 4).

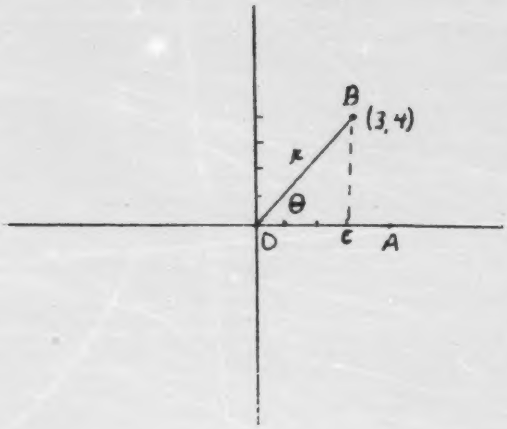


Figure 3

Since point B has coordinates (3, 4) then OC = 3 and CB = 4. Then by the Pythagorean Theorem:

$$r^2 = (OC)^2 + (CB)^2$$

$$r^2 = (3)^2 + (4)^2 \qquad \text{(Note that } r^2 = x^2 + y^2\text{)}$$

$$r^2 = 9 + 16$$

$$r^2 = 25$$

$$r = \sqrt{25}$$

$$r = 5$$

Then:

$$\sin \theta = \frac{y}{r}$$

$$= \frac{4}{5}$$

$= .8000$

$\cos \theta = \frac{x}{r}$

$= \frac{3}{5}$ (Since the x-coordinate for point B is 3 and r = 5)

$= .6000$

$\tan \theta = \frac{y}{x}$

$= \frac{4}{3}$

$= 1.3333$

$\cot \theta = \frac{x}{y}$

$= \frac{3}{4}$

$= .7500$

$\csc \theta = \frac{r}{y}$

$= \frac{5}{4}$

$= 1.2500$

$\sec \theta = \frac{r}{x}$

$= \frac{5}{3}$

$= 1.6667$

Example 2. Find the six trigonometric function values for angle θ as in Figure 4. This angle is an example of an angle in quadrant 2. The quadrants are numbered as indicated in Figure 4.

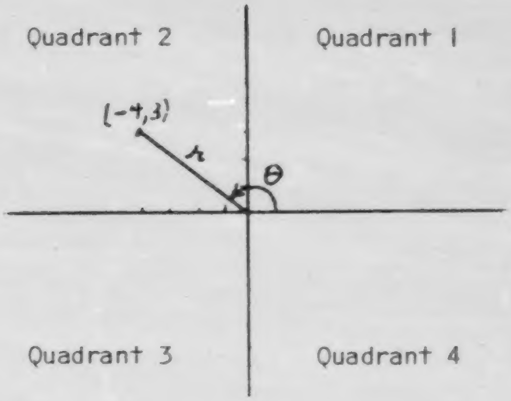

Quadrant 2
Quadrant 1
(-4,3)
r
θ
Quadrant 3
Quadrant 4

Figure 4

By the Pythagorean Theorem:

$$r^2 = (-4)^2 + (3)^2$$

$$r^2 = 16 + 9$$

$$r^2 = 25$$

$$r = \sqrt{25}$$

$$r = 5$$

Then:

$$\sin\theta = \frac{y}{r} = \frac{3}{5} = .6000$$

$$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5} = -.8000$$

$$\tan\theta = \frac{y}{x} = \frac{3}{-4} = -\frac{3}{4} = -.7500$$

$$\cot\theta = \frac{x}{y} = \frac{-4}{3} = -\frac{4}{3} = -1.3333$$

$$\csc\theta = \frac{r}{y} = \frac{5}{3} = 1.6667$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-4} = -\frac{5}{4} = -1.2500$$

Example 3. Find the six trigonometric function values for angle θ in Figure 5.

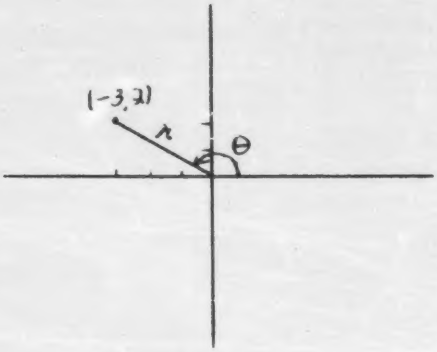


Figure 5

From the Pythagorean Theorem:

$$r^2 = x^2 + y^2$$

$$r^2 = (-3)^2 + (2)^2$$

$$r^2 = 9 + 4$$

$$r^2 = 13$$

$$r = \sqrt{13}$$

$r = 3.6056$ (From the tables)

Then:

$$\sin \theta = \frac{y}{r}$$

$$= \frac{2}{\sqrt{13}}$$

$$= \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{2\sqrt{13}}{13}$$

$$= \frac{2(3.6056)}{13}$$

$$= \frac{7.2112}{13}$$

$$= .5547$$

$$\cos\theta = \frac{x}{r}$$

$$= \frac{-3}{\sqrt{13}}$$

$$= \frac{-3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{-3\cdot\sqrt{13}}{13}$$

$$= \frac{-3(3.6056)}{13}$$

$$= \frac{-10.8168}{13}$$

$$= -\frac{10.8168}{13}$$

$$= -.8321$$

$$\tan\theta = \frac{y}{x}$$

$$= \frac{2}{-3}$$

$$= -\frac{2}{3}$$

$$= -.6667$$

$$\cot\theta = \frac{x}{y}$$

$$= \frac{-3}{2}$$

$$= -\frac{3}{2}$$

$$= -1.50000$$

$$\csc \theta = \frac{r}{y}$$

$$= \frac{\sqrt{13}}{2}$$

$$= \frac{3.6056}{2}$$

$$= 1.8028$$

$$\sec \theta = \frac{r}{x}$$

$$= \frac{13}{-3}$$

$$= -\frac{\sqrt{13}}{3}$$

$$= -\frac{3.6056}{3}$$

$$= -1.2019$$

Notice that in the examples above, in the second quadrant, sin θ and csc θ are positive, and cos θ, sec θ, tan θ, and cot θ are negative.

II. REFERENCE ANGLES

In Figure 6 note that sin θ = 3/5. Label ∠BOC as Ø. Notice that $m\,Ø = 180^\circ - m\,\theta$. Now assume that Ø would be placed in standard position as in Figure 7.

A point on the terminal side of Ø would then be (4, 3). Notice that sin Ø = 3/5. Then: sin θ = sin Ø.

For the cosine function, $\cos \theta = -4/5$ and $\cos \emptyset = 4/5$. Then: $\cos \theta = -(\cos \emptyset)$.

For the tangent function, $\tan \theta = -3/4$ and $\tan \emptyset = 3/4$. Then: $\tan \theta = -(\tan \emptyset)$.

For the cotangent function, $\cot \theta = -4/3$ and $\cot \emptyset = 4/3$. Then: $\cot \theta = -(\cot \emptyset)$.

For the cosecant function, $\csc \theta = 5/3$ and $\csc \emptyset = 5/3$. Then: $\csc \theta = \csc \emptyset$.

For the secant function, $\sec \theta = -5/4$ and $\sec \emptyset = 5/4$. Then: $\sec \theta = -(\sec \emptyset)$.

Now, to what use can we make of the above conclusions? Suppose that we wish to use the tables to find trigonometric function values for an angle between 90° and 180°, but the tables do not go above 90°. If we use the above results, we can solve the problem.

Example 4. Find the trigonometric function values for 130°.

First, find the reference angle $\emptyset$.

$\emptyset = 180^\circ - \theta$

$\emptyset = 180^\circ - 130^\circ$

$\emptyset = 50^\circ$

Then:

$\sin \theta = \sin \emptyset$

$\sin 130^\circ = \sin 50^\circ$

$= .76604$ (the table value for $\sin 50^\circ$)

$\cos \theta = -(\cos \emptyset)$

$\cos 130^\circ = -(\cos 50^\circ)$

$-.64279$

$\tan \theta = -(\tan \emptyset)$

$\tan 130^\circ = -(\tan 50^\circ)$

$= -1.1917$

$\cot \theta = -(\cot \emptyset)$

$\cot 130^\circ = -(\cot 50^\circ)$

$= -.83910$

$\csc \theta = \csc \emptyset$

$\csc 130^\circ = \csc 50^\circ$

$= 1.3054$

$\sec \theta = -(\sec \emptyset)$

$\sec 130^\circ = -(\sec 50^\circ)$

$= -1.5557$

Example 5. Find the six trigonometric function values for 140°.

First, find the reference angle Ø.

$\emptyset = 180^\circ - \theta$

$= 180^\circ - 140^\circ$

$= 40^\circ$

Then:

$\sin \theta = \sin \emptyset$

$\sin 140^\circ = \sin 40^\circ$

$= .64279$

$\cos \theta = -(\cos \emptyset)$

$\cos 140^\circ = -(\cos 40^\circ)$

$= -.76604$

$\tan \theta = -(\tan \phi)$

$\tan 140^{\circ} = -(\tan 40^{\circ})$

$= -.83910$

$\cot \theta = -(\cot \phi)$

$\cot 140^{\circ} = -(\cot 40^{\circ})$

$= -1.1917$

$\csc \theta = \csc \phi$

$\csc 140^{\circ} = \csc 40^{\circ}$

$= 1.5557$

$\sec \theta = -(\sec \phi)$

$\sec 140^{\circ} = -(\sec 40^{\circ})$

$= -1.3054$

Example 6. If $\cos \theta = -.5000$ and θ is in the second quadrant, find the value for θ.

First of all, find the reference angle ϕ such that $\cos \phi = .5000$.

From the tables:

If $\cos \phi = .5000$

then: $\phi = 60^{\circ}$

Then, note from Figure 6 that:

$\theta = 180^{\circ} - \phi$

$= 180^{\circ} - 60^{\circ}$

$= 120^{\circ}$

Therefore, if $\cos \theta = -.5000$, then $\theta = 120^{\circ}$.

Example 7. If $\tan \theta = -1.0000$ and θ is in the second quadrant, find θ.

Find the reference angle ϕ such that $\tan \phi = 1.0000$.

From the tables:

If $\tan \phi = 1.0000$

then: $\phi = 45^\circ$

Then:

$$\begin{aligned} \theta &= 180^\circ - \phi \\ &= 180^\circ - 45^\circ \\ &= 135^\circ \end{aligned}$$

Therefore, if $\tan \theta = -1.0000$, then $\theta = 135^\circ$.

Example 8. If $\sin \theta = .50000$ and θ is in the second quadrant, find θ.

Find the reference angle ϕ such that $\sin \phi = .50000$.

From the tables:

if $\sin \phi = .50000$

then: $\phi = 30^\circ$

Then:

$$\begin{aligned} \theta &= 180^\circ - 30^\circ \\ &= 150^\circ \end{aligned}$$

Therefore, if $\sin \theta = .50000$, then $\theta = 150^\circ$.

III. CONCLUDING REMARKS

The discussion in section II. and the resulting examples are for angles in the second quadrant. The results that $\sin \theta = \sin \phi$, $\cos \theta = -(\cos \phi)$, $\tan \theta = -(\tan \phi)$, $\cot \theta = -(\cot \phi)$, $\csc \theta = \csc \phi$, and $\sec \theta = -(\sec \phi)$ are true for an angle θ in the second quadrant. Similar to our procedure in arriving at these results, we could investigate angles in the third and the fourth quadrants. We would find that the results would be slightly different (the negative signs will be changed on various functions).

There are many different results that may be obtained for various angles in trigonometry. For example, it is true that for any angle θ:

$\sin(-\theta) = -(\sin\theta)$

$\cos(-\theta) = \cos\theta$

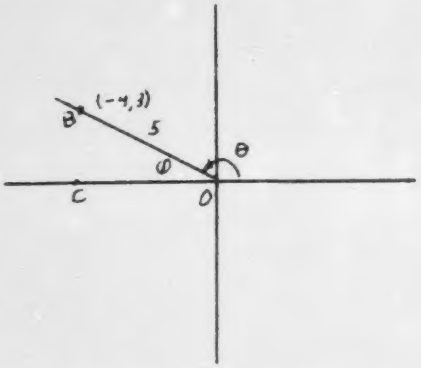
$\tan(-\theta) = -(\tan\theta)$

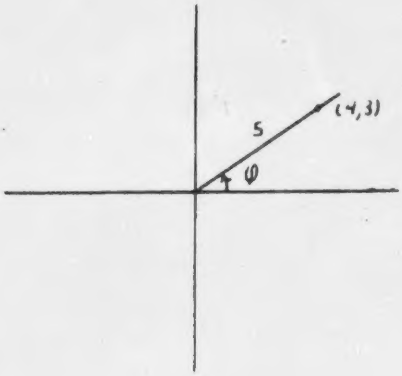
$\cot(-\theta) = -(\cot\theta)$

$\csc(-\theta) = -(\csc\theta)$

$\sec(-\theta) = \sec\theta$

If the student is interested in further study in trigonometry, he should consult a trigonometry textbook.





EXERCISES

1. Find the six trigonometric function values for θ with a point on the terminal side being:
   a. (-3, 4)
   b. (-2, 3)
   c. (-1, 1)

2. Find the six trigonometric function values for the following angles:
   a. 120°
   b. 135°
   c. 150°
   d. 170°

3. Find θ if:
   a. $\tan \theta = 1.0000$ (θ in the first quadrant)
   b. $\tan \theta = -.46631$ (θ in the second quadrant)
   c. $\sin \theta = .51504$ (θ in the second quadrant)
   d. $\cos \theta = -.95106$ (θ in the second quadrant)
   e. $\sec \theta = -1.7883$ (θ in the second quadrant)
   f. $\csc \theta = 1.1126$ (θ in the second quadrant)
   g. $\csc \theta = 1.1126$ (θ in the first quadrant)

ANSWERS

1.

	sin θ	cos θ	tan θ	cot θ	csc θ	sec θ
a.	.80000	-.60000	-1.3333	-.75000	1.2500	-1.6667
b.	.8321	-.5547	-1.5000	-.6667	1.2019	-1.8028
c.	.7071	-.7071	-1.0000	-1.0000	1.4142	-1.4142

2.

	sin θ	cos θ	tan θ	cot θ	csc θ	sec θ
a.	.86603	-.50000	-1.7320	-.57735	1.1547	-2.0000
b.	.70711	-.70711	-1.0000	-1.0000	1.4142	-1.4142
c.	.50000	-.86603	-.57735	-1.7320	2.0000	-1.1547
d.	.17365	-.98481	-.17633	-5.6713	5.7588	-1.0154

3. a. 45°

b. 155°

c. 149°

d. 162°

e. 124°

f. 116°

g. 64°

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements and Layouts

TECHNICAL INFORMATION TITLE: Solutions Involving Oblique Triangles

INTRODUCTION:
Many practical problems deal with angles which involve oblique triangles. In order to be able to measure angles and various dimensions of oblique triangles, it is essential to understand various special trigonometric laws involving oblique triangles.

OBJECTIVE:
To provide the student an opportunity to learn the principles of oblique triangles and their use in terms of measurements and layouts relating to the machine trades.

TECHNICAL INFORMATION:

An oblique triangle is a triangle in which there is no right angle. Therefore, we must resort to special trigonometric formulas to work with this type of triangle. From geometry we can classify oblique triangle problems into four categories according to the given information.

These four types are as follows: (what is listed is the given information)

1. Two sides and the angle opposite one of them
2. Three sides
3. Two sides and the included angle
4. Two angles and one side

I. THE LAW OF SINES

The first method by which we can work with oblique triangles is called

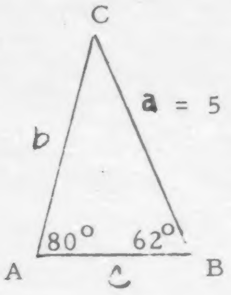
"The Law of Sines." This states that in any triangle ABC

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

Example 1. One side and two angles (the 4th category above)
Given: $a = 5$, $m\angle A = 80^\circ$, $m\angle B = 62^\circ$
Find: b, c, $m\angle C$

Since from geometry the sum of the measures of all three angles in any triangle is 180°, then:

$$\begin{aligned} m\angle C &= 180^\circ - (m\angle A + m\angle B) \\ &= 180^\circ - (80^\circ + 62^\circ) \\ &= 180^\circ - 142^\circ \\ &= 38^\circ \end{aligned}$$



Now, to find b:

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B}$$

$$\frac{a}{\sin\angle A}\cdot\sin\angle B = \frac{b}{\sin\angle B}\cdot\sin\angle B \qquad \text{(Multiply both sides by } \sin\angle B\text{)}$$

$$\frac{a(\sin\angle B)}{\sin\angle A} = b \qquad \text{(Multiplicative inverse, } \frac{a}{b}\cdot c = \frac{a\cdot c}{b}\text{)}$$

or $$b = \frac{a(\sin\angle B)}{\sin\angle A}$$

Therefore, in our problem:

$$b = \frac{5(\sin 62^\circ)}{\sin 80^\circ}$$

$$= \frac{5(.88295)}{.98481}$$

$$= \frac{4.4148}{.98481}$$

$b = 4.4828$ or 4.483 (to the nearest thousandth)

Now, to find c:

$$\frac{a}{\sin\angle A} = \frac{c}{\sin\angle C}$$

Similar to the previous procedure we can find that:

$$c = \frac{a(\sin\angle C)}{\sin\angle A}$$

In our problem:

$$c = \frac{5(\sin 38^\circ)}{\sin 80^\circ}$$

$$= \frac{5(.61566)}{.98481}$$

$$= \frac{3.07830}{.98481}$$

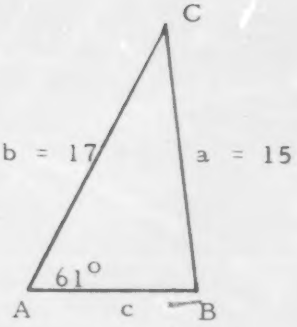
$c = 3.1258$ or 3.126 (to the nearest thousandth)

Example 2. Two sides and the angle opposite one of the sides (category I above)
Given: $a = 15$, $b = 17$, $m\angle A = 61^\circ$
Find: c, $m\angle B$, $m\angle C$

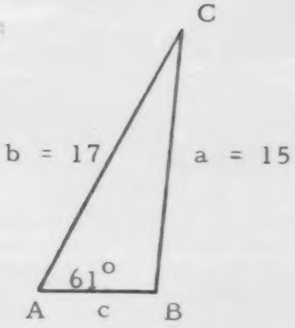
Note in the figure at the top of the following page that there are two possible triangles.

Now, to find $m\angle B$:

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B}$$



Triangle 1



Triangle 2

$a(\sin\angle B) = b(\sin\angle A)$ $\left(\frac{a}{b} = \frac{c}{d}\right.$ implies that $\left.a \cdot d = b \cdot c\right)$

$\frac{1}{a} \cdot a(\sin\angle B) = \frac{1}{a} \cdot b(\sin\angle A)$ (Multiply both sides by 1/a)

$\sin\angle B = \frac{b(\sin\angle A)}{a}$ (Multiplicative inverse, $\frac{a}{b} \cdot c = \frac{a \cdot c}{b}$)

Therefore, in our problem:

$$\sin\angle B = \frac{17(\sin 61^\circ)}{15}$$

$$= \frac{17(.87462)}{15}$$

$$= \frac{14.86854}{15}$$

$$\sin\angle B = .99124$$

Then, from the tables:

$m\angle B = 82^\circ 25'$ (to the nearest minute) in the first quadrant (for triangle 1)

$m\angle B = 180^\circ - 82^\circ 25'$ in the second quadrant (for triangle 2)

$= 179^\circ 60' - 82^\circ 25'$

$= 97^\circ 35'$ (for triangle 2)

Now, to find $m\angle C$:

For triangle 1:

$$\begin{aligned} m\angle C &= 180^\circ - (m\angle A + m\angle B) \\ &= 180^\circ - (61^\circ + 82^\circ 25') \\ &= 180^\circ - 143^\circ 25' \\ &= 179^\circ 60' - 143^\circ 25' \\ &= 36^\circ 35' \end{aligned}$$

For triangle 2:

$$\begin{aligned} m\angle C &= 180^\circ - (m\angle A + m\angle B) \\ &= 180^\circ - (61^\circ + 97^\circ 35') \\ &= 180^\circ - 158^\circ 35' \\ &= 179^\circ 60' - 158^\circ 35' \\ &= 21^\circ 25' \end{aligned}$$

Now, to find c:

As in the first example:

$$c = \frac{a(\sin\angle C)}{\sin\angle A}$$

For triangle 1:

$$\begin{aligned} c &= \frac{15(\sin 36^\circ 35')}{\sin 61^\circ} \\ &= \frac{15(.59599)}{.87462} \\ &= \frac{8.93985}{.87462} \\ &= 10.221 \quad \text{(to the nearest thousandth)} \end{aligned}$$

For triangle 2:

$$\begin{aligned} c &= \frac{15(\sin 21^\circ 25')}{\sin 61^\circ} \\ &= \frac{15(.36515)}{.87462} \end{aligned}$$

$$= \frac{5.47725}{.87462}$$

$$= 6.262 \quad \text{(to the nearest thousandth)}$$

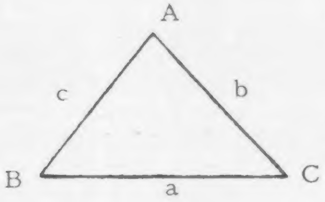
II. THE LAW OF COSINES

This Law states that in any triangle ABC:

$$a^2 = b^2 + c^2 - 2bc(\cos \angle A)$$

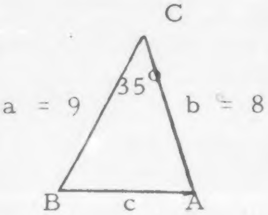
$$b^2 = a^2 + c^2 - 2ac(\cos \angle B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos \angle C)$$



Example 3. Two sides and the angle between them (category 3 above).
Given: $m\angle C = 35^\circ$, $a = 9$, $b = 8$
Find: $m\angle A$, $m\angle B$, c

To find c:



$$c^2 = a^2 + b^2 - 2ab(\cos \angle C)$$

$$= 9^2 + 8^2 - 2 \cdot 9 \cdot 8(\cos 35^\circ)$$

$$= 81 + 64 - 144(.81915)$$

$$= 145 - 117.9576$$

$$c^2 = 27.0424 \text{ or } 27.04 \text{ (to the nearest hundredth)}$$

Then: $c = 5.2$

To find $m\angle A$:

$$a^2 = b^2 + c^2 - 2bc(\cos\angle A)$$

$$a^2 - (b^2 + c^2) = (b^2 + c^2) - 2bc(\cos\angle A) - (b^2 + c^2)$$

(Subtract $b^2 + c^2$ from both sides)

$$a^2 - (b^2 + c^2) = -2bc(\cos\angle A) \qquad \text{(Additive inverse)}$$

$$\frac{1}{-2bc}\cdot(a^2 - (b^2 + c^2)) = \frac{1}{-2bc}\cdot -2bc(\cos\angle A) \qquad \text{(Multiply both sides by } 1/-2bc\text{)}$$

$$\frac{a^2 - (b^2 + c^2)}{-2bc} = \cos\angle A \qquad \left(\frac{a}{b}\cdot c = \frac{a\cdot c}{b}, \text{ multiplicative inverse}\right)$$

or $$\cos\angle A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Therefore, in our problem:

$$\cos\angle A = \frac{9^2 - (8^2 + (5.2)^2)}{-2\cdot 8\cdot (5.2)}$$

$$= \frac{81 - (64 + 27.04)}{-83.2}$$

$$= \frac{81 - 91.04}{-83.2}$$

$$= \frac{-10.04}{-83.2}$$

$$\cos\angle A = .12067$$

Then from the tables: $m\angle A = 83^\circ 4'$ (to the nearest minute)

Now, to find $m\angle B$:

$$m\angle B = 180^\circ - (m\angle A + m\angle C)$$

$$= 180^\circ - (83^\circ 4' + 35^\circ)$$

$$= 180^\circ - 118^\circ 4'$$

$$= 179^\circ 60' - 118^\circ 4'$$

$$= 61^\circ 56'$$

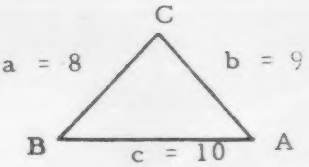
Example 4. All three sides given (category 2 above).
Given: $a = 8$, $b = 9$, $c = 10$
Find: $m\angle A$, $m\angle B$, $m\angle C$

To find $m\angle A$:

In example 3 above it was found that:

$$\cos\angle A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Therefore in our problem:



$$\cos\angle A = \frac{8^2 - (9^2 + 10^2)}{-2\cdot 9\cdot 10}$$

$$= \frac{64 - (81 + 100)}{-180}$$

$$= \frac{64 - 181}{-180}$$

$$= \frac{-117}{-180}$$

$$\cos\angle A = .65000$$

Then from the tables: $m\angle A = 49^\circ 28'$ (to the nearest minute)

To find $m\angle B$:

Similar to the procedure for deriving the formula for the $\cos\angle A$, we can find that:

$$\cos\angle B = \frac{b^2 - (a^2 + c^2)}{-2ac}$$

$$= \frac{9^2 - (8^2 + 10^2)}{-2\cdot 8\cdot 10}$$

$$= \frac{81 - (64 + 100)}{-160}$$

$$= \frac{-83}{-160}$$

$$\cos\angle B = .51875$$

Then from the tables: $m\angle B = 58^\circ 45'$ (to the nearest minute)

[redacted]

To find $m\angle C$:

$$
\begin{aligned}
m\angle C &= 180^\circ - (m\angle A + m\angle B) \\
&= 180^\circ - (49^\circ 28' + 58^\circ 45') \\
&= 180^\circ - 108^\circ 13' \\
&= 179^\circ 60' - 108^\circ 13' \\
&= 71^\circ 47'
\end{aligned}
$$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Measurements and Layouts

TECHNICAL ASSIGNMENT TITLE: Solution of Oblique Triangles

INTRODUCTION:

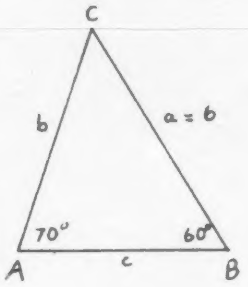
In some cases it is necessary to find the values of angles or sides of a triangle in which there is no 90° angle. A triangle in which there is no 90° (right) angle is called an oblique triangle.

OBJECTIVE:

To learn how to solve for measurements of sides and angles in oblique triangles.

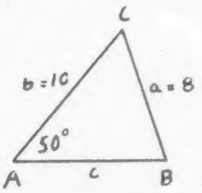
ASSIGNMENT:

1. Find:
   - a. b
   - b. c
   - c. $m\angle C$

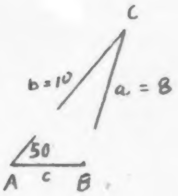


2. Find:
   - a. c
   - b. $m\angle B$
   - c. $m\angle C$

Note that there are two possible triangles.



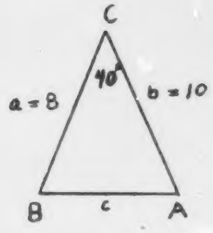
Triangle 1



Triangle 2

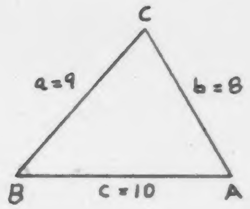
3. Find:

   a. c
   b. m∠A
   c. m∠B


C
40°
a = 8
b = 10
B
c
A

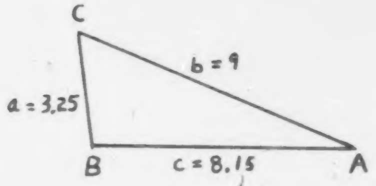
4. Find:

   a. m∠A
   b. m∠B
   c. m∠C


C
a = 9
b = 8
B
c = 10
A

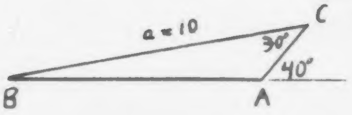
5. Find:

   a. m∠A
   b. m∠B
   c. m∠C


C
b = 9
a = 3.25
B
c = 8.15
A

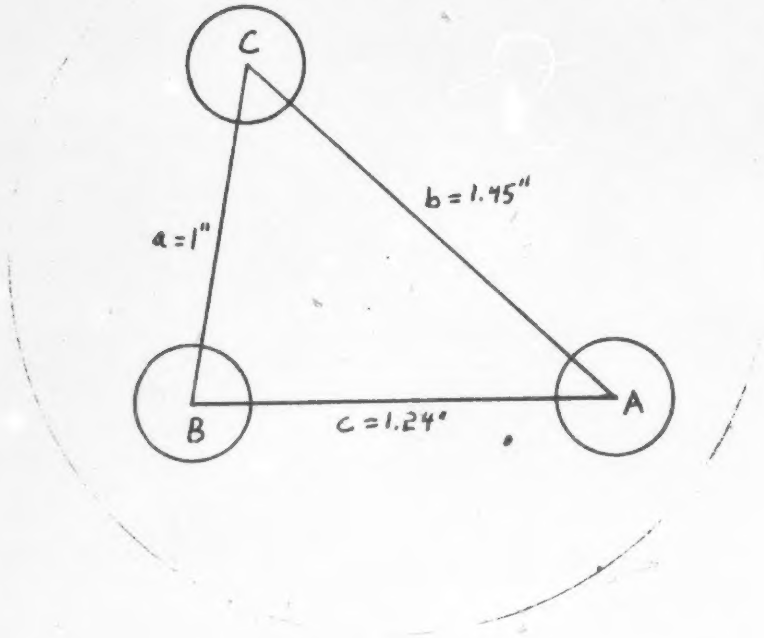
6. Find:

   a. m∠B
   b. b
   c. c


a = 10
C
30°
40°
B
A

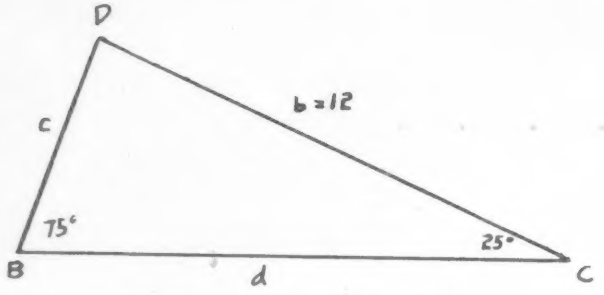
7. Find:

a. m A
b. m B
c. m C


C
a = 1"
b = 1.45"
B
c = 1.24"
A

8. Find:

a. m D
b. c
c. d


D
c
b = 12
75°
25°
B
d
C

ANSWERS

1.

a. $b = 5.530$
b. $c = 4.891$
c. $m\angle C = 50^\circ$

2.

	Triangle 1	Triangle 2
a.	$c = 8.734$	$c = 4.122$
b.	$m\angle B = 73^\circ 15'$	$m\angle B = 106^\circ 45'$
c.	$m\angle C = 56^\circ 45'$	$m\angle C = 23^\circ 15'$

3.

a. $c = 6.44$
b. $m\angle A = 53^\circ 3'$
c. $m\angle B = 86^\circ 57'$

4.

a. $m\angle A = 58^\circ 45'$
b. $m\angle B = 49^\circ 28'$
c. $m\angle C = 71^\circ 47'$

5.

a. $m\angle A = 21^\circ 6'$
b. $m\angle B = 94^\circ 21'$
c. $m\angle C = 64^\circ 33'$

6.

a. $m\angle B = 10^\circ$
b. $b = 2.702$
c. $c = 7.779$

7.

a. $m\angle A = 42^\circ 43'$
b. $m\angle B = 79^\circ 54'$
c. $m\angle C = 57^\circ 23'$

8.

a. $m\angle D = 80^\circ$
b. $c = 5.250$
c. $d = 12.235$

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machine

TECHNICAL INFORMATION TITLE: Calculations--Helix Angle and Lead for a Helical or Spiral Cut

INTRODUCTION:
In order to cut a spiral, other than with an end mill, it is necessary to set the table for the cutter to a certain angle measure.

OBJECTIVE:
To provide the student an opportunity to learn how to calculate the measure of the helix angle or the lead of the helix or spiral.

TECHNICAL INFORMATION:

In Figure I below L is called the lead of the helix or spiral. In Figure 2 a right triangle has been constructed such that one side has a length of L (the lead), and the other side has a length of C, where C is the circumference of the given cylinder containing the helix or spiral. We then define $\angle A$ to be the helix or spiral angle.

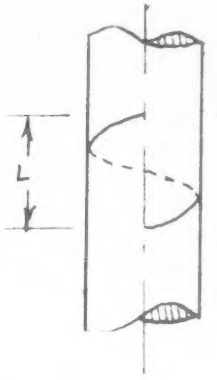

Figure I

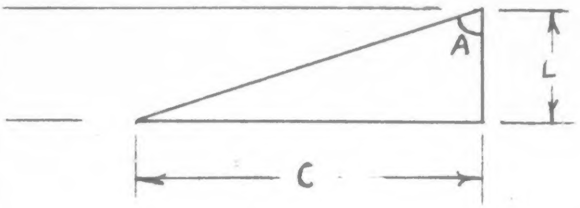

Figure 2

Thus:

$$\tan \angle A = \frac{C}{L}$$

or $\tan \angle A = \frac{\pi d}{L}$ (since $C = \pi d$, where d is the diameter of the cylinder)

APPLICATION OF THE RULE:

Example 1. Find the measure of the angle of the helix or spiral if the diameter of the blank is 2 inches and the lead is 20 inches.

$$\begin{aligned}\tan \angle A &= \frac{\pi d}{L}\\ &= \frac{(3.1416)2}{20}\\ &= \frac{(3.1416)2}{10 \cdot 2}\\ &= \frac{3.1416}{10}\\ &= .31416\end{aligned}$$

From the trig. tables:

$m \angle A = 17^\circ 26'$ (to the nearest minute)

Example 2. Find the lead if the circumference is 8 inches and the measure for the helix is 20°.

We need to solve the formula

$\tan \angle A = \frac{C}{L}$ for L.

$$\tan \angle A = \frac{C}{L}$$

$L(\tan \angle A) = L \cdot \frac{C}{L}$ (Multiply both sides by L)

$L(\tan \angle A) = C$ (Mult. inverse)

$L(\tan \angle A) \cdot \frac{1}{\tan \angle A} = C \cdot \frac{1}{\tan \angle A}$ (Mult. both sides by $1/\tan \angle A$)

$L = C \cdot \frac{1}{\tan \angle A}$ (Mult. inverse)

$$L = \frac{C}{\tan \angle A} \qquad \left(a \cdot \frac{b}{c} = \frac{a \cdot b}{c}\right)$$

Therefore, in Example 2.

$$L = \frac{C}{\tan \angle A}$$

$$= \frac{8}{\tan 20^\circ}$$

$$= \frac{8}{.36397}$$

$$= 21.98 \text{ in.}$$

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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Milling Machine

TECHNICAL ASSIGNMENT TITLE: Calculations--Helix Angle and/or Lead for a Helical or Spiral Cut

INTRODUCTION:

In order to mill a spiral using a universal mill, it is necessary to calculate the measure of the angle of the helix and also the lead.

OBJECTIVE:

To learn how to calculate the measure of the helix angle and the lead.

ASSIGNMENT:

1. Find the measure of the angle of the helix or spiral if the diameter of the blank is 3.500 in. and the lead is 15 in.

2. Find the lead if the circumference is 5.5000 in. and the measure of the angle is $18^\circ 20'$.

3. Find the measure of the angle of the helix or spiral if the diameter of the blank is 3.850 in. and the lead is 18 in.

4. Find the lead if the circumference is 7.750 in. and the helix angle is $15^\circ 40'$.

5. Find the measure of the spiral angle if the diameter is 1 1/4 in. and the lead of the spiral is 9.52 in.

6. Find the measure of the spiral angle of a spiral with a lead of 4.17 in. on work which has a diameter of 3/4 in.

7. What is the measure of the spiral angle on work 2 in. in diameter if the lead equals 12.00 in.?

8. Find the lead of a spiral if the diameter of the work is 1 1/4 in. and the measure of the spiral angle is $20^\circ 53'$.

9. Find the measure of the spiral angle if the lead is 26.57 in. and the diameter is 4 in.

10. Find the measure of the spiral angle if the lead is 3.70 in. and the diameter is 3/8 in.

ANSWERS

1. $36^{\circ}15'$ (to the nearest minute)
2. 16.60 in.
3. $33^{\circ}54'$ (to the nearest minute)
4. 27.63 in.
5. $22^{\circ}25'$ (to the nearest minute)
6. $29^{\circ}28'$ (to the nearest minute)
7. $27^{\circ}38'$ (to the nearest minute)
8. 10.29 in.
9. $25^{\circ}19'$ (to the nearest minute)
10. $17^{\circ}40'$ (to the nearest minute)

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TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Horizontal Milling Machine

TECHNICAL INFORMATION TITLE: Calculations--Gears for Spiral and Helical Milling

INTRODUCTION:

Often gears must be adapted and combined to formulate a revolving movement of the lead screw at a ratio which will be proper to mill a specified spiral.

OBJECTIVE:

To provide the student an opportunity to learn how to calculate the proper gears to be used for spiral and helical milling.

TECHNICAL INFORMATION:

The lead of the milling machine can be found by placing gears that have the same number of teeth on the feed screws and the worm shaft. The gears can be connected with idlers. Forty turns of the feed screw will turn the worm shaft forty times, which will revolve the index head forty times. The table will move ten inches. This is known as the lead of the machine. That is, the lead of the machine is 10.

The ratio may be expressed as follows:

$$\frac{\text{Lead of machine}}{\text{Lead of spiral desired}} = \frac{\text{Driving gears}}{\text{Driven gears}}$$

[redacted]

APPLICATION OF THE RULE:

Problem. Determine possible gears to use for a spiral with a lead of 12 inches. (The lead of the machine is 10 inches.)

$$\frac{\text{Lead of Machine}}{\text{Lead of spiral desired}} = \frac{\text{Driving gears}}{\text{Driven gears}}$$

$$\frac{10}{12} = \frac{\text{Driving gears}}{\text{Driven gears}}$$

Now, we find two fractions whose product is $\frac{10}{12}$.

For example, we may use 5/4 and 2/3 since:

$$\frac{5}{4} \times \frac{2}{3} = \frac{10}{12}$$

Next, multiply the numerator and the denominator of each fraction by a number such that the product will give numbers representing available gears.

For example:

$$\frac{5}{4} \times \frac{8}{8} = \frac{40}{32} \text{ and } \frac{2}{3} \times \frac{24}{24} = \frac{48}{72}$$

Now we have:

$$\frac{10}{12} = \frac{(5 \times 8) \times (2 \times 24)}{(4 \times 8) \times (3 \times 24)} = \frac{40 \times 48 \text{ (for driving gears)}}{32 \times 72 \text{ (for driven gears)}}$$

Therefore, we use:

Driving gears of 40 and 48 teeth.

Driven gears of 32 and 72 teeth.

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TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Horizontal Milling Machine

TECHNICAL ASSIGNMENT TITLE: Calculations--Gears for Spiral and Helical Milling

INTRODUCTION:

Specific gears are necessary to adapt a dividing head and the milling table with the proper ratio for a specified spiral.

OBJECTIVE:

To learn how to solve problems involving gear ratios.

ASSIGNMENT:

1. Find the possible gears to be used for a spiral with a 14 inch lead. Assume that the lead of the machine is 10.

2. Calculate the change gears required to cut a spiral with a lead of 16 inches. The lead of the machine is 10.

3. Gear up a milling machine to cut a spiral with a lead of 5 inches. The lead of the machine is 10.

4. Find the change gears required to cut a spiral with a lead of 9.6 in. The lead of the machine is 10.

5. Find the change gears required to cut a spiral with a lead of 11.2 in. The lead of the machine is 10.

6. If a worm is double threaded and rotates at 80 revolutions per minute, what is the speed of the worm gear if it has 40 teeth?

7. A speed reduction of 60 to 1 is required between a worm (helical) and worm gear. If the worm is triple threaded, how many teeth would the worm gear have?

ANSWERS

1. Driving gears: 50, 30
   Driven gears: 70, 30

2. Driving gears: 40, 24
   Driven gears: 24, 64

3. Driving gears: 56, 24
   Driven gears: 24, 28

4. Driving gears: 100, 30
   Driven gears: 40, 72

5. Driving gears: 50, 40
   Driven gears: 40, 56

6. 4 revolutions per minute

7. 180 teeth

MODERN MATHEMATICS
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OPERATION SHEET

OCCUPATIONAL AREA: Machine Trades

OPERATION: Setting Up and Cutting a Helical Gear

COURSE UNIT TITLE: Milling Machine--Cutting a Helical Gear

INTRODUCTION:

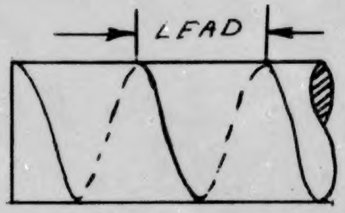
If a line is wound around a cylinder in a particular manner it is called a helix. The helical curve on a screw thread makes a definite number of complete turns within a specified distance along the axis of the thread. The teeth of a helical gear involve only a small section of a complete turn. The lead of a spiral is the distance advanced in one revolution measured parallel with its axis.

In milling a helical gear a combination of factors must be considered: (a) the gear specifications, (b) the setting of the index head, (c) the cutter, (d) the angle for the table setting, (e) the proper gears for the table train, and (f) the connection of the table train with the indexing head for revolution as the table is fed toward the cutter.

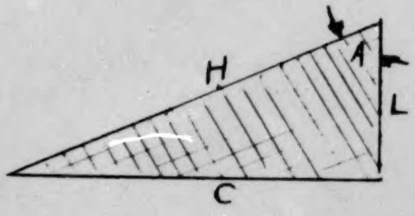
OBJECTIVE:

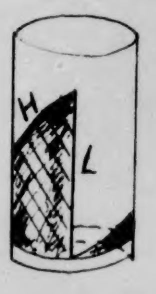
To provide the student practice in cutting a helical gear.

DRAWING:



C = Cir. of Cylinder
L = Lead of Spiral
H = Hypothenuse
A = Angle of Spiral
$\frac{C}{L}$ = Tan of ∠A





TOOLS AND MATERIALS REQUIRED:

Dial indicator
Mandrel
Gear tooth vernier
Dog
Miscellaneous wrenches
Gear blank

PROCEDURE:

(Operations)	(Related Information)
1. Press gear blank on mandrel.	1. Use arbor press.
2. Set up between index head and tailstock centers.	2. Use dog. Mount arbor between in such a way that the cutting action of the gear cutter will tend to tighten the blank on the mandrel.
3. Set up dividing head (index head) for proper divisions.	3. Refer to Technical Information Sheet on Indexing.
4. Put proper gears on table gear train.	4. Refer to Technical Information Sheet.
5. Set table to proper angle. Determine whether left or right hand.	5. Calculate helix angle. Refer to Technical Information Sheet.
6. Connect table train and index head.	6. Be sure to check by using hand lever before engaging automatic power feed.
7. Center cutter with work.	7. Touch top of gear blank and align.
8. Take first rough cut.	8. Allow approximately .015 in. for finishing cut.
9. Repeat cuts.	9. Complete cycle.
10. Set final depth and cut.	10. Repeat cycle.
11. Check.	11. Use gear tooth verniers.

MODERN MATHEMATICS
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TECHNICAL INFORMATION SHEET
(Lead In)

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Milling Machine

TECHNICAL INFORMATION TITLE: Coordinate System in Three Dimensions

INTRODUCTION:

Since most machine movements are generally at right angles to each other, and it is required that movements be calculated from some fixed reference, the logical thing to do is to utilize the relatively familiar Cartesian coordinate system.

The system provides a convenient means of locating points from two to three fixed lines, or planes through these lines, which are positioned at right angles to each other. Each line is called an axis.

OBJECTIVE:

This section should provide the student an opportunity to learn how to locate points in a three-dimensional coordinate system.

TECHNICAL INFORMATION:

Work in two dimensions with two axes is described in an earlier Technical Information Sheet. The student should review this information before proceding.

In work with three dimensions, a third axis, the z-axis, is utilized along with the x-axis and the y-axis. It is possible to establish the axes in various positions and to take either direction as the positive or negative direction. For example we could set up the axes as in Figure 1, Figure 2, or in Figure 3.

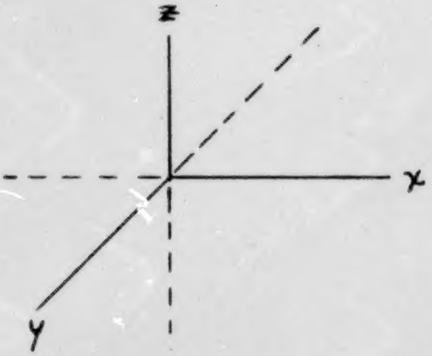


Figure 1

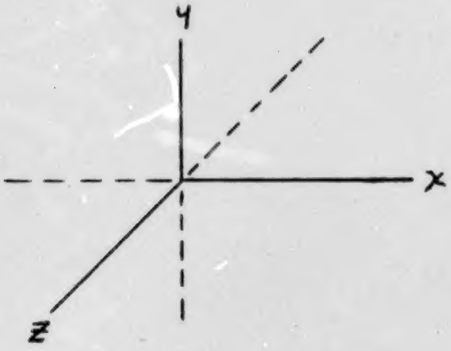


Figure 2

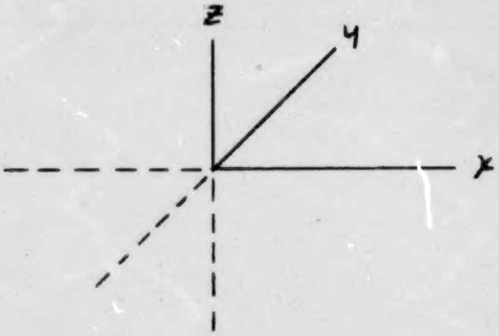


Figure 3

In each possible method of setting up the axes, each axis is perpendicular to the other two. Two of the axes are perpendicular to each other in the plane of the paper. The other axis should be thought of as coming straight out of the paper, perpendicular to the plane of the paper. The student might find it helpful to picture a corner of the room as the intersection of the three axes. In each figure above, the solid section of the axis is the positive end of the axis, and the dashed section of the axis is the negative end of the axis.

Example 1. Set up the axes as in Figure 4 and plot the point (1,2,3). 1 is the x-coordinate, 2 is the y-coordinate, and 3 is the z-coordinate.

Step 1 - Mark off 1 unit on the x-axis. Label this point A.

Step 2 - Draw the line parallel to the y-axis through point A.

Step 3 - Using the same scale as used on the y-axis, mark off two units on this parallel line (in the positive direction). Label this point B.

Step 4 - Draw the line through B parallel to the z-axis.

Step 5 - Using the same scale as used on the z-axis mark off three units on this parallel line (in the positive direction). Label this point C.

Step 6 - C is the desired point with coordinates (1, 2, 3).

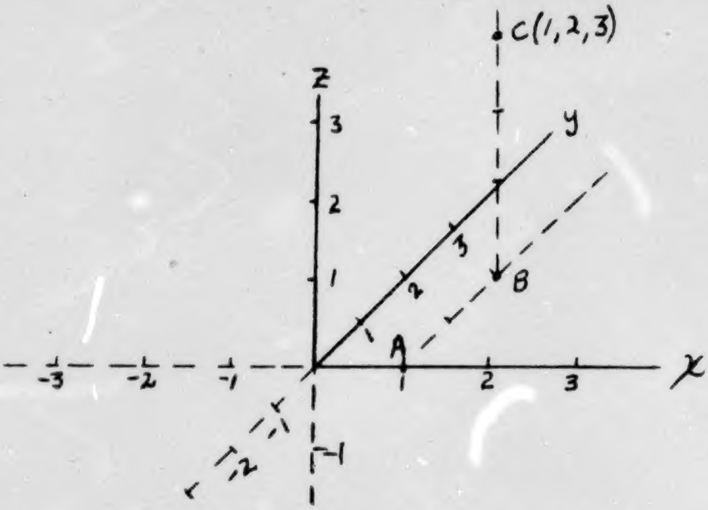


Figure 4

It sometimes helps to visualize the position of the point by "filling in the box" as follows in Figure 5.

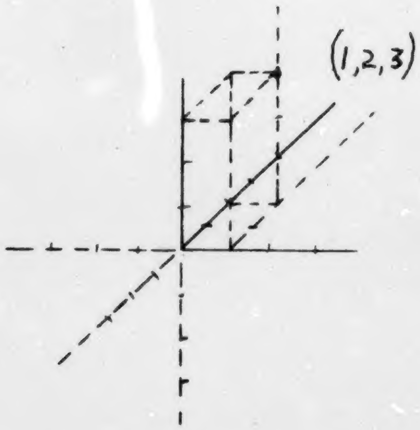


Figure 5

Example 2. Find the point with coordinates (2, -2, 4). See Figure 6.

Step 1 - Mark off two units on the x-axis. Label this point A.

Step 2 - Draw the line parallel to the y-axis through point A.

Step 3 - Using the same scale as used on the y-axis, mark off two units on this parallel line (in the negative direction). Label this point B.

Step 4 - Draw the line through B parallel to the z-axis.

Step 5 - Using the same scale as used on the z-axis, mark off four units on this parallel line (in the positive direction). Label this point C.

Step 6 - C is the desired point with coordinates (2, -2, 4).

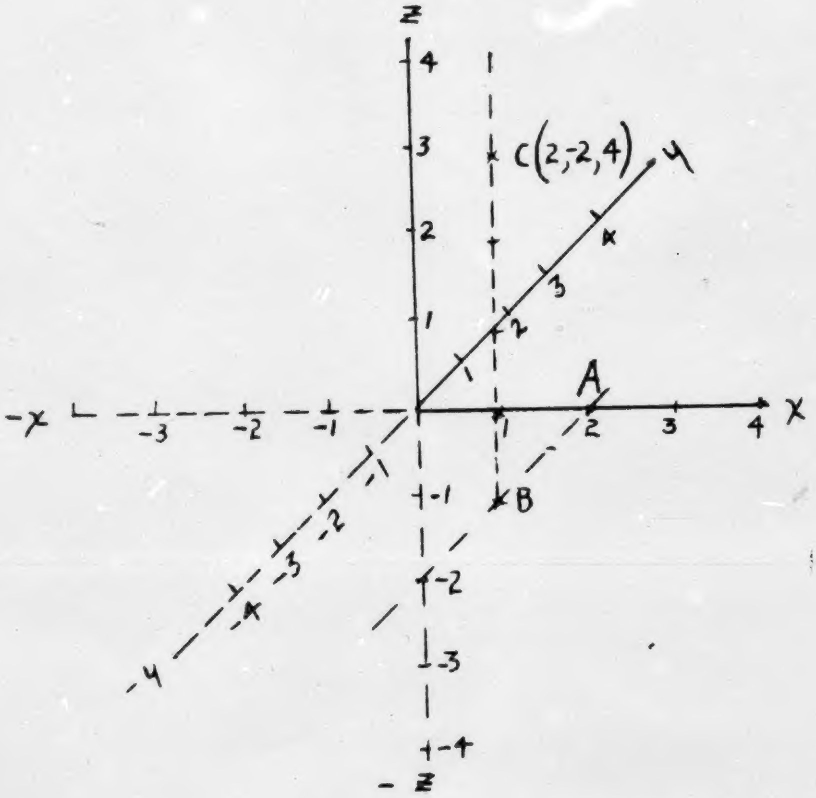

Z
4
3
2
1
-1
-2
-3
-4
- Z
C(2,-2,4)
Y
1
2
3
4
A
B
-X
-3
-2
-1
X
1
2
3
4
-1
-2
-3
-4
-Y

Figure 6

EXERCISES

Sketch the following points on an axis system designated by the teacher.

1. (2, 3, 1)
2. (4, 3, 6)
3. (1, -2, 3)
4. (-1, 3, 4)
5. (3, 5, -2)
6. (-3, 4, -2)
7. (3, -4, 1)
8. (-2, -3, -5)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Vertical Mill

TECHNICAL INFORMATION TITLE: Calculations--Incremental Values in x and y on the Circumference of a Circle (Introduction)

INTRODUCTION:

In calculating points on the circumference for a cutter path, it becomes necessary to calculate the increment in x and the direction (positive or negative) and the increment in y and the direction (positive or negative).

OBJECTIVE:

To provide the student an opportunity to determine, by calculations, the increments in x and y between two points on the circumference of a circle.

TECHNICAL INFORMATION:

There are various N/C systems all operating differently in terms of calculations, movements in x, y, and z, and origins. Two systems are generally accepted: the incremental system which measures the distances from one point to the next, and the absolute system whereby all the dimensional calculations in x and y are measured from the origin.

On sophisticated machine tool equipment, the hardware is designed with the capability to move in a path around the circle from point to point in the shortest distance. For closer tolerances, the points are spaced closer together.

Most all systems of N/C are digital rather than analog, because a closer control can be set up even though the curve will not appear as smooth. It depends on what the specifications are as to which system will be used.

APPLICATION OF THE RULE:

Example. Mill a circle with a 10 inch radius. Specifically, determine the increments in x and y between two points, P_1 and P_2 on the circumference of the circle in Figure 1.

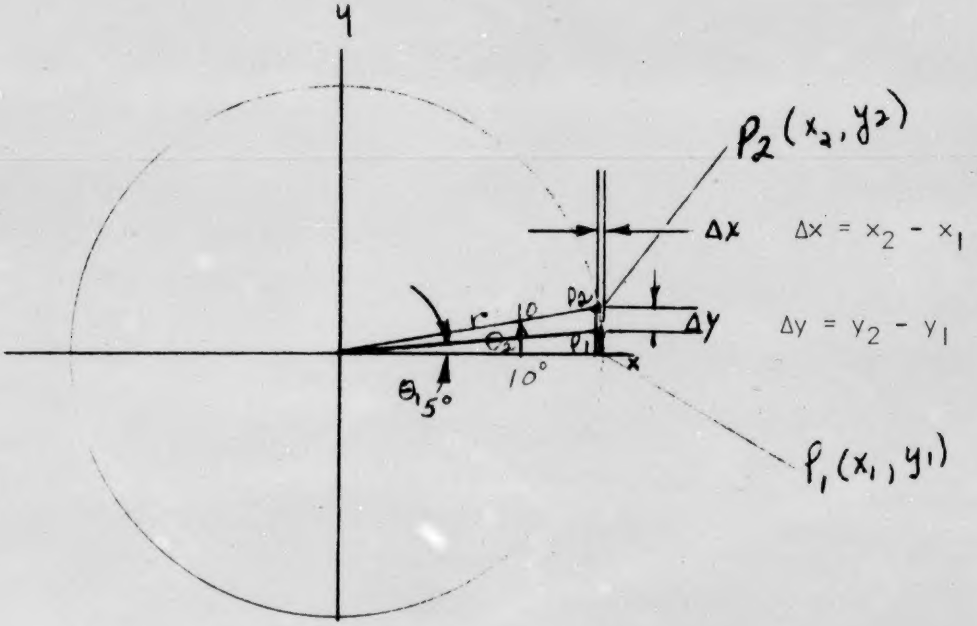


Figure 1

In Figure 1, find the increment in x and increment in y from point P_1 to P_2. The angle for point P_1 is 5°, and the angle for P_2 is 10°. The increment in x from P_1 to P_2 is very simply just the amount of change in the x value from P_1 to P_2. Similarly, the increment in y from P_1 to P_2 is very simply the amount of change in the y value from P_1 to P_2. We denote the change in x by Δx, and the change in y by Δy.

Notice that the difference in y is just the larger value of y (y_2) minus the smaller value of y (y_1). That is, $\Delta y = y_2 - y_1$. The difference in x (from P_1 to P_2) is $x_2 - x_1$. That is, $\Delta x = x_2 - x_1$. The difference is always taken as the coordinate at the second point minus the coordinate at the first point.

In order to find Δy, you need to find the values for y_1 and y_2 in the above problem.

From trigonometry, we know that:

$$\sin \theta_1 = \frac{y_1}{r}$$

$$r \cdot \sin \theta_1 = r \cdot \frac{y_1}{r} \qquad \text{(Mult. both sides by r)}$$

$$r \cdot \sin \theta_1 = y_1 \qquad \text{(Mult. inverse)}$$

Similarly:

$$\sin \theta_2 = \frac{y_2}{r}$$

$$r \cdot \sin \theta_2 = r \cdot \frac{y_2}{r} \qquad \text{(Mult. both sides by r)}$$

$$r \cdot \sin \theta_2 = y_2 \qquad \text{(Mult. inverse)}$$

Then:
$$\begin{aligned} \Delta y &= y_2 - y_1 \\ &= r \cdot \sin \theta_2 - r \cdot \sin \theta_1 \\ &= r(\sin \theta_2 - \sin \theta_1) \qquad \text{(Distributive property)} \end{aligned}$$

In order to find Δx, we must, first of all, find x_2 and x_1.

$$\cos \theta_1 = \frac{x_1}{r}$$

$$r \cdot \cos \theta_1 = r \cdot \frac{x_1}{r} \qquad \text{(Mult. both sides by r)}$$

$$r \cdot \cos \theta_1 = x_1 \qquad \text{(Mult. inverse)}$$

Similarly:

$$\cos\ \theta_2 = \frac{x_2}{r}$$

$$r\cdot\cos\ \theta_2 = r\cdot\frac{x_2}{r} \qquad \text{(Mult. both sides by r)}$$

$$r\cdot\cos\ \theta_2 = x_2 \qquad \text{(Mult. inverse)}$$

Then: $\Delta x = x_2 - x_1$

$$= r\cdot\cos\ \theta_2 - r\cdot\cos\ \theta_1$$

$$= r(\cos\ \theta_2 - \cos\ \theta_1) \qquad \text{(Distributive property)}$$

Now, if the radius is 10, $m\angle\theta_1 = 5^\circ$ and $m\angle\theta_2 = 10^\circ$, then

$\Delta y = r(\cos\ \theta_2 - \cos\ \theta_1)$

$$\Delta y = 10(\sin\ 10^\circ - \sin\ 5^\circ)$$

$$= 10(.17365 - .08715)$$

$$= 10(.08650)$$

$$= .8650$$

$\Delta x = r(\cos\ \theta_2 - \cos\ \theta_1)$

$$\Delta x = 10(\cos\ 10^\circ - \cos\ 5^\circ)$$

$$= 10(.98481 - .99619)$$

$$= 10(-.01138)$$

$$= -.1138$$

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Milling Machine

TECHNICAL INFORMATION TITLE: Calculations--Incremental Values in x and y on the Circumference of a Circle

INTRODUCTION:

In order to mill a circular object, utilizing a N/C Milling Machine, it is necessary to determine the values of increments in x and y on the circumference of a circle.

OBJECTIVE:

To provide the student an opportunity to learn how to determine values in x and y on the circumference of a circle.

TECHNICAL INFORMATION:

Assume that the angular measure between each of the points P_1, P_2, P_3, P_4, etc. in Figure 1 is 5°, and the radius of the circle is 5 inches. The data will first be determined for 1/8 of the circle. Then, the remainder will be found.

First of all, we will try to find the coordinates for points P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9 for the corresponding angles $\theta_0(0^\circ)$, $\theta_1(5^\circ)$, $\theta_2(10^\circ)$, $\theta_3(15^\circ)$, $\theta_4(20^\circ)$, $\theta_5(25^\circ)$, $\theta_6(30^\circ)$, $\theta_7(35^\circ)$, $\theta_8(40^\circ)$, $\theta_9(45^\circ)$.

The coordinates for point P_0 are (5, 0), since the radius is 5. Now, let us turn to the problem of finding the coordinates for point P_1.

$m\ \theta_0 = 0^\circ$ $P_0(x_0, y_0)$

$m\ \theta_1 = 5^\circ$ $P_1(x_1, y_1)$

$m\ \theta_2 = 10^\circ$ $P_2(x_2, y_2)$

$m\ \theta_3 = 15^\circ$ $P_3(x_3, y_3)$

$m\ \theta_4 = 20^\circ$ $P_4(x_4, y_4)$

$m\ \theta_5 = 25^\circ$ $P_5(x_5, y_5)$

$m\ \theta_6 = 30^\circ$ $P_6(x_6, y_6)$

$m\ \theta_7 = 35^\circ$ $P_7(x_7, y_7)$

$m\ \theta_8 = 40^\circ$ $P_8(x_8, y_8)$

$m\ \theta_9 = 45^\circ$ $P_9(x_9, y_9)$

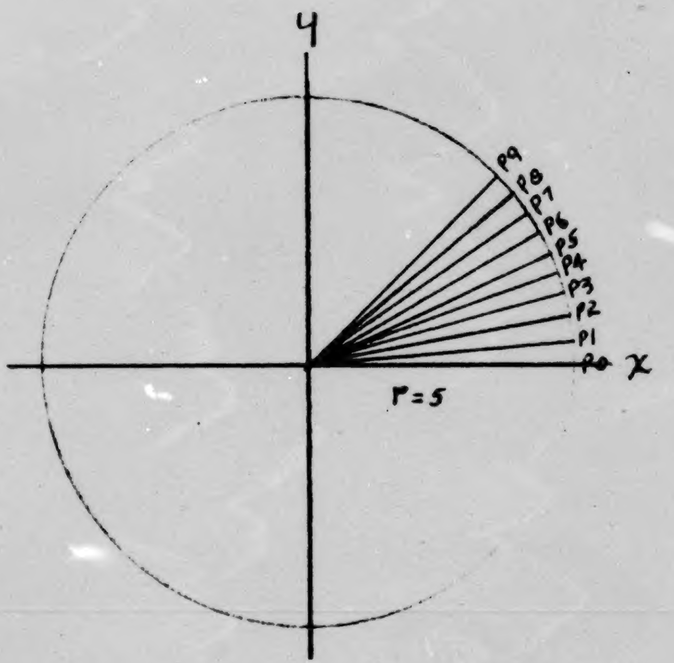

y
P9
P8
P7
P6
P5
P4
P3
P2
P1
P0
x
r=5

Figure 1

In Figure 1, let us look closely at angle θ_1. To do this let us blow up the size of this part of the Figure. This is done in Figure 2 below.

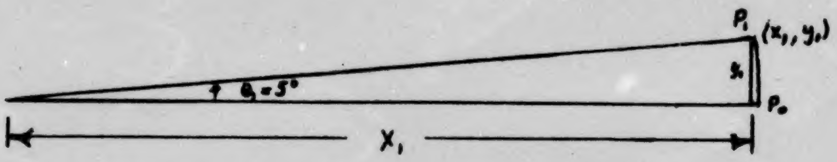

P1
(x1, y1)
y1
θ1=5°
P0
X1

Figure 2

From the figure:

$\sin \theta_1 = \frac{y_1}{r}$

$r(\sin \theta_1) = r \cdot \frac{y_1}{r}$ (Multiply both sides by r)

$r(\sin \theta_1) = y_1$ (Multiplicative inverse

Therefore:

$y_1 = r(\sin \theta_1)$

$= 5(\sin 5^\circ)$

$= 5(.08715)$

$= .43575$

Similarly:

$\cos \theta_1 = \frac{x_1}{r}$

$r(\cos \theta_1) = r \cdot \frac{x_1}{r}$ (Multiply both sides by r)

$r(\cos \theta_1) = x_1$ (Multiplicative inverse)

Therefore:

$x_1 = r(\cos \theta_1)$

$= 5(\cos 5^\circ)$

$= 5(.99619)$

$= 4.98095$

For Point P_2: (The process is almost identical except that the angle is now 10°)

$x_2 = r(\cos \theta_2)$

$= 5(\cos 10^\circ)$

$= 5(.98481)$

$y_2 = r(\sin \theta_2)$

$= 5(\sin 10^\circ)$

$= 5(.17365)$

$= 4.92405$ $\quad$ $= .86825$

For Point P_3

$$x_3 = r(\cos \theta_3) \qquad y_3 = r(\sin \theta_3)$$

$$= 5(\cos 15^\circ) \qquad = 5(\sin 15^\circ)$$

$$= 5(.96592) \qquad = 5(.25882)$$

$$= 4.82960 \qquad = 1.29410$$

Let us try to formulate a usable table to list these various values.

Point	Angle	sin θ	cos θ	y = 5(sin θ)	x = 5(cos θ)	Coordinates For Point
P_0	0°	.00000	1.00000	.00000	5.00000	(5.00000, .00000
P_1	5°	.08715	.99619	.43575	4.98095	(4.98095, .43575
P_2	10°	.17365	.98481	.86825	4.92405	(4.92405, .86825
P_3	15°	.25882	.96592	1.29410	4.82960	(4.82960, 1.294
P_4	20°	.34202	.93969	1.71010	4.69845	(4.69845, 1.710
P_5	25°	.42262	.90631	2.11310	4.53155	(4.53155, 2.113
P_6	30°	.50000	.86603	2.50000	4.33015	(4.33015, 2.5000
P_7	35°	.57358	.81915	2.86790	4.09575	(4.09575, 2.8679
P_8	40°	.64279	.77604	3.21395	3.83020	(8.83020, 3.2139
P_9	45°	.70711	.70711	3.53555	3.53555	(3.53555, 3.5355

Remember, Δy is the change in y from one point to the next, and Δx is the change in x from one point to the next.

Therefore:

$$\begin{aligned} \Delta y_1 &= y_1 - y_0 \\ &= .43575 - .00000 \\ &= .43575 \end{aligned}$$

$$\begin{aligned} \Delta x_1 &= x_1 - x_0 \\ &= 4.98095 - 5.00000 \\ &= -.01905 \end{aligned}$$

$$\begin{aligned} \Delta y_2 &= y_2 - y_1 \\ &= .86825 - .43575 \\ &= .43250 \end{aligned}$$

$$\begin{aligned} \Delta x_2 &= x_2 - x_1 \\ &= 4.92405 - 4.98095 \\ &= -.05690 \end{aligned}$$

$$\begin{aligned} \Delta y_3 &= y_3 - y_2 \\ &= 1.29410 - .86825 \\ &= .42585 \end{aligned}$$

$$\begin{aligned} \Delta x_3 &= x_3 - x_2 \\ &= 4.82960 - 4.92405 \\ &= -.09445 \end{aligned}$$

Again, let us try to construct a table to help.

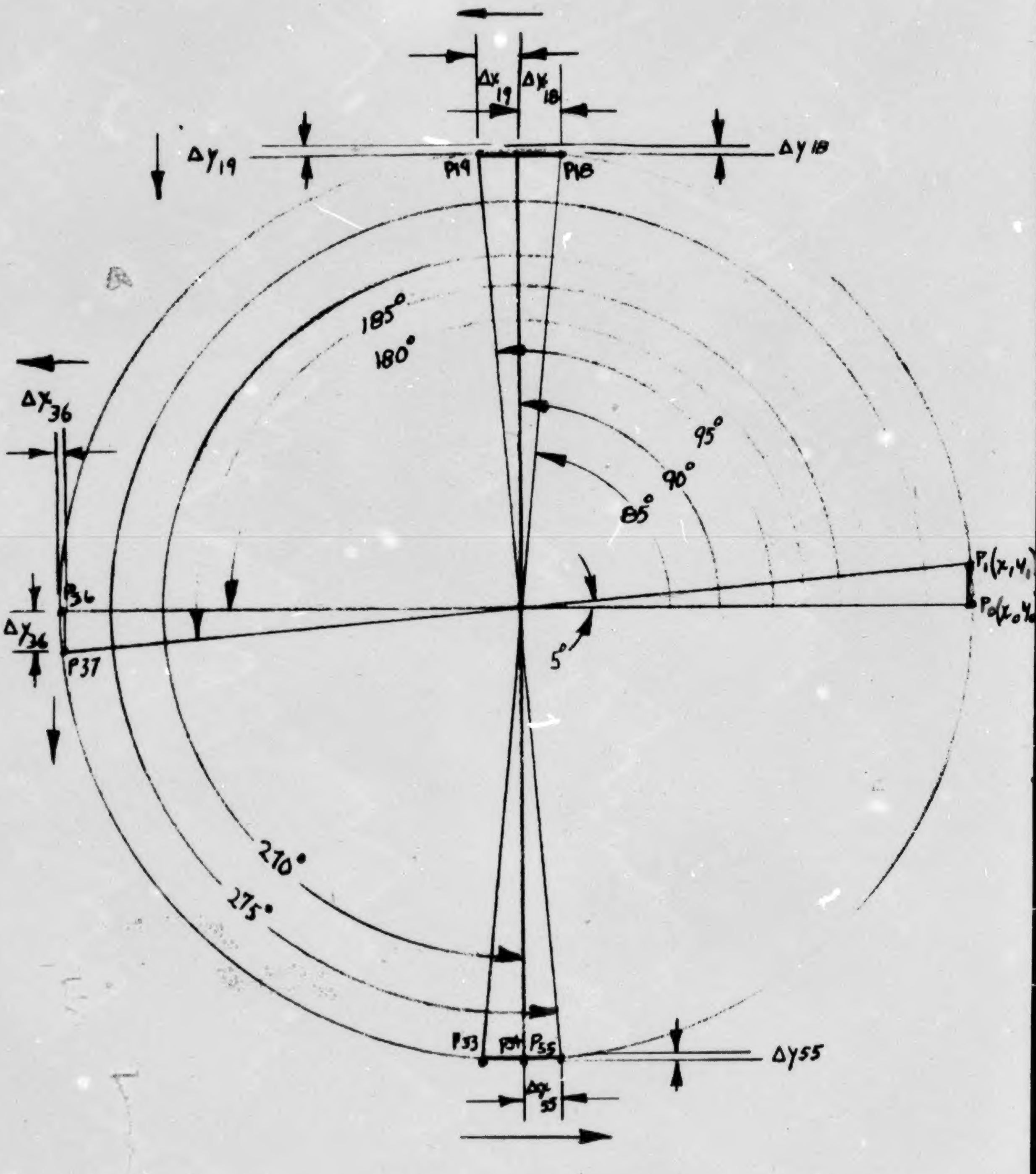

Δx19
Δx18
Δy19
Δy18
P19
P18
185°
180°
95°
90°
85°
Δx36
P36
Δy36
P37
5°
P1(x1 y1)
P0(x0 y0)
270°
275°
P53
P54
P55
Δy55
Δx55

Figure 3

Point	Coordinates	Δy	Δx
P_0	(5.00000, .00000)		
P_1	(4.98095, .43575)	$\Delta y_1 = .43575$	$\Delta x_1 = -.01905$ ✓
P_2	(4.92405, .86825)	$\Delta y_2 = .43250$	$\Delta x_2 = -.05690$
P_3	(4.82960, 1.29410)	$\Delta y_3 = .42585$	$\Delta x_3 = -.09445$
P_4	(4.69845, 1.71010)	$\Delta y_4 = .41600$	$\Delta x_4 = -.13115$
P_5	(4.53155, 2.11310)	$\Delta y_5 = .40300$	$\Delta x_5 = -.16690$
P_6	(4.33015, 2.50000)	$\Delta y_6 = .38690$	$\Delta x_6 = -.20140$
P_7	(4.09575, 2.86790)	$\Delta y_7 = .36790$	$\Delta x_7 = -.23440$
P_8	(3.83020, 3.21395)	$\Delta y_8 = .34605$	$\Delta x_8 = -.26555$
P_9	(3.53555, 3.53555)	$\Delta y_9 = .32160$	$\Delta x_9 = -.29465$

Now, let us continue to find values for the angles from 45° to 90°.

Point	Angle	$\sin \theta$	$\cos \theta$	$y = 5(\sin \theta)$	$x = 5(\cos \theta)$	Coordinates For Point
P_9	45°	.70711	.70711	3.53555	3.53555	(3.53555, 3.53555)
P_{10}	50°	.76604	.64279	3.83020	3.21395	(3.21395, 3.83020)
P_{11}	55°	.81915	.57358	4.09575	2.86790	(2.86790, 4.09575)
P_{12}	60°	.86603	.50000	4.33015	2.50000	(2.50000, 4.33015)
P_{13}	65°	.90631	.42262	4.53155	2.11310	(2.11310, 4.53155)
P_{14}	70°	.93969	.34202	4.69845	1.71010	(1.71010, 4.69845)
P_{15}	75°	.96592	.25882	4.82960	1.29410	(1.29410, 4.82960)
P_{16}	80°	.98481	.17265	4.92405	.86825	(.86825, 4.29405)
P_{17}	85°	.99619	.08715	4.98095	.43575	(.43575, 4.98095)
P_{18}	90°	1.00000	.00000	5.00000	.00000	(.00000, 5.00000)

Point	Coordinates	Δy	Δx
P_9	(3.53555, 3.53555)	Δy_9 = .32160	Δx_9 = -.29465
P_{10}	(3.21395, 3.83020)	Δy_{10} = .29465	Δx_{10} = -.32160
P_{11}	(2.86790, 4.09575)	Δy_{11} = .26555	Δx_{11} = -.34605
P_{12}	(2.50000, 4.33015)	Δy_{12} = .23440	Δx_{12} = -.36790
P_{13}	(2.11310, 4.53155)	Δy_{13} = .20140	Δx_{13} = -.38690
P_{14}	(1.71010, 4.69845)	Δy_{14} = .16690	Δx_{14} = -.40300
P_{15}	(1.29410, 4.82960)	Δy_{15} = .13115	Δx_{15} = -.41600
P_{16}	(.86825, 4.92405)	Δy_{16} = .09445	Δx_{16} = -.42585
P_{17}	(.43575, 4.98095)	Δy_{17} = .05690	Δx_{17} = -.43250
P_{18}	(.00000, 5.00000)	Δy_{18} = .01905	Δx_{18} = -.43575

To continue into the second quadrant, all we have to notice is that the coordinates of P_{19} will be identical to the coordinates for P_{17} except the x coordinate for P_{19} will be the negative of the x coordinate for P_{17}. Likewise the coordinates for P_{20} will be identical to the coordinates for P_{16} except for the change in sign for the x values. This same pattern will be true for all points in the second quadrant.

Point	Angle	Coordinates For Point
P_{19}	95°	(-.43575, 4.98095)
P_{20}	100°	(-.86825, 4.92405)
P_{21}	105°	(-1.29410, 4.82960)
P_{22}	110°	(-1.71010, 4.69845)
P_{23}	115°	
P_{24}	120°	

P_{25}	125°
P_{26}	130°
P_{27}	135°
P_{28}	140°
P_{29}	145°
P_{30}	150°
P_{31}	155°
P_{32}	160°
P_{33}	165°
P_{34}	170°
P_{35}	175°
P_{36}	180°

Now, we can evaluate the values for Δy and Δx for angles in the second quadrant.

Point	Coordinates	Δy	Δx
P_{18}	(.00000, 5.00000)	$\Delta y_{18} = .01905$	$\Delta x_{18} = -.43575$
P_{19}	(-.43575, 4.98095)	$\Delta y_{19} = -.01905$	$\Delta x_{19} = -.43575$
P_{20}	(-.86825, 4.92405)	$\Delta y_{20} = -.05690$	$\Delta x_{20} = -.43250$
P_{21}	(-1.29410, 4.82960)	$\Delta y_{21} = -.09445$	$\Delta x_{21} = -.42585$
P_{22}	(-1.71010, 4.69845)	$\Delta y_{22} = -.13115$	$\Delta x_{22} = -.41600$
P_{23}			
P_{24}			
P_{25}			
P_{26}			
P_{27}			

P_{28}

P_{29}

P_{30}

P_{31}

P_{32}

P_{33}

P_{34}

P_{35}

P_{36}

Note that in the above table the values for Δy for P_{19} and P_{18} are identical except for the sign. The values for Δx for P_{19} and P_{18} are identical. This is true since the values for x are decreasing from point to point in the second quadrant just as they were in the first quadrant. However, in the first quadrant, y increases from point to point, whereas in the second quadrant, y decreases from point to point. The student should complete the above tables to make sure he understands the procedure.

Now, let us turn to the third quadrant. Here, the coordinates for 185° will correspond to the coordinates for 5° except that both the x and y coordinates are negative for 185°. Since in the third quadrant the x values are increasing from point to point, the values for Δx should now be positive. The value of Δx_{37} will be the same as that for $-\Delta x_1$. In the third quadrant the values of y are decreasing. Therefore, the values for Δy should be negative. The value for Δy_{37} should be the same as that for Δy_1 except that the value will be negative.

Point	Angle	Coordinates For Point
P_{36}	180°	(-5.00000, .00000)
P_{37}	185°	(-4.98095, -.43575)
P_{38}	190°	(-4.92405, -.86825)
P_{39}	195°	(-4.82960, -1.29410
P_{40}	200°	
P_{41}	205°	
P_{42}	210°	
P_{43}	215°	
P_{44}	220°	
P_{45}	225°	
P_{46}	230°	
P_{47}	235°	
P_{48}	240°	
P_{49}	245°	
P_{50}	250°	
P_{51}	255°	
P_{52}	260°	
P_{53}	265°	
P_{54}	270°	

Point	Coordinates	Δy	Δx
P_{36}	(-5.00000, .00000)		
P_{37}	(-4.98095, -.43575)	$\Delta y_{37} = -.43575$	$\Delta x_{37} = .01905$
P_{38}	(-4.92405, -.86825)	$\Delta y_{38} = -.43250$	$\Delta x_{38} = .05690$

P_{39}	$(-4.82960, -1.29410)$	$\Delta y_{39} = -.42585$	$\Delta x_{39} = .09445$
P_{40}			
P_{41}			
P_{42}			
P_{43}			
P_{44}			
P_{45}			
P_{46}			
P_{47}			
P_{48}			
P_{49}			
P_{50}			
P_{51}			
P_{52}			
P_{53}			
P_{54}			

The student should complete the above tables.

In the fourth quadrant, the coordinates for 275° will be the same as those for 85° except that the y coordinate for 275° will be the negative of the y value for 85°. The x coordinates will be identical. Since in the fourth quadrant the x values are increasing from point to point, the values for Δx should be positive. Since the y values are also increasing from point to point, the values for Δy should be positive. The value for Δx_{55} should be the same as Δx_{18} except for the difference in sign. The value for Δy_{55} should be identical to the value for Δy_{18}.

Point	Angle	Coordinates For Point
P_{55}	275°	(.43575, -4.98095)
P_{56}	280°	(.86825, -4.92405)
P_{57}	285°	(1.29410, -4.82960)
P_{58}	290°	
P_{59}	295°	
P_{60}	300°	
P_{61}	305°	
P_{62}	310°	
P_{63}	315°	
P_{64}	320°	
P_{65}	325°	
P_{66}	330°	
P_{67}	335°	
P_{68}	340°	
P_{69}	345°	
P_{70}	350°	
P_{71}	355°	
P_{72}	360°	

Point	Coordinates	Δy	Δx
P_{54}	(.00000, -5.00000)		
P_{55}	(.43575, -4.98095)	$\Delta y_{55} = .01905$	$\Delta x_{55} = .43575$
P_{56}	(.86825, -4.92405)	$\Delta y_{56} = .05690$	$\Delta x_{56} = .43250$
P_{57}	(1.29410, -4.82960)	$\Delta y_{57} = .09445$	$\Delta x_{57} = .42585$
P_{58}	(1.71010, -4.69845)	$\Delta y_{58} = .13115$	$\Delta x_{58} = .41600$

P_{59}

P_{60}

P_{61}

P_{62}

P_{63}

P_{64}

P_{65}

P_{66}

P_{67}

P_{68}

P_{69}

P_{70}

P_{71}

P_{72}

The student should complete the above tables.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Vertical Milling Machine

TECHNICAL ASSIGNMENT TITLE: Incremental Values in x and y on the Circumference of a Circle

INTRODUCTION:

Increments in x and y and the coordinates of specific points on the circumference of a circle become very essential for certain types of N/C equipment.

OBJECTIVE:

To learn how to calculate coordinates and incremental values in x and in y for points on the circumference of a circle.

ASSIGNMENT:

1. In Figure 1 if $\theta = 5^\circ$ find the coordinates for points P_0, P_1, P_2, P_3, and P_4. Also, find $\Delta x_1 = x_1 - x_0$, $\Delta x_2 = x_2 - x_1$, $\Delta x_3 = x_3 - x_2$, $\Delta x_4 = x_4 - x_3$, $\Delta y_1 = y_1 - y_0$, $\Delta y_2 = y_2 - y_1$, $\Delta y_3 = y_3 - y_2$, $\Delta y_4 = y_4 - y_3$.

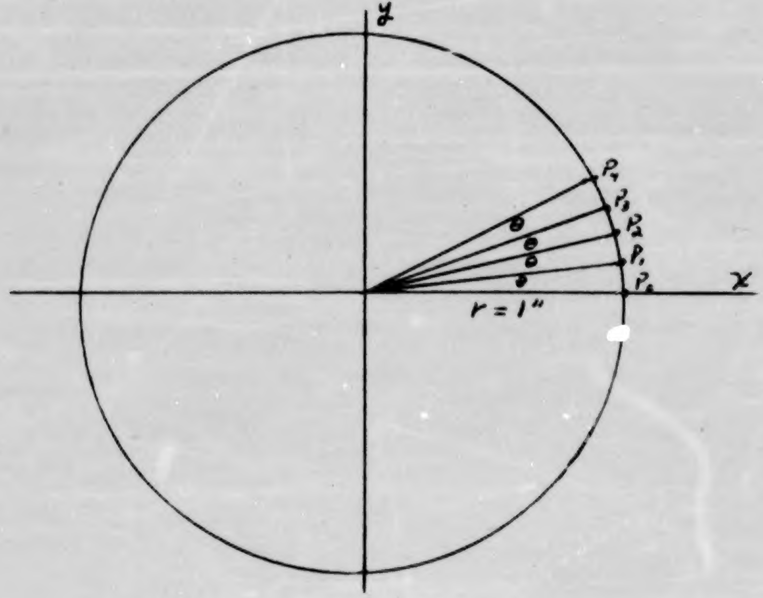


Figure 1

[redacted]

2. Referring to Figure I and problem I, do the same problem if $\theta = 4^\circ$.

3. Referring to Figure I and problem I, do the same problem if $\theta = 6^\circ$ and the radius is 2 in.

4. Referring to Figure I and problem I, do the same problem if $\theta = 2^\circ$ and the radius is 4 in.

ANSWERS

1.	P_0:	(1, 0)	$\Delta x_1 = -.00381$	$\Delta y_1 = .08715$
	P_1:	(.99619, .08715)	$\Delta x_2 = -.01138$	$\Delta y_2 = .08650$
	P_2:	(.98481, .17365)	$\Delta x_3 = -.01889$	$\Delta y_3 = .08517$
	P_3:	(.96592, .25882)	$\Delta x_4 = -.02623$	$\Delta y_4 = .08320$
	P_4:	(.93969, .34202)		
2.	P_0:	(1, 0)	$\Delta x_1 = -.00244$	$\Delta y_i = .06976$
	P_1:	(.99756, .06976)	$\Delta x_2 = -.00729$	$\Delta y_2 = .06941$
	P_2:	(.99027, .13917)	$\Delta x_3 = -.01212$	$\Delta y_3 = .06874$
	P_3:	(.97815, .20791)	$\Delta x_4 = -.01689$	$\Delta y_4 = .06773$
	P_4:	(.96126, .27564)		
3.	P_0:	(2, 0)	$\Delta x_1 = -.01096$	$\Delta y_1 = .20906$
	P_1:	(1.98904, .20906)	$\Delta x_2 = -.03274$	$\Delta y_2 = .20676$
	P_2:	(1.95630, .41582)	$\Delta x_3 = -.05418$	$\Delta y_3 = .20222$
	P_3:	(1.90212, .61804)	$\Delta x_4 = -.07504$	$\Delta y_4 = .19544$
	P_4:	(1.82708, .81348)		
4.	P_0:	(4, 0)	$\Delta x_1 = -.00244$	$\Delta y_1 = .13960$
	P_1:	(3.99756, .13960)	$\Delta x_2 = -.00732$	$\Delta y_2 = .13944$
	P_2:	(3.99024, .27904)	$\Delta x_3 = -.01216$	$\Delta y_3 = .13908$
	P_3:	(3.97808, .41812)	$\Delta x_4 = -.01700$	$\Delta y_4 = .13856$
	P_4:	(3.96108, .55668)		

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL INFORMATION SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Milling Machine

TECHNICAL INFORMATION TITLE: Angular Calculations for N/C

INTRODUCTION:
When it is necessary to mill an angular path with an N/C Milling Machine, we must determine increments in x and in y on the angular line.

OBJECTIVE:
To provide the student an opportunity to learn the principles of manular programming of a N/C machine in terms of an angular movement.

TECHNICAL INFORMATION:

Some N/C systems are not sophisticated enough to mill a continuous angular path in a straight line movement; but instead, the path is determined by a series of increments in x and in y.

In order to program a path one must know the ratio of the sides of a desired angle. By knowing the lengths (the increments in x and in y) and the ratio of the sides to each other for a desired angle, a path will be programmed within the tolerances of a given system.

Example. Find the points on the line in Figure I in terms of the x and y coordinates and the increments in x and in y from one point to the next. The angle is 32°.

Consider any point (x, y) on the terminal side of an angle of measure 32°.

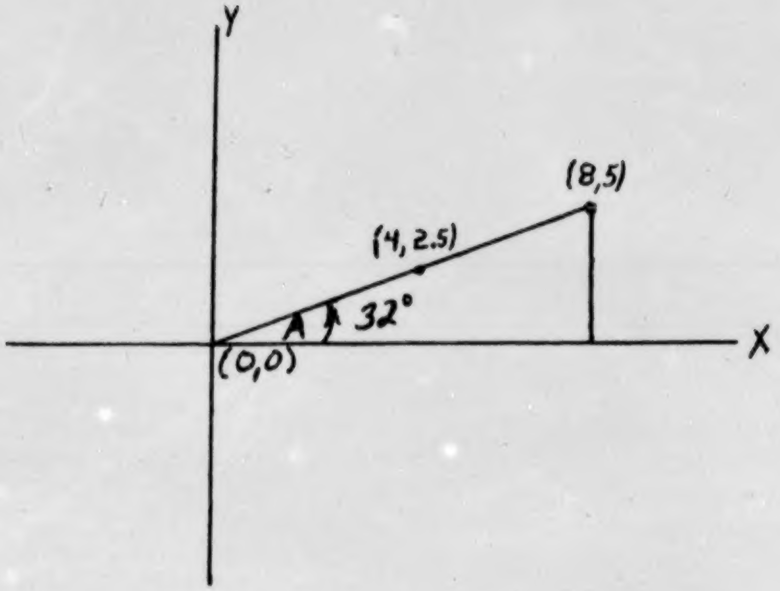


Figure I

Since we are interested in finding the ratio of x to y, let us use the tangent function.

$$\frac{y}{x} = \tan \angle A$$

$$= \tan 32^{\circ}$$

$$= .625$$ (Here, the value of tan 32° was taken to the nearest thousandth.)

Therefore, for any point (x, y) on the terminal side, the ratio of y to x is .625. If x is now chosen to be any value we can find y by solving the formula for y.

$$\frac{y}{x} = .625$$

$$\frac{y}{x} \cdot x = .625x$$ (multiply both sides by x)

$$y = .625x$$ (multiplicative inverse)

For example, if x = 8, then

$y = .625(8)$

$= 5$

Therefore, one point is (8, 5).

If $x = 4$, then $y = .625(4) = 2.5$

Therefore, another point is (4, 2.5).

Notice, then, that any point on the line can be found by simply selecting a specific x value and then finding the resulting y value.

If x is taken as .040, then $y = .625(.040) = .025$

Now, let us designate the point (0, 0) by (x_0, y_0) and the point (.040, .025) by (x_1, y_1). Then:

$$y_1 - y_0 = .025 - 0$$
$$= .025$$

or Δy (the difference in y) = .025

$$x_1 - x_0 = .040 - 0$$

or Δx (the difference in x) = .040

It these same differences are taken from one point to the next, then there will be $\frac{8.000}{.040}$ sequences in x and y to mill from the point (0, 0) to the point (8, 5).

$$\text{sequences in x (also y)} = \frac{8.000}{.040}$$
$$= 200$$

(Note that it is also true that $\frac{5.000}{.025} = 200$)

See Figure 2 and the following table.

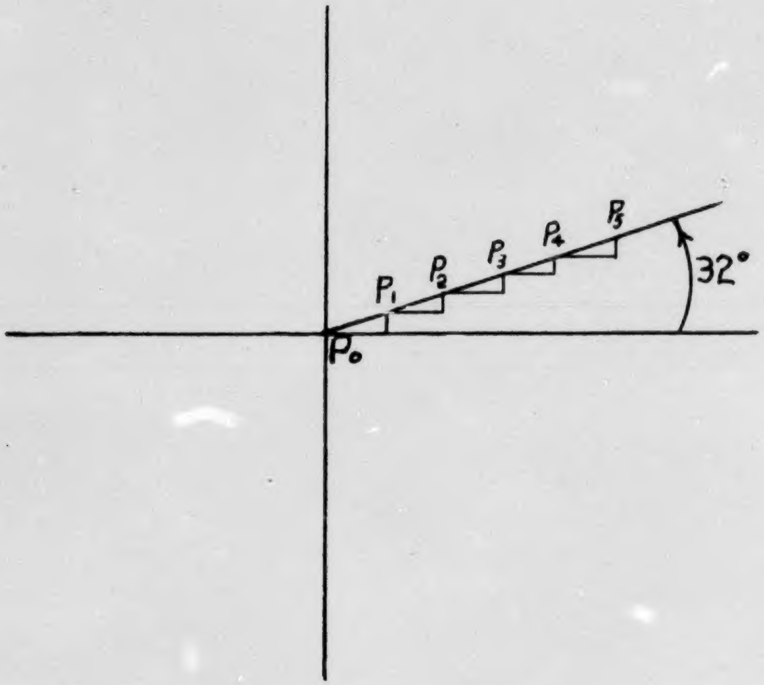


Figure 2

Point	x-increment	y-increment	Point Coordinates
P_0			(0.000, 0.000)
P_1	.040	.025	(0.040, 0.025)
P_2	.040	.025	(0.080, 0.050)
P_3	.040	.025	(0.120, 0.075)
P_4	.040	.025	(0.160, 0.100)
.	.	.	.
.	.	.	.
.	.	.	.
P_{200}	.040	.025	(8.000, 5.000)

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

TECHNICAL ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: N/C Milling Machine

TECHNICAL ASSIGNMENT TITLE: Angular Calculations for N/C

INTRODUCTION:

It is necessary to compute the values of the increments in x and in y for a desired angular movement for a mill table.

ASSIGNMENT:

1. Find the value for each increment in x and the value for each increment in y in Figure I if there are to be 80 equal divisions.

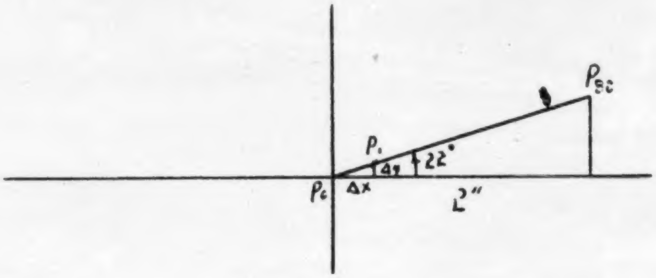


Figure I

2. Referring back to problem I and Figure I, do the same problem if the angle is changed to 30°.

3. Referring back to problem 1 and Figure 1, do the same problem if the angle is changed to 60°.

4. Find the value for each increment in x and the value for each increment in y in Figure 2 if there are to be 100 equal divisions from P_0 to P_{100}.

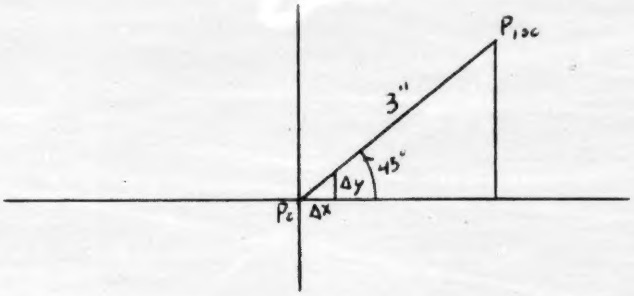

P_{100}
3"
45°
Δy
P_0
Δx

Figure 2

5. Referring to Figure 2 and problem 4, do the same problem if the angle is 58°.

6. If Figure 3 find the value for each increment in x and the value for each increment in y if there are to be 125 equal divisions from P_0 to P_{125}.

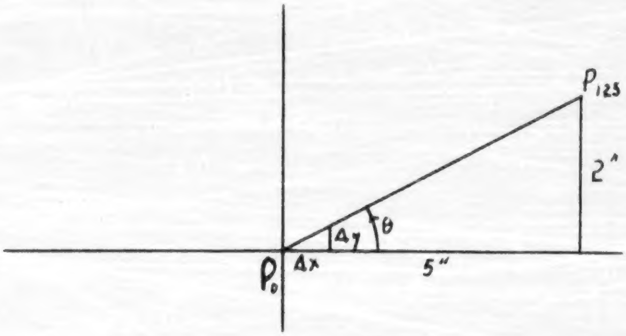

P_{125}
2"
θ
Δy
P_0
Δx
5"

Figure 3

7. In Figure 4, if each $\Delta x = .015$ in., what is the value for each Δy? How many equal divisions will be made from 0 to P?

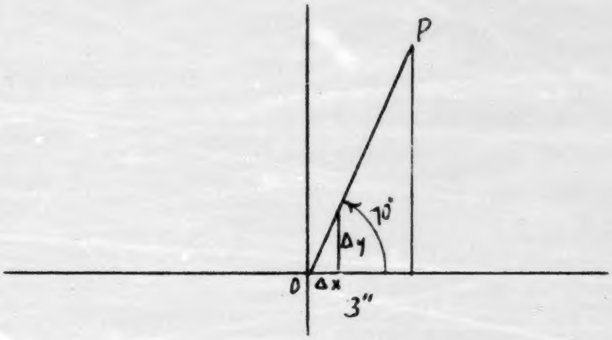


Figure 4

ANSWERS

1. Each Δx = .025 in.
   Each Δy = .010 in. (to the nearest thousandth)
2. Each Δx = .025 in.
   Each Δy = .014 in. (to the nearest thousandth)
3. Each Δx = .025 in.
   Each Δy = .043 in. (to the nearest thousandth)
4. Each Δx = .021 in. (to the nearest thousandth)
   Each Δy = .021 in. (to the nearest thousandth)
5. Each Δx = .016 in. (to the nearest thousandth)
   Each Δy = .025 in. (to the nearest thousandth)
6. Each Δx = .040 in.
   Each Δy = .016 in.
7. Each Δy = .041 in. (to the nearest thousandth)
   There will be 200 equal divisions.

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Layout and Bench Work

JOB TITLE: Plate Layout

INTRODUCTION:

Layout is vitally important when making dies, machine parts, fixtures, etc. One of the fundamental factors to remember is that an accumulative error is to be avoided. Too often reference edges are not used for all measurements.

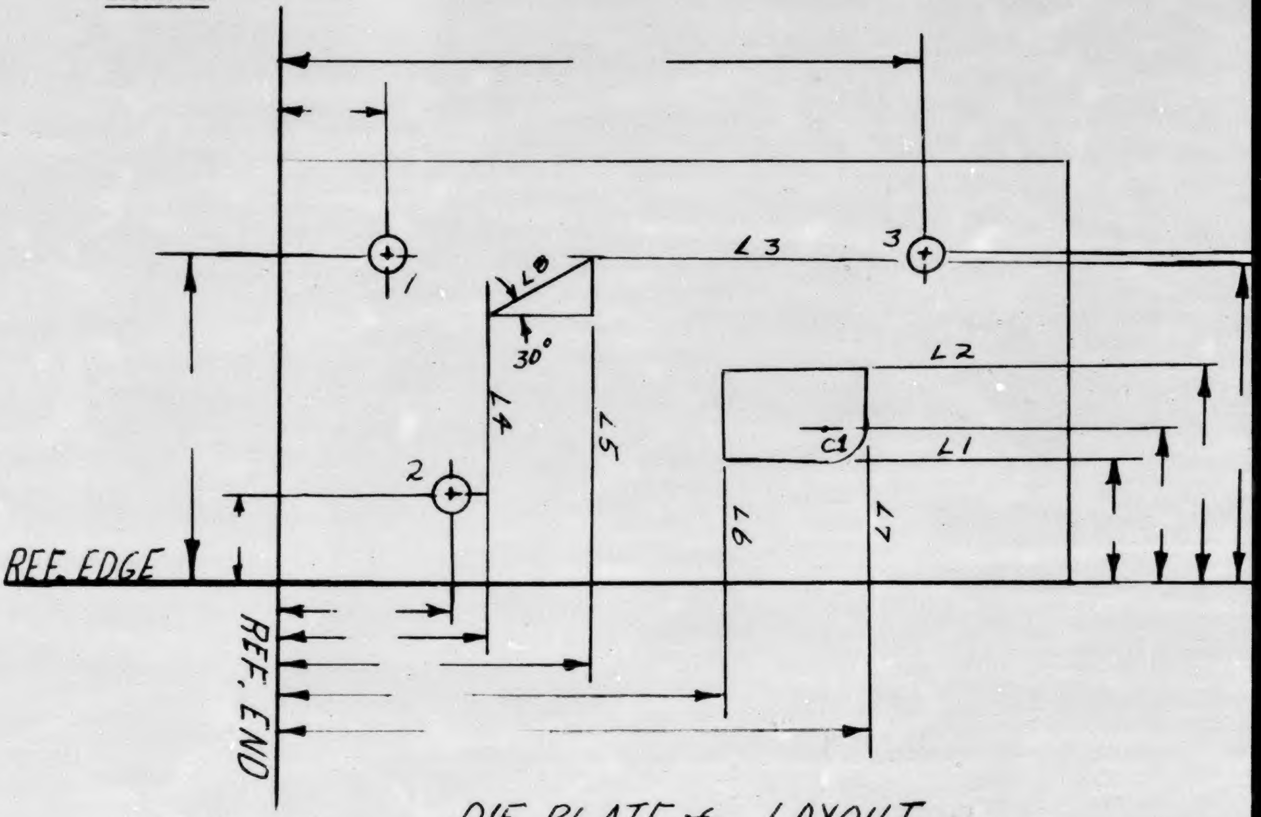
OBJECTIVE:

To provide the student an opportunity to apply the skills of laying out locations, angles, lines, and circles on a plate.

JOB SPECIFICATIONS:

Please follow the procedure carefully and use available tools for accurate results. Locations, lines, circles, and the angle are scribed from the reference edge and the reference end as indicated. This should prevent accumulative errors. The procedure is based on an absolute system rather than an incremental system. In the absolute system all measurements are made from the reference edge and reference end. This is opposed to an incremental type of system where measurements are taken from one position to the next, then to the next, etc. The use of an incremental type of system usually results in errors being accumulated.

DRAWING:


1
2
3
L3
L8
30°
L2
L4
L5
C1
L1
L6
L7
REF. EDGE
REF. END

DIE PLATE for LAYOUT

TOOLS:

Angle plate
Scriber
Gage blocks and scriber
Surface gage
Planer gage and scriber
Vernier protractor

Dykon blueing
Center punch
Power saw
Surface grinder
Mill

End mill
Vernier height gage
Cadillac gage
Protractor
Combination square

MATERIALS:

Plate steel of suitable type steel for a die plate

PROCEDURE:

(Operations)	(Related Information)
1. Procure plate material.	1. Refer to print.
2. Measure rough stock.	2. Allow stock for clean-up on ends and edges.
3. Cut rough stock.	3. Use power saw.
4. Mill first edge.	4. Use vertical end mill.
5. Mill first end.	5. Use vertical end mill.
6. Grind first face.	6. Use surface grinder.
7. Grind first edge.	7. Use angle plate and surface grinder.
8. Grind first end.	8. Use angle plate and surface grinder.
9. Locate no. 1 hole.	9. Measure from reference edge and reference end. Use angle plate, vernier height gage (or equivalent), and scribe line.
10. Locate no. 2 hole.	10. Same as 9.
11. Locate no. 3 hole.	11. Same as 9.
12. Locate circle 1.	12. Same as 9.
13. Lay out line 1 (L1) and scribe.	13. Measure from reference edge. Use angle plate and height gage with scriber.
14. Lay out line 2 (L2) and scribe.	14. Same as 13.
15. Lay out line 3 (L3) and scribe.	15. Same as 13.
16. Lay out line 4 (L4) and scribe.	16. Measure from reference end. Use angle plate and vernier height gage with scriber.
17. Lay out line 5 (L5) and scribe.	17. Same as 16.
18. Lay out line 6 (L6) and scribe.	18. Same as 16.
19. Lay out line 7 (L7) and scribe.	19. Same as 16.
20. Lay out line 8 (L8) and scribe.	20. Use protractor. Use either reference edge or reference end starting at intersection of line 3 and line 4. Set for 30°.

QUESTIONS:

1. What is an absolute system of measurement? Give an example.

2. What is an incremental system of measurement? Give an example.

3. What is a reference edge? What is a reference line?

4. What is an accumulative error? Give an example.

5. Which of the following systems is more accurate: abolute or incremental? Why?

6. In the layout of this plate, why do we use an angle plate?

7. Which of the following will be the most accurate: (a) combination square and surface gage, (b) vernier height gage, (c) gage blocks and gage block scriber, or (d) planer gage?

SELF-EVALUATION:

1. Did you understand all of the terms?

2. Did you ask questions?

3. Did you make any mistakes?

4. Did you correct your mistakes?

5. What grade would you give yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Square Threads--Lathe

JOB TITLE: Set-up Jack

PART TITLE: Screw (Square Threaded) for Set-up Jack

INTRODUCTION:

Because of the strength of the thread, the square thread is used for jack screws and other devices where maximum transmission of power is needed. The sides of a square thread are perpendicular to the center axis of the thread. Friction involved with square threads is reduced to a minimum.

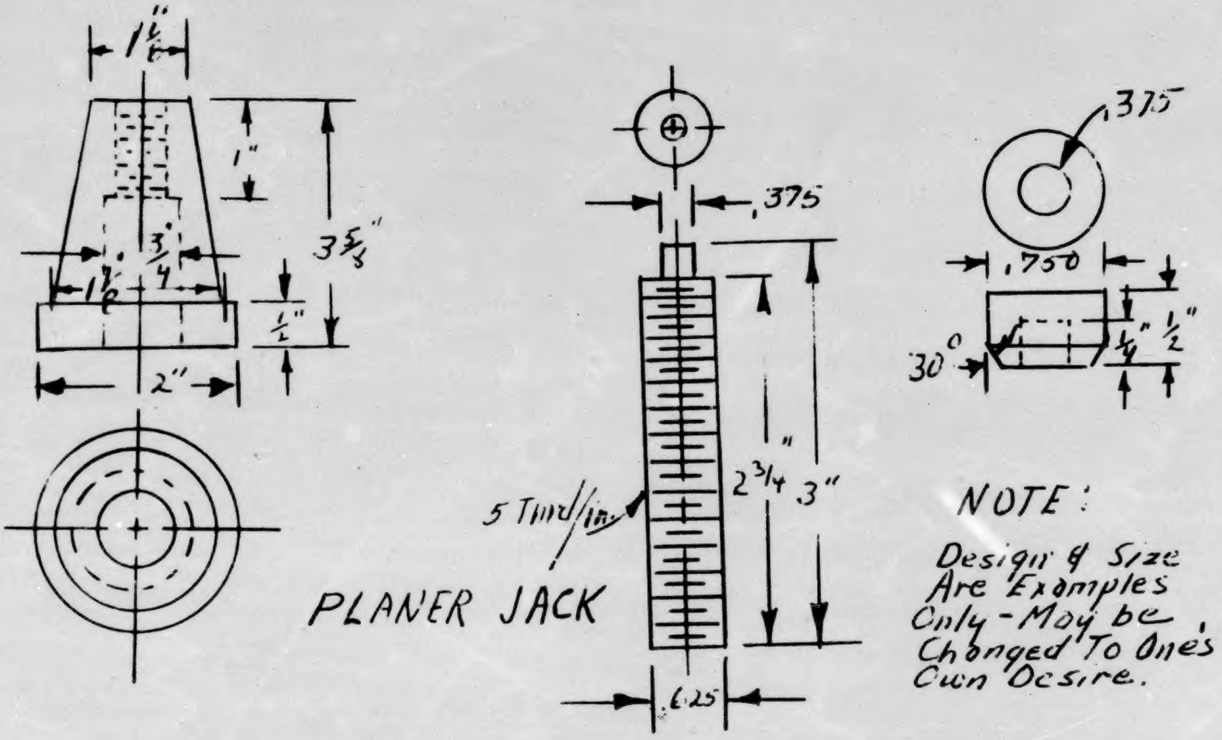
OBJECTIVE:

To provide the student an opportunity to apply the skill of cutting a square thread on a lathe.

JOB SPECIFICATIONS:

The student should refer to the drawing and carefully follow the steps of procedure. In producing the thread some machinists will use a narrow tool as a roughing tool and then follow with a form tool to relieve the cutting pressure and prevent springing of the work. Turn the outside diameter between centers and use a follower rest for long threads if needed.

DRAWING:


1 1/8"
1"
3 5/8"
3/4
1 1/8
1/2"
2"
.375
2 3/4"
3"
5 Thrd/in.
.625
.375
.750
30°
1/4"
1/2"

PLANER JACK

NOTE:
Design & Size
Are Examples
Only - May be
Changed To One's
Own Desire.

TOOLS:

Roughing tool bit
Form ground tool bit
Lathe
Dog
Drive plate

Drill chuck
Center drill
Lathe tool bit for O. D.
Lathe tool bit for I. D.
Rule

Lathe centers
Cutting oil
Micrometer
Vernier

MATERIALS:

High speed steel or alloy steel

PROCEDURE:

(Operations)	(Related Information)
1. Procure proper material.	1. Bill of materials should be made from the print.

2. Measure rough stock.	2. Allow sufficient stock for removal of one center drilled hole.
3. Saw.	3. Use power saw.
4. Face first end.	4. Use facing tool. Face off only enough stock to clean up end.
5. Center drill first end.	5. Use center drill of correct size.
6. Face second end.	6. Same as no. 4.
7. Center drill second end.	7. Same as no. 5.
8. Mount between centers and turn large O. D.	8. Use roughing and finishing cuts to bring to exact size.
9. Turn root diameter at recess.	9. Calculate root diameter. Refer to Technical Information Sheet.
10. Set compound rest.	10. Set compound rest parallel to axis of piece.
11. Set tool bit.	11. Set on center.
12. Set quick change gear box.	12. Set for proper number of threads.
13. Set machine.	13. Set feed lever on neutral and engage thread dial.
14. Make trial run.	14. Use pencil to mark thread path. Check number of threads.
15. Repeat cuts.	15. Feed with crossfeed .002 to .003 in. Finish with lighter cuts.
16. Check for size.	16. Check with appropriate measuring tools or with matching internal threads.

QUESTIONS:

1. What is the formula for the depth of a square thread?
2. At what angle do you set the compound rest?
3. Do you use a roughing tool? Why?
4. What is the width of the thread tool?

SELF-EVALUATION:

1. Did you perform your job safely and carefully?
2. Did you have any questions before starting your job? Did you ask about them?

3. Did you make any mistakes? Did you correct them?

4. What grade would you give yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Acme Threads--Lathe

JOB TITLE: Vise

PART TITLE: Vise Screw with Acme Threads

INTRODUCTION:

Acme threads are used to produce traversing movements on machine tools, steam valves, vises, and other similar applications. Although Acme threads are not quite as strong as the square threads, they are easier to machine.

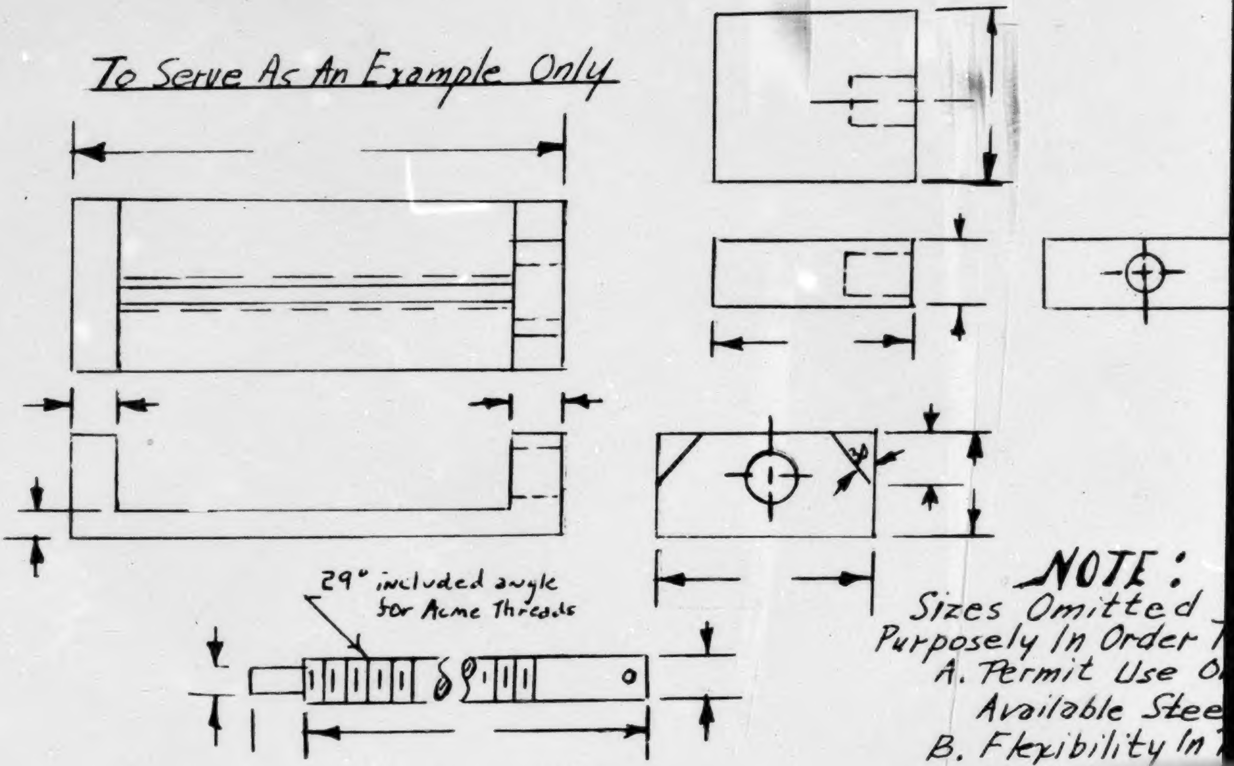
OBJECTIVE:

To provide the student an opportunity to apply the skills of cutting an Acme thread for a vise screw.

JOB SPECIFICATIONS:

Acme threads have a 29° included thread angle as shown in the drawing. There are two basic thread standards for 29° Acme threads: (a) the old standards and (b) the new standards introduced in 1952. These two are not interchangeable. Check drawing and procedure carefully.

DRAWING:


To Serve As An Example Only

NOTE:
Sizes Omitted
Purposely In Order
A. Permit Use O
Available Stee
B. Flexibility In
Of design & Siz

VISE DRAWING

TOOLS:

Acme standard 29° thread tool gage
Micrometer
Gear tooth vernier
Acme ground tool bit
Lathe
Dog
Drive plate
Drill chuck
Center drill
Rule
Lathe centers
Cutting Oil

MATERIALS:

High speed steel or alloy steel

PROCEDURE:

(Operations)	(Related Information)
1. Procure material.	1. Write Bill of Materials.
2. Measure rough stock.	2. Allow for cutting out center on one end.
3. Saw to desired length.	3. Use power band saw.
4. Secure in three jaw chuck.	4. Extend 1/2 in. beyond chuck.
5. Face first end.	5. Face off only enough to clean up.
6. Center drill first end.	6. Center drill carefully before removing from chuck.
7. Face second end.	7. Same as no. 5.
8. Center drill second end.	8. Same as no. 6.
9. Mount between centers and turn O.D.	9. Use roughing cuts and fine finish cut.
10. Turn root diameter at end of thread.	10. Calculate root diameter. See Technical Information Sheet. Then use cut-off tool for making recess.
11. Set compound rest.	11. Be sure to set to 14 1/2 degrees to the right.
12. Set up threading tool bit in holder.	12. Mount tool bit such that only an adequate amount of the threading tool projects from the tool holder.
13. Set up tool holder and tool bit.	13. Set on longitudinal center and square to work. Use Acme thread gage against tailstock sleeve.
14. Set up machine for proper number of threads--quick change gear box.	14. Check number of threads per inch on drawing.
15. Set up machine and engage thread chasing dial.	15. Refer to specific machine for odd and even thread settings.
16. Take trial run.	16. Use pencil for trial cut. Check to be sure the gear setting is correct for the number of threads.
17. Repeat cuts.	17. Use compound feed. Take .002 to .003 in. cut.
18. Use finish type cuts for finishing.	18. Finish cuts should be .001 in. Clean up with no feed.
19. Check for size.	19. Use outside calipers and telescope gage or other appropriate instruments for checking minor diameter accurately.

QUESTIONS:

1. What is the measure of the included angle for Acme threads?
2. At what degree setting should the compound rest be set?
3. Why is the Acme thread used? Give example for its use.
4. What type of gage is used?
5. How can the Acme thread be checked?

SELF-EVALUATION:

1. Did you follow the procedure safely and carefully?
2. Did you have any questions before you started? Did you clear up these questions?
3. Did you make any mistakes? Did you correct them?
4. What grade would you give yourself?

MODERN MATHEMATICS
As Applied To
THE MACHINE TRADES

JOB ASSIGNMENT SHEET

OCCUPATIONAL AREA: Machine Trades

COURSE UNIT TITLE: Helical Gears--Milling Machine

JOB TITLE: Small Arbor Press

PART TITLE: Helical Gear

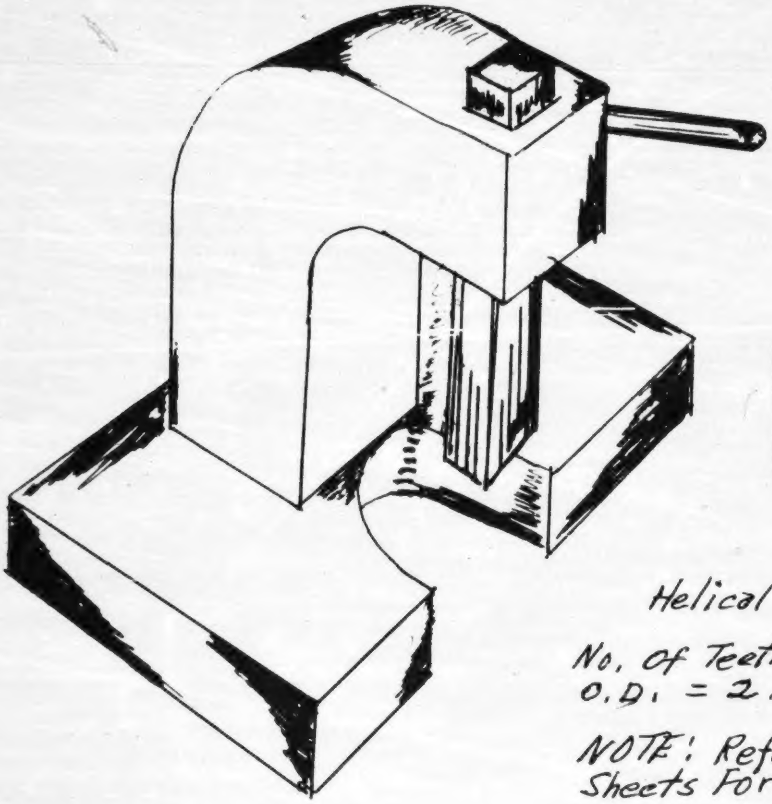
INTRODUCTION:
In milling a helical gear it is necessary to use a universal mill or a universal head on a mill to set up the helix angle. Also, it is necessary to gear up an index head for rotation as the table is fed toward the cutter.

OBJECTIVE:
To provide the student an opportunity to apply the skill of cutting a helical gear on a milling machine.

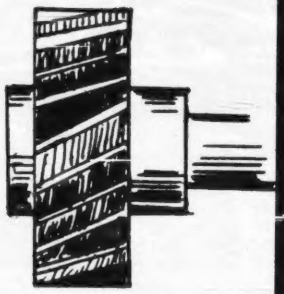
JOB SPECIFICATIONS:

Please follow the procedure carefully. The blank for the gear is turned on a lathe. The dividing head must be geared to the milling machine table travel. Use proper gear trains for the desired helix angle. Set the table for the proper helix angle as indicated.

DRAWING:



ARBOR PRESS



Helical Gear Specificati

No. Of Teeth = 10
O.D. = 2.000 Inches

NOTE: Refer To Technical Sheets For All Other Data

TOOLS:

- Lathe tool bits
- Micrometer
- Mill gear cutter
- Index head for universal milling machine
- Gear tooth vernier
- Rule
- Mandrel
- Dog

MATERIALS:

Steel--S.A.E. 1020 or !!40

PROCEDURE:

(Operations)	
1. Procure material.	1. Write Bill of Material from blueprint or drawing.
2. Measure rough stock.	2. Allow adequate material for facing.
3. Saw.	3. Use power saw.
4. Secure blank in 3 jaw lathe chuck.	4. Allow to protrude from chuck approximately 1/2 inch.
5. Face first end.	5. Remove only enough material to clean up the end.
6. Center drill.	6. Use correct size center drill. Note following drilling operation.
7. Drill.	7. Leave approximately .015 in. for reaming.
8. Ream.	8. Use slow R.P.M. and cutting oil.
9. Press on mandrel.	9. Use arbor press. Press on tightly.
10. Secure mandrel between lathe centers.	10. Use dog on large end of tapered mandrel. Cutting action should tend to tighten gear blank on mandrel.
11. Face sides.	11. Use pointed type side facing tool. Be careful not to cut into mandrel.
12. Turn O.D.	12. Check closely.
13. Secure between dividing head (index head) centers.	13. Indicated alignment using "last word" indicator if available.
14. Set index head for proper divisions.	14. Calculate. Refer to Technical Information Sheet on Indexing.
15. Mount proper gear cutter on mill arbor.	15. Check a textbook for proper gear cutter selection.
16. Set proper gears on mill train.	16. Check chart.
17. Connect table and index head.	17. Check carefully for proper gearing before turning on power feed.
18. Set table for proper helix angle.	18. Calculate helix angle. Refer to Technical Information Sheet.
19. Set gear cutter on center of gear blank.	19. Touch and move crossfeed. Then line in center of spot.
20. Dry run entire set up.	20. Spot each division. Should come to original start.

[redacted]

21. Set depth of first cut.	21. Allow .015 in. for finishing cut.
22. Repeat cuts for first cycle.	22. Always push sectors. Don't pull.
23. Set finish cut.	23. Tighten up vertical and crossfeed locks.
24. Repeat entire routine.	24. Use extreme care.
25. Check.	25. Inspect.

QUESTIONS:

1. How did you determine the number of teeth?
2. Was the diameter given?
3. What is the formula for diametral pitch?
4. What was the number of the cutter for the specific diametral pitch?
5. What is the depth of the tooth? Is it indicated on the cutter?
6. What data is given on a gear cutter?
7. What data is needed to calculate the helix angle?
8. What gears are used for the table train?
9. How are they chosen (calculated)?
10. How do you calculate the number of divisions on the index head?
11. How do you calculate the number of turns?
12. How do you calculate what circle to use on the index head?
13. How do you calculate how many spaces or holes are needed on the circle of the index head?

SELF-EVALUATION:

1. Did you analyze the entire procedure for cutting a helical gear?
2. Did you ask any questions?
3. Did you make any mistakes? Why?
4. Did you correct your mistakes?
5. What grade would you reward yourself?

The following is a partial listing of books that might be utilized for additional study in the machine trades and in modern mathematics.

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